Aristarchus of Samos

Using the Duration of a Lunar Eclipse to Measure the Size of the Moon

Reproduce Aristarchus's calculation (with a modern improvement)

Aristarchus of Samos, 310-230 BC, imagined a Sun-centered universe. He did calculations to estimate the size and distance of the Sun. Unfortunately, the Sun is too far away to measure its parallax angles. See the discussion on p.

Here is what I call the pie-crust formula (to make it memorable):

$$s = 2\pi r \frac{\theta}{360^\circ} = \frac{r\theta}{57.3^\circ}$$

In this formula, s is an arc length, r is a radius, and θ is an angle.

Things that go into Aristarchus's calculation

The duration of the lunar month: 30 days or 720 hours. Aristarchus liked round numbers.

The duration of a lunar eclipse: 3 hours. Another round number.

From the above two facts, the Earth's shadow fills 3/720 of the Moon's orbit. How many degrees is this?

The Size of the Umbra (A modern improvement on Aristarchus's calculation)

Aristarchus knew the diameter of the Earth. It is about 12.7 x 10^6 m. He just used that for the size of the umbra, because he didn't know how much the Earth's umbra tapered. We can improve on his calculation by using that out at the Moon's orbit, the Earth's umbra has tapered to 3/4 the size of the Earth.

Just using the diameter of the Earth and the fraction that the umbra tapers, what is the size s of the umbra out at the Moon's orbit? (Round answer to one decimal place.)

Make a Quality Diagram

It always helps to make a quality diagram of what you are trying to understand. By quality, I mean circles should look pretty circular, straight lines should look pretty straight, and if possible, try to make the diagram to scale.

So make a quality diagram showing what is happening. Make sure that r, θ, and s are labeled, and label the umbra and the penumbra.

Rearranging the Pie Crust Formula

Whenever we have three unknowns, we can solve for one in terms of the other two.

Since s and θ are known, solve the pie crust formula for the unknown r. Don't plug in the numbers yet.

Get the Distance to the Moon

You have found the angle θ that the shadow appears to fill. You know the size s of the Earth's umbra out at the Moon's orbit.

Plug in these values for your formula for r. You get:

$r =$ _____ x 10^6 meters

Official Answer for the Distance to the Moon

$r = 383 \times 10^6$ meters

A second application of the pie crust formula: The Diameter of the Moon

The Moon appears to fill about half a pinky and a pinky-width at arms length is about 1 degree. So the Moon takes up about 0.5º. To be a bit more accurate, it takes up 31' (arc-minutes) or 31/60 or 0.52º.

As a second application of the pie crust formula, plug in θ=0.52º and the official value for r into the pie crust formula. What is the diameter of the Moon?