Solar Eccentricity

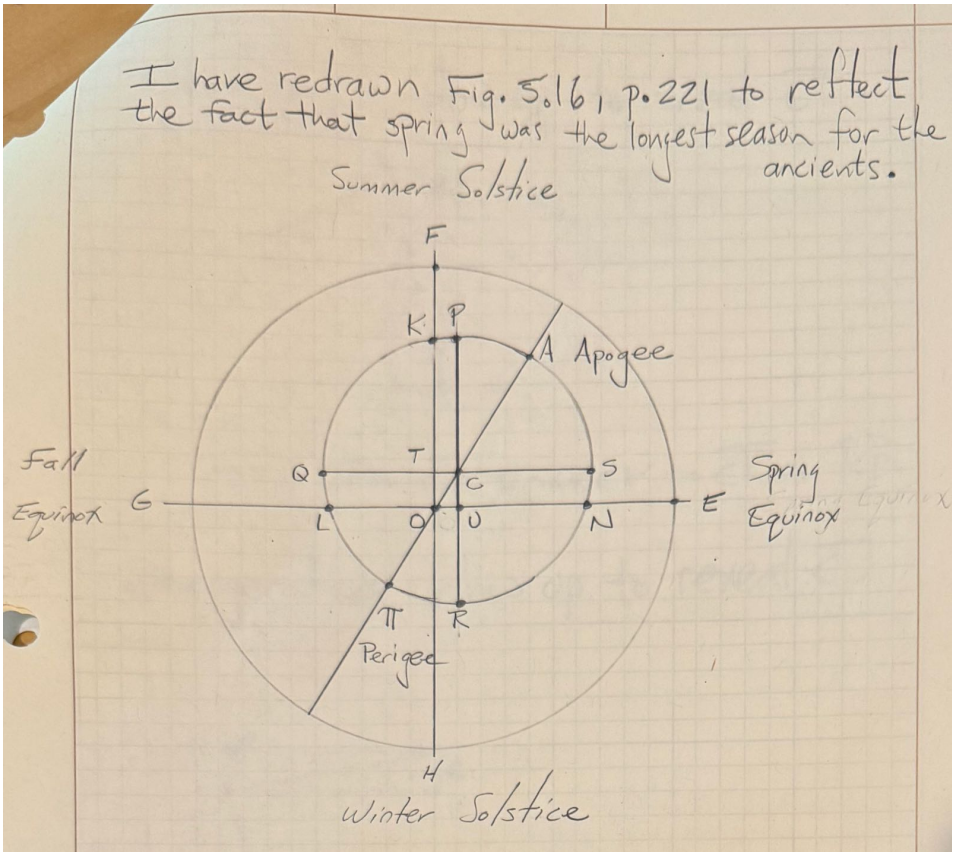
The idea was to do Exercise 5.6, on the Solar Theory, together in class. This document will help us get started efficiently.

Input Data — The Big Arcs — Spring and Summer

On p. 223, we are given that spring in 140 B.C. was about 94.5 days, and summer was 92.5 days.

Converting these to degrees, the Sun's arc of spring KPN is $360° \frac{94.5}{365.25} = 93.142°$ Similarly, the Sun's arc of summer KQL is $360° \frac{92.5}{365.25} = 91.170°$.

Please refer to my re-drawn version of Fig. 5.16, p. 221:



Geometry — The Little Arcs — QL, SN, and PK

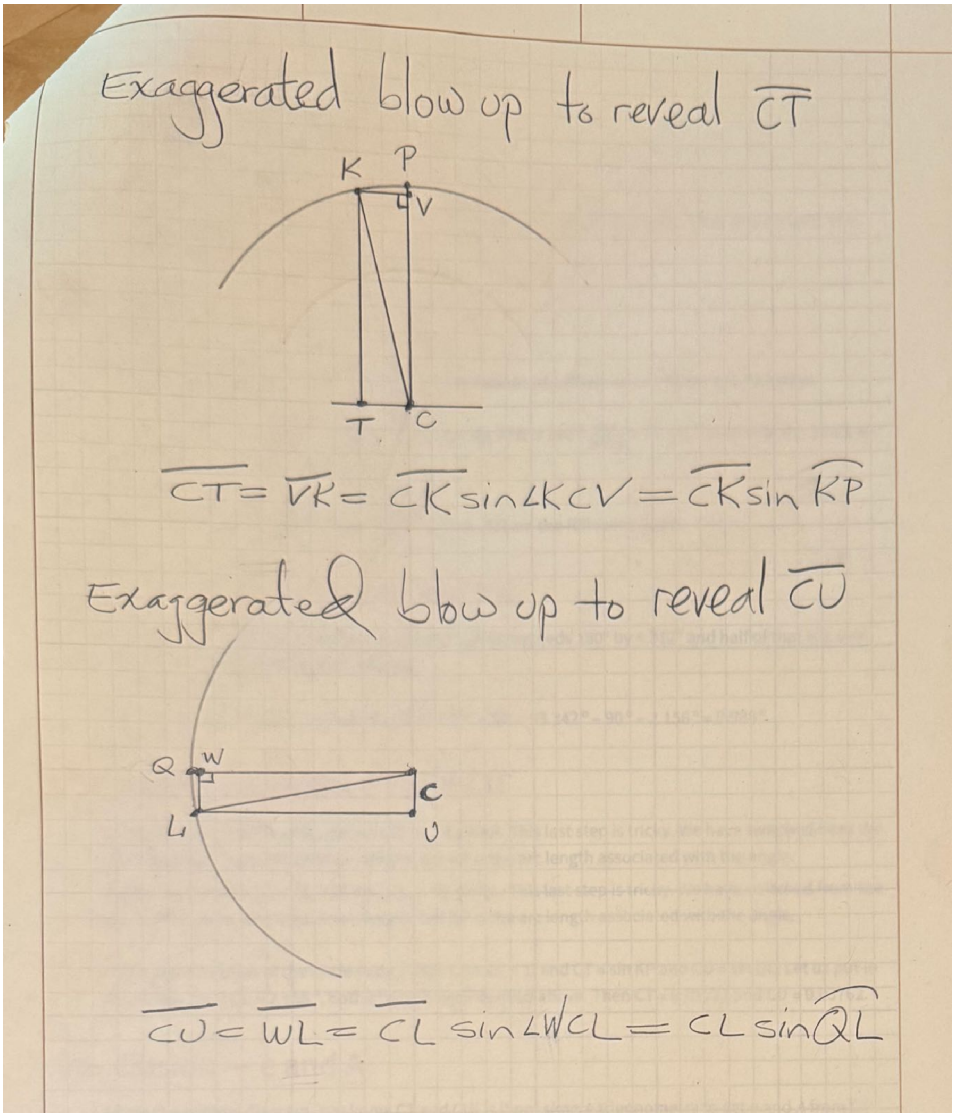
The sum of spring and summer arcs is 184.312° which exceeds 180° by 4.312° and half of that is 2.156°, and that is the lengths of arcs QL and SN.

The next step is to get arc PK, and PK = KPN – 90° – SN = 93.142° – 90° – 2.156° = 0.986°.

We now have the little arcs. What we really want though is the little lengths, CT and CU.

Geometry — Little Lengths — CT and CU

We have the little arcs, but what we really want is the little lengths. You wouldn't be way off in saying that CT = KP and CU = QL, however, Hipparchus and the Greeks in general liked geometry, so they got CT and CU exactly. Here are two exaggerated blow-ups to help see their method:



We notice that CT = VK = CK sin KCV = CK sin KP. This last step is tricky. We have switched from the sin of an angle to the sin of an arc length, but KP is the arc length associated with the angle.

We also notice that CU = QL = CL sin QCL = CL sin QL.

If we take the radius of the circle to be 1 then CK = CL = 1, and CT = sin KP and CU = sin QL. Let us put in KP = 0.986 ° and QL = 2.156 °, both of which were derived above. Then CT = 0.01721 and CU = 0.03762.

If you don't take CK = CL = 1, you have to do a little more work, but after the little bit of additional work, they just divide out of the expressions for *e* and *A*.

Conclusion — *e* and *A*

Study the original diagram. You know CT and CU! Is it not simple trigonometry to get *e* and *A* from CT and CU? What are the formulae?

To help yourself get the formulae, draw just the box CTOU. Draw it big and as a rectangle and able that you know CT and CU.

Where are *e* and *A* in your box? What are the formulae for *e* and *A*?

At the top of p. 224 are Hipparchus's results. After you get your formulae and plug in, do you agree?

Addendum — Fall and Winter

As a final worthy point for discussion, how did we get an answer without knowing the lengths of fall and winter?

It turns out that in this model,

fall + spring = length of year / 2

so we didn't need to know fall because we knew spring and the length of the year, and knowing fall wouldn't tell us anything new.

Similarly, in this model

winter + summer = length of year / 2

so we didn't need to know winter either.

We know from Kepler that Ptolemy's model isn't perfect. However, it is so close to right that the fall + spring and winter + summer totals are in fact quite accurate.