Relativity and Black Holes — Syllabus

Unofficial/Short Course Title: Black Holes

Fall 2024, Deep Springs College, ***Prof. Brian Hill***

Overview

More than anyone else since Einstein, John Archibald Wheeler has influenced our understanding of spacetime. He was the person that pressed relativists to take seriously the Schwarzschild solution into the interior of the event horizon. He is a co-author (with Misner, Thorne, and Taylor, among others) on the greatest textbooks in the field and his students have gone on to greatly advance the subject, including black hole thermodynamics and quantum mechanics in curved spacetime. We will work through Taylor and Wheeler's descriptions of two things: (1) special relativity, which is how space and time are related in the absence of gravity, (2) the behavior of spacetime around a black hole. The latter is also entirely relevant for tight orbits around ordinary stars, and in fact the first evidence for general relativity listed by Einstein was the explanation of the deviation of Mercury's orbit from the ellipse expected from Newtonian gravity.

The intertwining of space and time is highly counterintuitive. The intertwined fabric is called "spacetime." Our story will begin with Galileo who formulated the version of relativity that Newton built upon, and which lasted for almost three centuries until Lorentz, Poincaré, and Einstein formulated special relativity. Our tale will continue with Einstein's General Relativity where we will explore Schwarzschild's exact solution of Einstein's equations surrounding a black hole. Time permitting, we will conclude with a special topic where advancement has been recently made, such as gravitational wave detection or the evidence for dark energy.

Prerequisite / Joining the Class

I will use mathematics that is commonly taught in AP Calculus. If this is material you have gotten rusty on, I will remind you of the key results and their meaning as we use them, but at a pace that would be unmanageably fast if you have never seen the material before.

Texts

We are going to use two texts. Very generously, Taylor, Wheeler, and Bertschinger have made PDFs of their textbooks completely free:

- ***Spacetime Physics, 2nd Edition*, https://www.eftaylor.com/spacetimephysics/** — Softcover copies are still readily available used
- ***Exploring Black Holes, 2nd Edition*, https://www.eftaylor.com/exploringblackholes/** — However, the original is simultaneously more concise and more intuitive, so we will work from used copies of the 1st edition, which are a bit hard to find now

Grading

- 45% assignments
- 15% (45% total) for each of three exams, dates to be determined, but coming at about the 5th, 9th, and 14th week of classes
- 10% preparation for class and leadership of course

Problem Sets / Handouts / Being Neat and Organized

There will be problem sets due almost every class, limited only by how quickly I can assign, write solutions, and grade. The more problems you do the better. In addition to the problem sets and their solutions, there will be exams and exam solutions. To be organized, locate a three-ring binder and a three-ring hole punch, and file everything chronologically. Actually, reverse-chronological is the most convenient, because you then naturally open your binder to what you are currently working on. Problem sets should be *neat* and on standard 8 1/2 x 11 paper. Multi-page problem sets — and most will be multi-page — should be stapled. The nicest technical work is facilitated by engineering pads, such as these **Roaring Spring Engineering Pads at Amazon** (which are pretty expensive unless you buy by the case), and done with a mechanical pencil, a ruler, and an eraser at hand.

Absences (and late work)

The College's policies on absences (and late work) are applicable. Refer to the Deep Springs Handbook.

Daily Schedule Term 2

Week 1 — Galilean Relativity — Invariance of the Interval

- Thursday, Aug. 29 — We did a **calculus self-assessment** — We went over the **syllabus** — You read pp. 1-15 of Chapter 1 of *Spacetime Physics, 2nd Edition* — Go to the excercises at the end of the Chapter and choose an exercise to work out and present (we chose to present 1-3 and 1-4) — Also read what Galileo's character **Salvatius has to say** about Galilean relativity — I have written up and will talk you through **my own introduction** to speed, velocity, and Galilean Relativity which is less elegant than Galileo's but uses the reliable crutch of Cartesian coordinate systems

Week 2 — Free-Float Frames — Time Dilation Illustrated in Muon and Pion Decay

- Monday, Sep. 2 — Read and be prepared to discuss *Spacetime Physics* to p. 30 — Problem Set 1 to be turned in at the beginning of Monday's class is **here** — I played **Chris Hadfield's cover of David Bowie's "A Space Oddity"** as an important illustration of what it is like to be in a free-float frame — I proved **Galilean Addition of Velocities** which establishes the near impossibility that the speed of light could possibly be constant — And yet the speed of light is constant in any frame(!), and this follows easily from the invariance of the interval
- Thursday, Sep. 5 — Read and be prepared to discuss *Spacetime Physics* to the end of Chapter 2 (to p. 44) — I will use the first two terms of the Maclaurin series to present an important approximation method that you will be using a lot — Then two groups will present their detailed analyses of the falling railway coach — **Here are the variables** for analyzing the horizontal problem (Rania and Rebecca), and **here are similar variables** for analyzing the vertical problem (Walker, Will, and Eden) — Both groups should be seeking approximate expressions for $d(t)$ where initially $d(t)=d_I$, and we are really only interested in what happens in a short time Δt — **Problem Set 2** for Thursday

Week 3 — Time Dilation — The Relativity of Simultaneity — Length Contraction

- Monday, Sep. 9 — Read and be prepared to discuss *Spacetime Physics* to p. 65 — Problem Set 3 to be turned in at the beginning of Monday's class is **here** — **Maclaurin and Taylor series graphs** — Theory behind the coefficients in the Maclaurin and Taylor Series — Overview of the three biggest consequences of special relativity: (1) Time Dilation, (2) Length Contraction, (3) The Relativity of Simultaneity — Derivation of the first effect: Time Dilation (Kel and Jeremy) — The γ factor notation — Invariance of the transverse dimension via the Rockets-with-Paintbrushes thought experiment — The Relativity of Simultaneity via Einstein's Lightning-Strikes-Train thought experiment
- Thursday, Sep. 12 — Read and be prepared to discuss *Spacetime Physics* to p. 77 (the end of Chapter 3) — Problem Set 4 to be turned in at the beginning of Thursday's class is **here** — Derivation of Length Contraction by Sasha and Eli — **Length contraction board photo** — **Rania's Length Contraction handout** — The Pole-in-the-Barn Paradox — A Relativity of Simultaneity derivation using **a variant of Einstein's Lightning-Strikes-Train thought experiment**

Week 4 — Presentations of Interesting Problems — Start The Lorentz Transformation Formulas

- Monday, Sept. 16 — To give you time to consolidate the first three chapters, there is no new reading assignment due for Friday

- Thursday, Sept. 19 — Read Chapter L to p. 102, which is the derivation of the Lorentz Transformation formulas summarized in Eq. L-10a on p. 102

Week 5 — Finish The Lorentz Transformation Formulas — Exam 1

- Monday, Sept. 23 — Finish Chapter L — Problem Set 5, with due date delayed to the beginning of Monday's class, is **here**
- Thursday, Sept. 26 — **Exam 1**

Week 6 — World-Lines of Accelerating Particles — The Light-Cone and Causality

- Monday, Sept. 30 — If you feel solid about Chapters 1, 2, 3, and L, there is not much new in Chapter 4 — If you feel you need to cement in some ideas with the twin paradox as the example then Chapter 4 is great — Everyone, whether feeling solid or not, should pay close attention to pp. 127-130 of Chapter 4 — Then continue studying through Section 5.5, p. 148, of Chapter 5 — Problem Set 6, due at the beginning of Monday's class, is **here** and in your file folders along with tracing graph paper and a **hyperbola template**
- Thursday, Oct. 3 — Finish Studying Chapter 5 — Problem Set 7, due at the beginning of Thursday's class, is **here**

Week 7 — More Presentations of Interesting Problems — Non-Relativistic Momentum and Energy — $E=mc^2$

- Monday, Oct. 7 — Study all of Chapter 6 and also Sections 7.1 and 7.2 to p. 195 — Organize yourselves into three groups of three for presentations on three problems from Chapters, 5, 6, and 7, such as 5-7 (The Runner-on-The-Train Paradox), 6-6 (Relativistic Race-Walking), and 7-3 (The Kinetic Energy of a Freight Train) — Target 10 minutes plus 5 minutes for questions per presentation
- Thursday, Oct. 10 — You can continue studying Chapter 7 which is on relativistic momentum and energy — **Worksheet I**: The Runner-on-the-Train Paradox — **Worksheet II**: Relativistic Race-Walking — **Worksheet III**: $E=mc^2$ and the Energy to Raise a Freight Train a Mile High — **Non-Relativistic Momentum and Energy**

Daily Schedule Term 3

Week 8 — Relativistic Momentum and Energy

- Monday, Oct. 28 — No class (last day of Term 2-3 break)
- Thursday, Oct. 31 — Continue your study of Chapter 7 of *Spacetime Physics* to the end of Section 7.5, p. 206 — Problem Set 8, with due date delayed to the beginning of this Thursday's class, is **here**

Week 9 — Start *Exploring Black Holes* — Spherical Polar Coordinates — Reduced Radius — Schwarzschild Metric

- Monday, Nov. 4 — Study Chapter 1 of *Exploring Black Holes* through Section 1-7, p. 1-13 — Problem Set 9 is **here** and in your file folders and two groups of you have volunteered for presentations as noted in the problem set — Discussion of the Principle of Extremal Aging — Spherical polar coordinates — The “reduced” radius
- Thursday, Nov. 7 — Study Chapter 2 of *Exploring Black Holes* through p. 2-20 — Problem Set 10 is **here** and in your file folders and a group (consisting of Eden and Sasha) will present the full Black Hole Company Construction Problem

Week 10 — Finish Introducing the Schwarzschild Metric

- Monday, Nov. 11 — Problem Set 11 is **here** and in your file folders — Finish reading Chapter 2 — We discussed the relation between E, p, h, λ, and ν for photons, and contrasted it with the relation between E, p, and m for massive particles* — Presentation: Walker and Rebecca, the Zeno's Paradox problem
- Thursday, Nov. 14 — Problem Set 12 is **here** and in your file folders — No new reading assignment

(just continue to consolidate your understanding of Chapter 2) — Presentation: Eli and Eden, the Chesapeake Bay problem

Week 11 — Exam 2 — Start Plunging

- Monday, Nov. 18 — **Exam 2** covering everything from Chapter 4 to Section 7.5 of *Spacetime Physics*, and Chapters 1 and 2 of *Exploring Black Holes*
- Thursday, Nov. 21 — Study Chapter 3 through Section 3-6 (through p. 3-18) — We need two groups of two presenters from among those that didn't present the last two times, so that would be Jeremy, Kel, Rania, and Sasha — The two topics I can imagine being good to present are Sample Problem 2 (p. 3-22) and Sample Problem 3 (p. 3-25) — Problem Set 13 is **here** and in your file folders

Week 12 — Finish Plunging — Start Rain Coordinates

- Monday, Nov. 25 — Finish your study of Chapter 3 (pp. 3-19 to 3-26) and then continue to page B-13 of Chapter B — Problem Set 14 is **here** and in your file folders
- Thursday, Nov. 28 — No class - Shakespeare Festival

Week 13 — Finish Rain Coordinates — Start Orbiting

- Monday, Dec. 2 — Finish Rain Coordinates — Rebecca requested we read the material referenced at the end of Problem 6 on p. 3-30 of *Exploring Black Holes* — Specifically, the material referenced was pp. 422-448 of *Black Holes and Time Warps,* by Kip Thorne — Also read the remainder of Chapter B of *Exploring Black Holes* for Monday — Problem Set 15 for Monday (that does nothing more than flesh out the readings a bit) is **here** and in your file folders
- Thursday, Dec. 5 — Start Orbiting — Study Chapter 4 of *Exploring Black Holes* through Section 4-5 (just through p. 4-10) — For Problem Set 16 do Problems 1 and 2 on. p. 4-28 — In class derived the relativistic version of angular momentum conservation — And we did a lightning derivation of the Newtonian effective potential for motion in a spherically symmetric potential

Week 14 — Finish Orbiting

- Monday, Dec. 9 — Continue your study Chapter 4 of *Exploring Black Holes* through Section 4-7 (through p. 4-20) — For Problem Set 17 do Problems 3 and 4 on. p. 4-29 and 4-30 — We derived Eq. 30 on p. 4-15 and did Problem 6 on p. 4-31 in class, and that concludes our study of Chapter 4
- Thursday, Dec. 12 — Study Sections 5-1 to 5-5 of Chapter 5 (through p. 5-10) — For Problem Set 18 do Problem 5 on p. 5-33 (you can use Eq. 25 from Section 5-6 blindly) — We will do a complete derivation of Eq. 25 in Section 5-6 (the effective potential for light) and work through some of the consequences pictured in Section 5-7

Week 15 — Exam 3 — Cosmology

- Monday, Dec. 16 — **Exam 3** — Covering *Exploring Black Holes Chapters* 3, B, and 4
- Thursday, Dec. 19 — Cosmology — In 1998, by studying distant supernova, it was determined that the universe is accelerating (expanding faster now than in the past) — All the way back in 1917, Einstein had introduced a "cosmological constant" term that has the effect of causing the universe to expand — Quantum field theorists interpret the cosmological constant as the "vacuum energy density" and cosmologists refer to the same thing the "dark energy" — In our final class, we will introduce the metric of the universe (the Friedmann-Robertson-Walker metric), explore its solutions, and close with a brief look at the experimental evidence that the universe contains dark energy — Buckle up for Cosmology

This course would have been nothing without the clarity and brilliance of ***John Archibald Wheeler*** *1911-2008 and the precious textbooks in which he shared his insights.*