Black Holes, Worksheet III for Thursday, Oct. 10

$E = mc^2$ and the Energy to Raise a Freight Train a Mile High (a riff on Taylor and Wheeler, Problem 7-3)

Units of Energy

The most common scientific unit for energy is the Joule, abbreviated J. System Internationale (SI) units are designed to make things convenient. Wherever possible, the conversion factor is simply 1. So, for example,

$$1\text{ J} = 1\,\frac{\text{kg m}^2}{\text{s}^2}$$

If you look at that formula, you'll see that $\frac{\text{m}^2}{\text{s}^2} = \left(\frac{\text{m}}{\text{s}}\right)^2$is in it. We have a conversion factor between meters and seconds: $c = 1 = \frac{3*10^8\text{ m}}{\text{s}}$. So that means we can convert between kilograms and Joules!

$$1\text{J} = \frac{1\text{ J}}{1^2} = \frac{1\text{ J}}{c^2}$$

Of course, we can go big (or go home) with our viewpoint that $c = 1$. Then there is no need to convert at all! Just measure energy in kilograms (or grams or milligrams) and don't bother converting, *unless someone tells a story in SI units or demands an answer in Joules.*

Let's do an energy problem, but not the same one as 7-3, since Eden, Eli, and Sasha already made us a very nice handout for Problem 7-3. How about instead we blow a freight train a mile high? Actually, how about we gently raise a freight train a mile high?

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1. Energy to Raise a Freight Train a Mile High

A mile is about 1600 meters. Let's call that h. The acceleration of gravity is usually denoted, g, and it is about $g = 10\text{ m}/\text{s}^2$. The energy required to raise an object of mass M to a height h is $E = M\,g\,h$. In 7-3, Taylor and Wheeler estimated the mass of a freight train as $5*10^6$ kg. To make things come out tidier, I am going to use $M = 9*10^6$ kg which is appropriate for a 90-car freight train where each car weighs 100,000 kg. Using $M = 9*10^6$ kg, $g = 10\text{ m}/\text{s}^2$, and $h = 1600$ m, what is

$E = M\,g\,h =$ (make life easy; answer in Joules)

2. $E = m c^2$

In Problem 1, there was no need for any conversion factors! The story was told in SI units and you reported your answer in Joules. One of the two most famous equations in physics says that mass can be converted to energy and vice versa. At some extremely deep level, mass is energy! So we could have reported the answer for E in Problem 1 in units of mass. We just need the magic conversion factor c^2.

(a)Using whatever you got in Problem 1 for E, and $c^2 = 1 = 9 * 10^{16}\ \frac{\text{m}^2}{\text{s}^2}$, what is

$$E = \frac{E}{1} = \frac{E}{c^2} =$$

(b) Your answer to (a) is probably in kg. Multiply by $1 = \frac{10^6\ \text{mg}}{\text{kg}}$ to convert it to milligrams. What is

$$E = 1 \cdot E = \frac{10^6\ \text{mg}}{\text{kg}}\ E =$$

That’s how much mass has to be converted to energy to raise a 90-car freight train a mile high. You can see from such a simple calculation why the nuclear physicists became so concerned once the neutron was discovered and the possibility of chain reactions became less and less theoretical.

THE BAKER TEST DURING OPERATION CROSSROADS, A SERIES OF TWO NUCLEAR WEAPONS TESTS CONDUCTED BY THE UNITED STATES AT BIKINI ATOLL.