Kirchhoff’s Laws

Analog Electronics, 2026-01-23

In the previous handout on 2026-01-20, you saw Kirchhoff’s First Law (the Current Law). In this handout, we discover Kirchhoff’s Second Law (the Voltage Law).

A Little More About Voltage

Voltage is a lot like height, or more precisely like the combination gh where g is the acceleration of gravity and h is height.

Honestly, it is not terribly helpful to think of voltage as being like height for two reasons:

(1a) Height is uni-directional (always up) wherever we are standing, and (1b) it always changes at the same rate (and the rate is 9.8 Joules per kg per meter, usually written as just $9.8\,\mathrm{m}/\mathrm{s}^2$). Whereas the direction of increasing voltage can be in any direction, and it can have any rate of change.

(2) Mass is always positive, which means that things always want to fall from higher heights to lower heights. There are no anti-gravity devices and masses never fall up. Whereas there are both positive and negative charges, and negative charges feel a force in the direction of *higher* voltage.

The analogy between height and voltage can be made more precise, and it certainly isn’t wrong, but making it more precise is not our goal. We are just trying to have some intuition about voltage. Here are two ways that voltage is like height:

(3) If you carry something from one height h_1 to another height h_2, the amount of energy required only depends on the mass and the height difference, and it is $mg(h_2 - h_1)$. Similarly, if you move some charge from one voltage, V_1 to another voltage V_2, the energy required only depends on the charge and the voltage difference, and it is $q(V_2 - V_1)$. Here the intuition is perfect, except (as already noted in (2)), charges can be negative!

(4) If you carry something from an initial height h_i to a final height h_f, you can’t avoid, no matter how hard you try by finding alternate routes, to change the amount of height. There is a similar statement for voltage, but let me rewrite what I just said about voltage as a law!

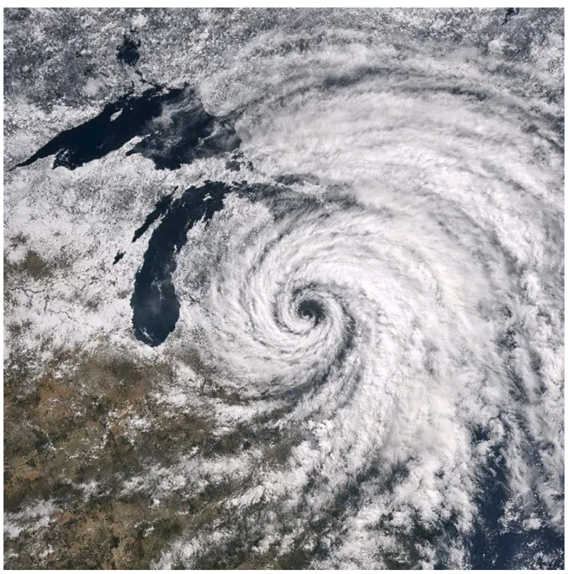
The Anti-Escher Law for Heights

The fourth property of height stated above can be restated as follows:

If you start at an initial place with height h_i and finish at a final place with height h_f, and along the way you visit n places with heights $h_1, h_2, h_3, \ldots, h_n$, then all the height changes cannot avoid changing the net amount of height, $h_f - h_i$. In equations:

$$h_1 - h_i + h_2 - h_1 + h_3 - h_2 + \ldots + h_n - h_{n-1} + h_f - h_n = h_f - h_i$$

This seems inescapable! It is just a property of differences. But I assure you that for some forces, like the cyclonic force of the wind in a large storm system, there are paths you can take that always go with the wind, and other paths you can take that always go against it.



Fighting gravity is not like fighting a cyclone. If you ascend some height, at some point you can descend that same amount. Let's capture that in equations.

Suppose you start and end at the same place so that $h_i = h_f$, and let's call the starting and ending height h_0. Then the statement about heights becomes (check the substitution and cancellation I am claiming!):

$$h_1 - h_0 + h_2 - h_1 + h_3 - h_2 + \ldots + h_n - h_{n-1} + h_0 - h_n = 0$$

Let's rearrange slightly and add some parenthesis just for clarity:

$$(h_0 - h_n) + (h_1 - h_0) + (h_2 - h_1) + (h_3 - h_2) + \ldots + (h_n - h_{n-1}) = 0$$

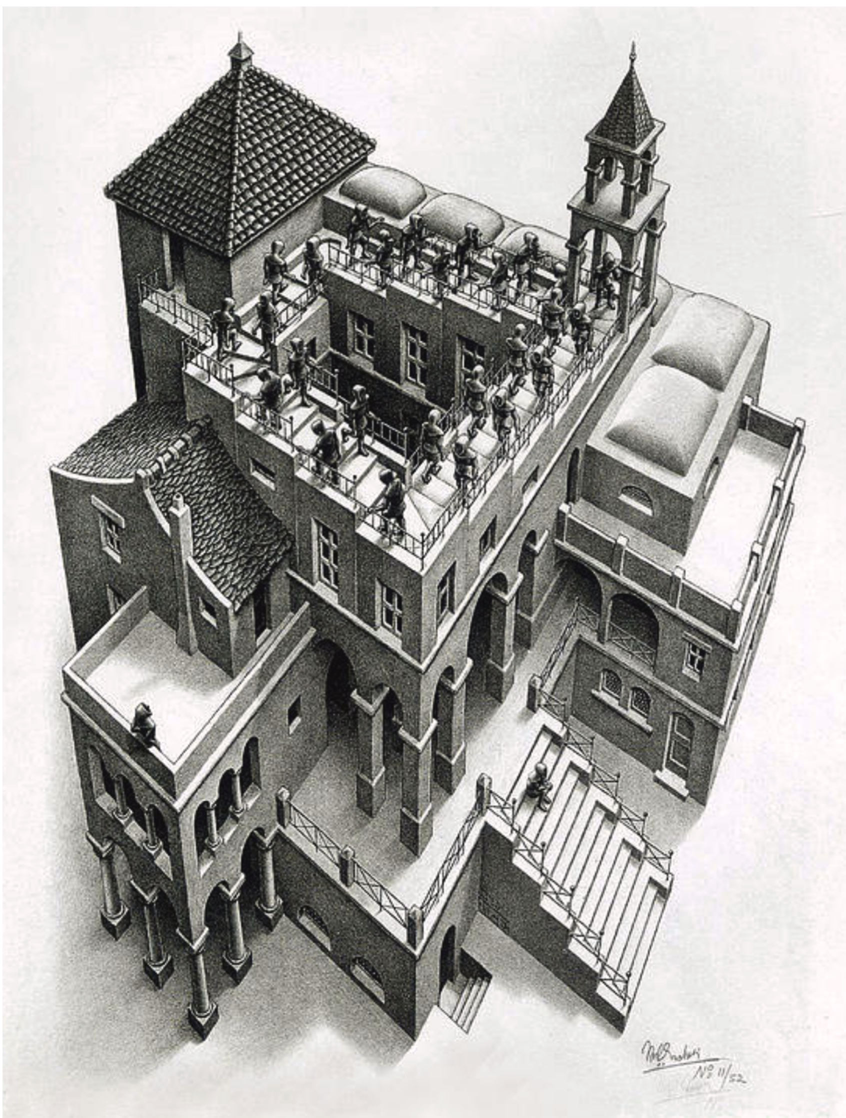
Let's define variables for each of these height differences, and use a fancy font to make the equations more impressive:

$\mathcal{H}_0 \equiv h_0 - h_n$
$\mathcal{H}_1 \equiv h_1 - h_0$
$\mathcal{H}_2 \equiv h_2 - h_1$
...
$\mathcal{H}_n \equiv h_n - h_{n-1}$

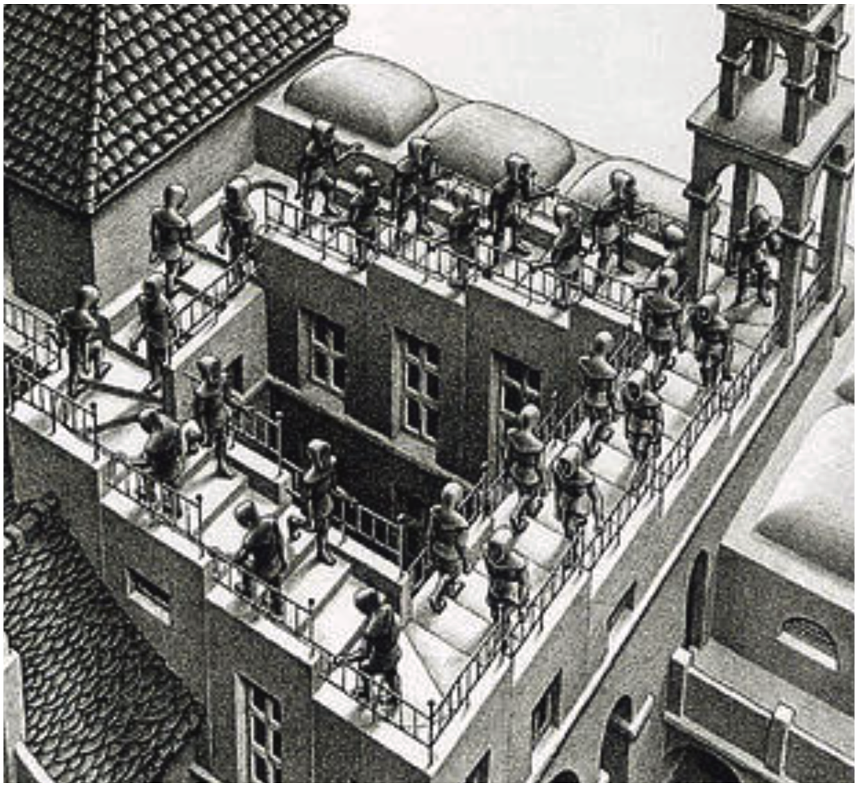
Then our statement about heights becomes:

$\mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2 + \ldots + \mathcal{H}_n = 0$

I am going to call this the Anti-Escher Law for heights, because M.C. Escher is famous for drawings of many types, including this:



Let’s zoom in on the stooped people always walking clockwise up the stars, and the more rested people always walking counterclockwise down the stairs:



According to our equation about heights, this can never happen. If one of the people walking counter-clockwise climbs and climbs and gets back to the same point, they must have had a place where they got to descend the same amount as they climbed.

Kirchhoff’s Second Law

If we understand $\mathcal{V}_0, \mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_n$ in exactly the same way as we understood $\mathcal{H}_0, \mathcal{H}_1, \mathcal{H}_2, \ldots, \mathcal{H}_n$, that is to say, if we understand each of the voltages as voltage differences between two places in a loop, indexed from 0, ..., n, then the equation

$$\mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2 + \ldots + \mathcal{H}_n = 0$$

is perfectly analogous to:

$$\mathcal{V}_0 + \mathcal{V}_1 + \mathcal{V}_2 + \ldots + \mathcal{V}_n = 0$$

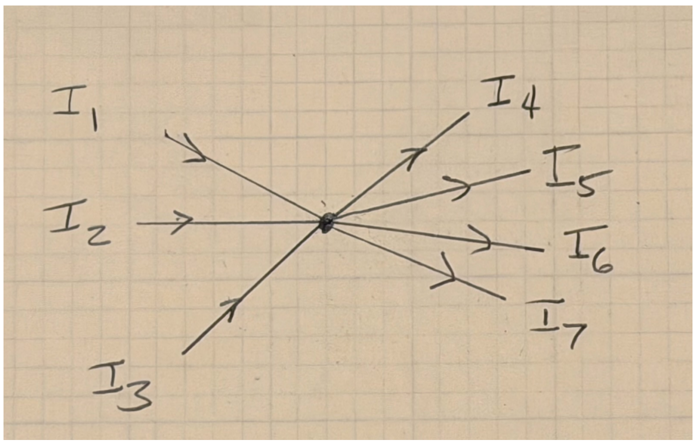
This is Kirchhoff’s Second Law (the Voltage Law).

Conclusion

You now have both of Kirchhoff's Laws! Let's us summarize them, in the way that they are usually stated.

Kirchhoff's First Law

All the current going into a junction must equal all the current going out of a junction. In this diagram,



$I_1 + I_2 + I_3 = I_4 + I_5 + I_6 + I_7$. In words, the sum of I_1 to I_3 must equal the sum of I_4 to I_7.

Actually, Kirchhoff's First Law is usually stated by saying *all the currents going into a junction must add up to zero,* and interpreting—of course!?—currents going out with a minus sign—like you would with money leaving your bank account. So you usually see,

$I_1 + I_2 + \ldots + I_n = 0$

with all of the currents oriented inward and some of them negative.

Alternate names for Kirchhoff's First Law are Kirchhoff's Current Law and Kirchhoff's Junction Law. Take your pick what you want to call it.

Kirchhoff's Second Law

All the voltages as you go around *any loop in a circuit must add up to zero.*

$\mathcal{V}_0 + \mathcal{V}_1 + \mathcal{V}_2 + \ldots + \mathcal{V}_n = 0$

This is Kirchhoff's Second Law, or Kirchhoff's Voltage Law, or Kirchhoff's Loop Law. Take your pick on what you want to call that too.

I most often say "Junction" and "Loop" rather than "First" and "Second," or "Current and "Voltage."