Astronomy — PS 11 — Galaxies

June 8, 2021

In Chapters 15 and 16, Pasachoff & Filippenko covered our galaxy (The Milky Way Galaxy), and other galaxies. Here are some problems that go with those chapters.

Our Solar System Orbiting Our Galaxy

Our Solar System is orbiting the Galaxy, about once every 250,000,000 years. To complete this circle (it actually might be an ellipse) with radius of about 26,000 light-years in that time, it is going about 230 km/s.

1. The question was raised in class whether the gravitational forces holding the Solar System together are large compared to the tidal forces on the Solar System. Here is a basic attack on this question, focused on Jupiter. The approach I am going to have you use just computes accelerations. You don't need to know the mass of Jupiter or the mass of the Sun or the mass of the Galaxy. Of course once you have the accelerations, you could get the force on Jupiter from the acceleration using $F = ma$.

(a) Compute the acceleration of Jupiter that represents its circular motion around the Sun, using the formula

$$a = \frac{v^2}{r} = \frac{(2\,\pi\,r\,/\,P)^2}{r} = \frac{4\,\pi^2\,r}{P^2}$$

Use $r = 42$ light–minutes, and $P = 12$ years.

There is no need to convert units. Your answer for acceleration will be in light-minutes/years^2, which is a perfectly valid unit for acceleration.

(b) Using the same formula as in (a), compute the acceleration of our whole solar system, including Jupiter, that represents the Solar System's circular motion around the center of the Milky Way Galaxy.

For this part, use $r = 26\,000$ light–years, and P =250,000,000 years.

Before you plug in 26,000, convert it to light-minutes, by multiplying by 365.25 * 24 * 60. That way your answer to this part will also be in light–minutes / year^2.

(c) What is the ratio of the acceleration you got in (b) to the acceleration you got in (a)?

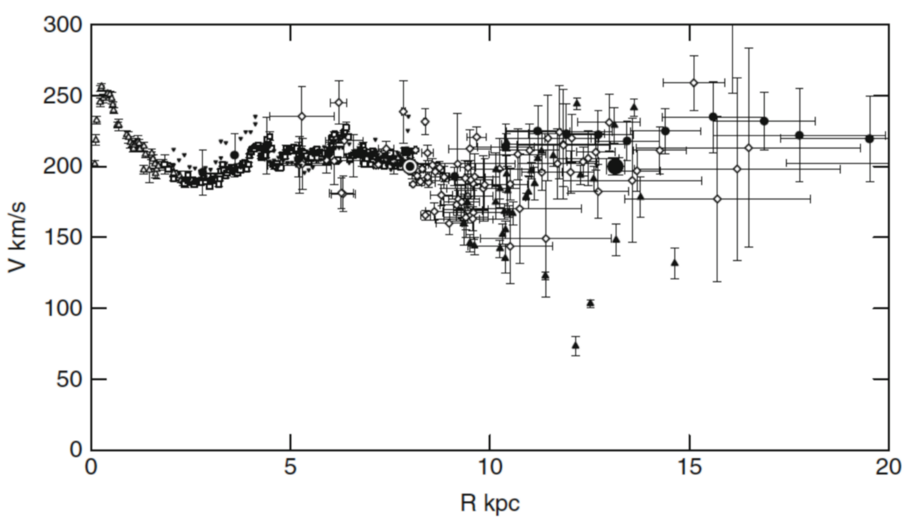
(d) What you computed in (b) is a valid ratio, but it way overestimates how much orbiting the Milky Way is tearing our Solar System apart.

Only tidal forces can tear things apart. The question is, how much different is the acceleration of Jupiter if it is 42 light-minutes further from the center of the Galaxy than the Sun is? It is the difference in accelerations that represents the tidal forces that are tearing the Solar System apart.

In (b) you had a value for r in light-minutes. The formula for the difference in the acceleration at a slightly larger radius is $\Delta a/a = -2\,\Delta r/r$. So compute $\Delta r/r$ with $\Delta r = 42$ light–minutes and multiply that by 2. That is how much weaker (fractionally) the acceleration. (The minus sign just says the tidal force is weaker with increasing distance.)

(e) Whatever you got in (c), multiply it by whatever fractional change you got in (d). Finally, we have an answer for how negligible tidal forces are at a distance of 26,000 light-years compared to the forces that keep Jupiter in orbit around the Sun.

Galactic Rotation Curves and the Winding Problem



2. Above is actual data for the rotation curves of our Galaxy. If you are curious, the whole paper is here: https://arxiv.org/pdf/0811.0859.pdf. There are many papers on this subject. We are going to use this data to understand the winding problem.

(a) Our Solar System is the slightly different looking circle at about 8kpc. According to this data, it looks like we are going 200 km/s. Using 8kpc as the radius, converting that to km and multiplying by 2 π to get a circumference, *C*. Then compute the period *P* that says how long it will take us to orbit the Milky Way:

$$P = \frac{C}{v} = \frac{2 \pi r}{v}.$$

(b) If you did everything in (a) using km and km/sec, then your answer to (a) is in seconds. Convert seconds to years (by dividing by 365.25*24*60*60).

(c) Consider a star orbiting at 4kpc (half our distance from the center of the Galaxy). Repeat what you did in parts (a) and (b) using the data for this new radius. The galactic rotation data above is so close to flat, you might as well use 200 km/s again for this object's speed.

(d) Consider a star orbiting at 16kpc (twice our distance from the center of the Galaxy). Repeat what you did in parts (a) and (b) using the data for this new radius. Out at that distance, the data is very uncertain, but perhaps *v* is closer to 230 km/s.

(e) If our Galaxy is 13 billion years old, how many times has each of the three stars (the one at 4 kpc, our Sun at 8 kpc, and the one at 16 kpc) gone around the Galaxy? If you imagined that all three of these stars were part of the same spiral arm, you can see that the inner part of the arm should have wound up many more times than the outer part, but that is not what we see. This is the winding problem. Density wave theory is one answer to the winding problem.

Cepheids in the Spiral Nebula

3. In Section 16.1, Pasachoff and Filippenko discuss Hubble's discovery of Cepheids in the spiral nebula. You also have a copy of his 1925 paper. Let's re-do Hubble's calculation for one of the stars in Table II of the 1925 paper.

TABLE II.

Cepheids in M 31.

Var. No.	Period in Days.	Log. P.	Photographic Magnitude, Max.
5	50·17	1·70	18·4
7	45·04	1·65	18·15
16	41·14	1·61	18·6
9	38	1·58	18·3
1	31·41	1·50	18·2
12	22·03	1·34	19·0
13	22	1·34	19·0
10	21·5	1·33	18·75
2	20·10	1·30	18·5
17	18·77	1·28	18·55
18	18·54	1·27	18·9
14	18	1·26	19·1

(a) Let's use variable star #16. It has a 41-day period, and log of the period is about 1.6. Go to page 342 of the textbook and use figure 11-28(a) to find the maximum brightness of a variable with this period. This gives you an absolute magnitude M_V.

(b) What you did in (a) is applied Leavitt's Law (the period-luminosity relationship) to find out how bright this Cepheid would be if it were at the standard distance of 10 parsecs. Go back to Table II of Hubble's paper, and find the apparent magnitude (Hubble labeled the column “Photographic Magnitude, Max.” of Cepheid #16. Call that m_V. Then compute the difference $m_V - M_V$.

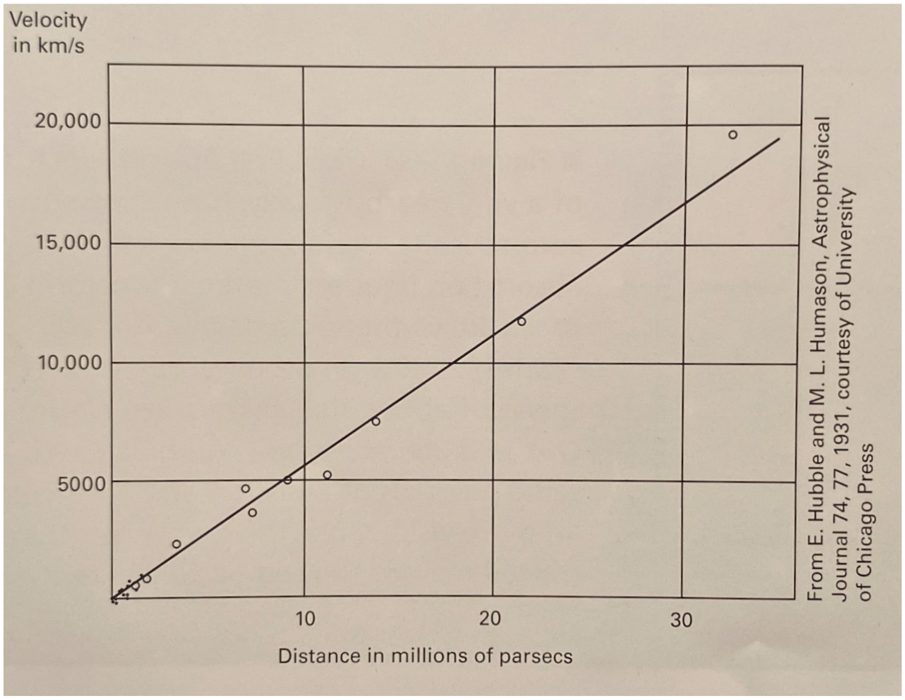
(c) Solving the formula $\frac{d}{10\text{ parsecs}} = 10^{\frac{1}{5}(m_V - M_V)}$ for d, find the distance to variable star #16 in M31. Your answer for the distance will be in parsecs. You have found the distance that makes this star look so terribly faint.

(d) Convert your answer to (c) from parsecs to light-years by multiplying to 3.26.

The official modern value for the distance to M31 is 2.537 million light-years. If you didn't get something that is *sort*of* in the ballpark go back and check your work. We used Hubble's data that is nearly 100 years old to get our answer! Look at Figure 16-6(b) and compare the fuzzy data he had to work with compared to the images we have now in Figure 16-6(e).

Hubble Plots, the Hubble Constant, and Hubble’s Law

4. In Section 16.7, Pasachoff and Filippenko discuss Hubble’s 1929 paper, which you also have.



(a) Above, I have reproduced the 1931 data in Figure 16-34(b) of your book. What is the recession velocity of the galaxy that is at 22 million parsecs?

(b) Estimate the recession velocity in km/s of a galaxy if it were at 30 Megaparsecs.

(c) The Hubble constant is the slope of this graph. Hubble's units for the slope of the line were km/s/Megaparsec. Divide the recession velocity you got in (b) by 30 Megaparsecs to get the slope of the line.

(d) You can also go the other way. Imagine the spectroscopy of a galaxy shows a redshift, and you convert that to km/sec using the Doppler shift formula and get 15,000 km/sec. If you know that recession velocity, predict the distance to the galaxy in Megaparsecs.

Turning Doppler Shift into Distance

5. Do problem 41 on p. 520 from the textbook. To get an answer, you will first have to convert the redshift to a recession velocity using the Doppler shift formula. Then you will use the Hubble constant to turn the recession velocity into a distance.