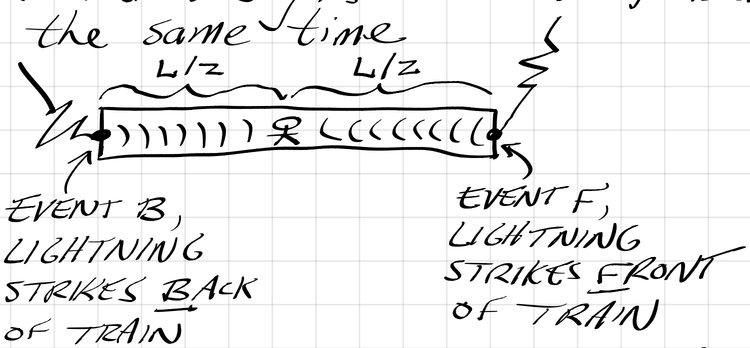
Lightning Strikes Train

Einstein's version of this thought experiment has the lightning striking both ends of the train at the same time according to the observer whose frame is the embankment.

I'm going to flip the thought experiment to one where the lightning strikes both ends of the train at the same time according to the observer whose frame moves with the train. Here we go...

In the frame of the train, if both ends of the train are struck simultaneously, and somebody is sitting in the middle of the train, then the flash from the back and the flash from the front arrive at the same time



Passenger in middle receives flashes from both back and front a time $L/2$ later.

Let's call the frame-of-the-train coordinates (t', x') and let's choose those so that Event B occurs at $(0,0)$ and Event F occurs at $(0,L)$.

$t'_B = 0$ $\qquad$ $t'_F = 0$

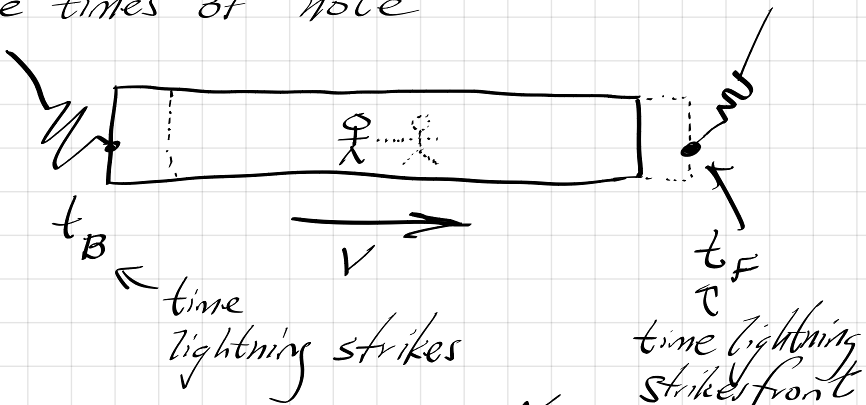
$x'_B = 0$ $\qquad$ $x'_F = L$

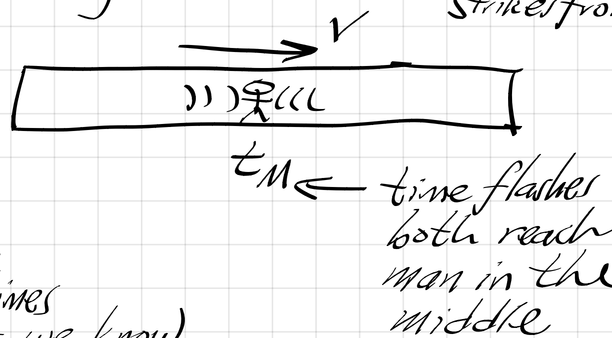
All good so far, now let's look at all of this from the viewpoint of an observer who is at rest with respect to the embankment.

It must be that for that observer the lightning struck the back of the train first, because the flash from that lightning strike has to catch up with the person in the middle, whereas the flash from the front has to go less distance to meet the person in the middle.

How much later did the lightning strike the front is the question.

In the embankment coordinates, there are three times of note





We have done it enough times now that we know

$$t_M - t_B = \frac{L}{2\gamma} + v(t_M - t_B)$$

and

$$t_M - t_F = \frac{L}{2\gamma} - v(t_M - t_F)$$

In other problems we have called $t_M - t_B = t_{forward}$ and $t_M - t_F = t_{backward}$

The only possible surprise is that $\frac{L}{2\gamma}$ appears in the formulae, but that is the Lorentz contracted length that the observer in the embankment frame measures half the train length to be.

With the usual rearranging,

$$t_M - t_B = \frac{L/2\gamma}{1-V}$$

and

$$t_M - t_F = \frac{L/2\gamma}{1+V}$$

Subtract second equation from first and get

$$t_F - t_B = \frac{L}{2\gamma}\left(\frac{1}{1-V} - \frac{1}{1+V}\right) = \frac{L}{2\gamma}\,\frac{1+V-(1-V)}{1-V^2}$$

$$= \frac{L}{\gamma}\,\frac{V}{1-V^2} = \frac{L}{\gamma} V\gamma^2 = L\gamma V$$

Let's choose $t_B = 0$ and $x_B = 0$

Then $t_F = L\gamma V$

Also $x_F = \frac{L}{\gamma} + L\gamma V^2$

SUMMARY

	TRAIN FRAME	EMBANKMENT FRAME
EVENT B	$(0, 0)$	$(0, 0)$
EVENT F	$(0, L)$	$\left(L\gamma V, \frac{L}{\gamma} + L\gamma V^2\right)$