Black Holes, Problem Set 3 for Monday, Sept. 9

Study and be prepared to discuss *Spacetime Physics* to p. 65 (you can stop at the "Speeding-Train Thought Experiment"). *So you have a chance to see how much better (or maybe not better!) you work if you have high-quality materials to work with, I put some engineering paper in each of your file folders. It is meant to be written on on one side only. The graph paper grid is meant to just barely show through to your work.*

Problem 1 — Maclaurin Series

In class we computed the zeroth and first terms in the Maclaurin series for $f(x) = \frac{1}{(1-x)^2}$. We got $f(x) \approx 1 + 2x$.

(a) In general (not just for the function we did), the zeroth and first terms in the Maclaurin series are $f(x) = f(0) + f'(0) \cdot x$. What is the next term in the Maclaurin series? What is the nth term?

(b) Use the next term in the series to make a better approximation to $f(x) = \frac{1}{(1-x)^2}$.

Problem 2 — Taylor Series

Sometimes you don't want to approximate a function near $x = 0$. Instead, you want to approximate it at some other value. In that case, you use Taylor series:

$f(x) = f(x_0) + f'(x_0) \cdot (x - x_0) + \ldots$

(a) What is the next term in the Taylor series? What is the nth term in the Taylor series?

(b) Let's apply this to the function $f(x) = \cos(\pi x)$ with $x_0 = 1$. What is $f(1)$? What is $f'(1)$? What is $f''(1)$?

(c) Use what you just got in parts (a) and (b) to get the first three terms in an approximation to $\cos(\pi x)$ near $x = 1$. Note, if you did it right, one of the three terms will be 0.

Problem 3 — Relativity and Swimming

Do Problem 3-1 on p. 78 of *Spacetime Physics.*

Problem 4 — Many Unpowered Rockets

Do Problem 3-5 on pp. 78-79 of *Spacetime Physics.*