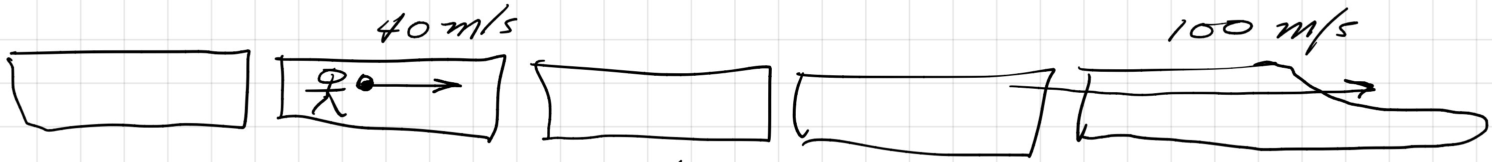
Example and Derivation of
Galilean Addition of Velocities

Example

A bullet train is going 100 m/s. On the train is a pitcher who can throw 40 m/s.



An observer is watching this from the side of the tracks by looking through the train windows. It seems intuitive that the observer says the baseball is going $140 \frac{m}{s}$. How would we derive this?

Wait $\Delta t = 1s$. The train will have gone 100 m. On the train, the ball will have gone 40 m. By adding these distances, we say that the ball has gone 140 m. 140 m in 1 second is $140 \frac{m}{s}$. The bottom line is because distances add, velocities add.

Derivation

Instead of using example values, I could have used variables. The train's speed is V_T. The pitcher's fastball pitch speed is V_B. The distance traveled by the train in Δt is $V_T \Delta t$. The distance traveled by the ball according to the people on the train is $V_B \Delta t$.

For the observer on the side of the tracks, because distances add, the distance the ball travels is $V_T \Delta t + V_B \Delta t$. Since this happens in Δt, the observer on the side of the tracks computes the ball's speed as

$$V = \frac{d}{t} = \frac{V_T \cancel{\Delta t} + V_B \cancel{\Delta t}}{\cancel{\Delta t}} = V_T + V_B$$

Although this argument seems airtight, we will soon learn that it is wrong, and our first counter-example is the photons coming out of a flashlight in Mary's rocket ship, as observed in John's lab.

going c

going 0.5c

Not what is observed by John! $c + 0.5c = 1.5c$