Black Holes, Problem Set 11 for Monday, Nov. 11

Reading from *Exploring Black Holes*

Finish Reading Chapter 2. This is a long chapter. We'll be spending a total of 2 1/2 classes on it, I expect.

Presentations

Eli and Eden, see the Chesapeake Bay problem below.

Walker and Rebecca, see the Zeno's Paradox problem below.

For Problem Set 11

Problem 1 — The North Pole Construction Company

The circumference of the Earth is almost exactly 40,000 km. This is not a historical accident. $C = 40\,000$ km was the number the Parisians were aiming for when Napoleon's astronomers defined the meter and planned that it would be exactly 10,000,000 meters from the North Pole to the Equator on a line passing through Paris.

In what follows. It is sometimes more convenient to use the constant R, the radius of Earth, where $R \equiv 40\,000\ \mathrm{km}/2\,\pi$.

The North Pole Construction Company has its headquarters at the North Pole, and they contracted to build concentric circles around the North Pole spaced by 666.67 km until they got "one-third of the way from the North Pole to the Equator" (5 * 666.67 is 3333.3).

If you like to think in terms of lines of latitude, they were contracted to build 5 concentric circles at latitudes 84°, 78°, 72°, 66°, and 60°.

A crafty lawyer at the North Pole Construction Company who had heard about the concept of reduced radius noticed that at latitude 60° the circumference of the wall would be $\cos 60\,° * 2\,\pi * R$.

Since $\cos 60\,° = 1/2$, he noticed that at latitude 60° the circle built at that latitude would have circumference C/2. Therefore — in some legalistic sense — he could argue that latitude 60° is already halfway to the equator not one-third.

A meeting was called with the engineers and they were told to save a tremendous amount of materials and money by building circles of circumference:

$C_1 = \frac{2\pi R}{15}$
$C_2 = 2\,\frac{2\pi R}{15}$
$C_3 = 3\,\frac{2\pi R}{15}$
$C_4 = 4\,\frac{2\pi R}{15}$
$C_5 = 5\,\frac{2\pi R}{15} = \frac{2\pi R}{3} = \frac{C}{3}$

Notice that in the reduced radius sense, these walls are equally spaced, and the fifth circle is at 1/3 the reduced radius of the Equator.

The crafty lawyer believed that he could maintain in court that the fifth circle was — in a legalistic sense involving a bunch of mumbo jumbo about “reduced radius” that the judges and opposition lawyers would be snowed by — “one-third of the way from the North Pole to the Equator.”

A. What is the total length of the walls built this way? E.g., what is $L = C_1 + C_2 + C_3 + C_4 + C_5$? Feel free to leave R in your answer.

B. At what five latitudes were these five circles built? You will need a calculator with inverse trig functions. Answer in conventional latitudes, the way we measure them on Earth, from the Equator — not in radians from the North Pole the way physicists would be more likely to answer.

C. If the original interpretation of the contract had been honored, with concentric circles at latitudes 84°, 78°, 72°, 66°, and 60°, what would the five circumferences have been? Feel free to leave R in your answer.

D. What would the total length L have been with the original interpretation? Again, feel free to leave R in your answer.

E. What was the percentage savings that the company could thank the crafty lawyer for? E.g., what is

$$\frac{\text{answer to D} - \text{answer to A}}{\text{answer to D}} \cdot 100\% \; ?$$

NOTE: Now the $R's$ you left in your answers to A and D will cancel out and you will be left with a percentage savings.

Problem 2 — The Chesapeake Bay Problem

This is Problem 7 on p. 2-47 to 2-48.

For everyone: Don't use the infinitesimal notation. Figure out what these things mean in terms of small changes Δr and Δr_{shell}. Make drawings that distinguish what these small changes refer to.

For those who are ***not presenting:*** You only have to do Part A.

For Eli and Eden, you will present the remainder of the Problem.

Problem 3 — The Zeno’s Paradox Problem

This is Problem 9 on p. 2-49.

For those who are ***not presenting:*** You may assume the result of the integral that is quoted in the problem statement on p. 2-49.

For Walker and Rebecca: Let’s augment what everyone else is doing by attacking this integral as many ways as we can come up with. Our goal will be to remind people of (a) changes of variables as a way of getting an integral into a standard form, (b) the relationship between indefinite and definite integrals, (c) the idea of integrable singularities (where the integrand blows up, but sufficiently gently that the integral doesn’t blow up), and (d) various properties of logarithms and other bits of algebra we might need to get our answer into a reasonable form.

Problem 4 — This isn’t a Problem ;) — Name: ____________________

We didn’t get time in the Nov. 7 class to read and debate your first round of questions from Chapter 2, partly because it took me longer to do a complete job of explaining the various formulae for photons, which are particles we take quite seriously, and which will be doing their best to escape from black holes, unsuccessfully if they are inside the event horizon, $r = 2\,M$. ***Sorry!*** We will rectify the failure to get to your questions!

In the meantime, we’ll still collect questions from the remainder of the Chapter 2. We’ll make sure to get to discussing and debating questions in both the Nov. 11 and Nov. 14 classes.

Tear off this sheet and use the space below to write your contribution to the second round of questions.