AU, Parsec, Light-Year, Luminosity, Intensity, Absolute Magnitude, and the HR Diagram

Astronomical Unit

The “astronomical unit” or AU is the “average distance” from the earth to the Sun. Since “average” is an imprecise term when you are talking about an ellipse, it is now *defined* to be 1.495978707 x 10^{11} meters. In round numbers, 150,000,000 km or 93 million miles.

Light-Year

The light-year is how far light goes in a year. Nothing goes faster than light, and light of all kinds — gamma rays, X-rays, ultraviolet, visible light (ROY G BIV), infrared, microwaves and radio waves — always goes the speed of light, so the speed of light is extremely fundamental. The speed of light is so fundamental that since 1960, the meter is defined by the speed of light. Up until the 1950's, the meter was defined by official copies of meter bars that were kept in various locations such as the US National Institute of Standards and Technology (NIST). Now that the speed of light defines the meter, and the definition of the meter is how far light goes in:

$\frac{1}{299{,}792{,}458}$ of a second

it follows that the speed of light is exactly 299,792,458 meters per second. In round numbers, 3 x 10^8 m/s or 300,000 km/sec.

The year needs defining too. Is it a leap year or a regular year (366 days or 365 days)? People decided to compromise and since a leap year usually comes every four years, they settled on 365.25 days.

Value for the Light-Year

Now that we know the speed of light and what is meant by a year, we can compute the light year (distance = speed * time):

$$299{,}792{,}458\ \frac{\text{meters}}{\text{second}} * 365.25 * 24 * 60 * 60 \text{ seconds} = 9.4607304725808 * 10^{15} \text{ meters}$$

and that is *exact* thanks to the fact that both the meter and the year are now *defined*, not measured, values.

Light-Year Compared to AU

It takes light only 500 seconds or 8 1/3 minutes to get from the Sun to the Earth, so obviously the light-year is a lot more than the AU. How much more? It's 9.4607304725808 * 10^{15} / 1.495978707 x 10^{11} = 63241.077084 times as much.

Parallax and the Parsec

Astronomers use the parsec more often than the light-year. Perhaps that will change, but for now, we also need to know the parsec. The parsec is defined as follows: *One parsec is how far away something has to be so that in angle, it appears to move exactly one arc-second as you move one astronomical unit.*

One arc second is tiny. It is 1/60th of 1/60th of 1/360th of the way around a circle (because there are 60 arc-seconds in an arc-minute, and 60 arc-minutes in one degree and 360 degrees in a full circle). That's $\frac{1}{1{,}296{,}000}$ of a circle. The apparent movement (the change in angle) is the "parallax angle." For the parallax angle to be less than a millionth of a circle of radius 1 A.U., a parsec must be an extremely large distance. A full circle has radius $2\,\pi r$ so $\frac{1}{1{,}296{,}000}$ of a circle has length:

$$1\text{ AU} = \frac{1}{1{,}296{,}000}\,2\,\pi r$$

Solving for r, we have the parsec:

$$1\text{ parsec} = 1\text{ AU}\,\frac{1{,}296{,}000}{2\pi} = 1\text{ AU} * 206\,265 = 3.0856776 * 10^{16}\,m$$

If you thought it was slightly cheesy to use the pie-crust formula, you could just as well have used:

$$\frac{1\text{ AU}}{1\text{ parsec}} = \tan\,\frac{1}{3600\,°}$$

but the angle is so tiny it doesn't make any difference even out at the 7th decimal place, and in any case, the International Astronomical Union has *defined* the parsec to be 1 AU $\frac{1{,}296{,}000}{2\pi}$ rather than using $\tan\,\frac{1}{3600\,°}$.

Parsec Compared to Light-Year

How does the parsec compare to the light-year?

$$\frac{3.0856776 * 10^{16}}{9.4607304725808 * 10^{15}} \approx 3.26$$

It is good to remember that the parsec is more than three light years. The distance to the nearest star is also handy to know. It is Proxima Centauri, and it is 4.22 light-years away, or 1.29 parsecs.

Luminosity and Intensity

Luminosity, usually denoted L, is a star's power output. Power is energy / time. In MKS units that is Joules/second which is more commonly know as Watts.
Intensity, usually denoted as I by physicists, but which is b in our textbook (see p. 326), is power per unit area. In MKS units that is Watts/m^2. If you are a distance R from a star, its intensity is

$$b = \frac{L}{4\pi R^2}$$

Relationship between Intensity and Magnitude (or casually, "brightness")

There must be a relationship between intensity and magnitude, since they are both measures of what physicists call intensity and what we casually call "brightness." It's actually a little more complicated than I am about to write, but the details aren't worth getting into:

$$b = I_0\, 100^{-m/5}$$

I_0 is a standard intensity that the International Astronomical Union has agreed is:

$$I_0 = \frac{3.0128\times 10^{28}\ \text{Watts}}{4\pi\,(10\ \text{parsec})^2}$$

You will never need I_0 because we are going to compute ratios of brightnesses, and it disappears out of the ratios.

Absolute Magnitude

Suppose a star is at a distance R. Then we already have one formula for its brightness:

$$b = \frac{L}{4\pi R^2}$$

Imagine moving this star to a standard distance of 10 parsecs (without changing its luminosity). In that case, its intensity would be:

$$b_{\text{at 10 parsecs}} = \frac{L}{4\pi\,(10\ \text{parsec})^2}$$

Suppose its magnitude at R was m and its magnitude at the standard distance of 10 parsecs is M. M is by definition the "absolute magnitude." It is its magnitude at the standard distance.

From the prior section we have

$$b = I_0\, 100^{-m/5}$$

and

$$b_{\text{at 10 parsecs}} = I_0\, 100^{-M/5}$$

Combine the two formulas for b and the two formulas for $b_{\text{at 10 parsecs}}$:

$$\frac{L}{4\,\pi\,R^2} = I_0\, 100^{-m/5}$$

$$\frac{L}{4\,\pi\,(10\text{ parsec})^2} = I_0\, 100^{-M/5}$$

Now take the ratio of these two formulas. Notice that L, $\frac{1}{4\,\pi}$, and I_0 is in both formulas so it drops out of the ratio:

$$\frac{1/R^2}{1/(10\text{ parsec})^2} = \frac{100^{-m/5}}{100^{-M/5}}$$

To summarize, we now have formula we can solve for M, the "absolute magnitude." It is the magnitude of the star if it were at the standard distance of 10 parsecs.

Solving for *M* and the HR Diagram

In the previous section, I derived

$$\frac{1/R^2}{1/(10\text{ parsec})^2} = \frac{100^{-m/5}}{100^{-M/5}}$$

Usually we know R from astrometry, and the apparent magnitude m from looking at the star, and we want to know M.

We can take the square root of the equation and we get:

$$\frac{1/R}{1/(10\text{ parsec})} = \frac{10^{-m/5}}{10^{-M/5}} = 10^{(M-m)/5}$$

We can take log base 10 of that equation and we get:

$\log_{10} \frac{10\ \text{parsec}}{R} = (M - m)/5$ <==== this is as far as I got at about 12:20 in class

Solve for *M* and get:

$M = m + 5\ \log_{10} \frac{10\ \text{parsec}}{R}$ <===== this is the formula you need and that I was going to do using Sirius as the example

Let's try it for the Sun:

$m = -26.74$

$R = 0.000004848$ parsecs

$M = -26.74\ + 5 \log_{10} \frac{10\ \text{parsec}}{0.000004848\ \text{parsecs}} = 4.83$

We have just discovered that by far the brightest object in our sky (the Sun) would barely be visible even at a location like Deep Springs if it were at the standard distance of 10 parsecs. In an urban area or even a suburban area you would have no chance of seeing it due to light pollution. In such areas you are lucky to see magnitude 3 stars.

Now you have what you need to fill out the HR diagram that I wanted to fill out in class.