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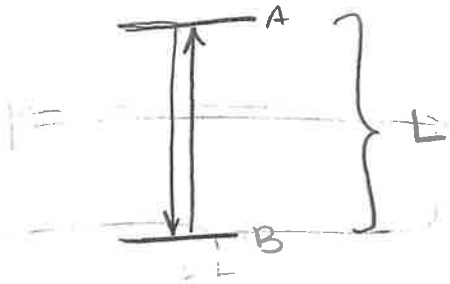
Black Holes

Length Contraction (presented by Sasha & Eli)



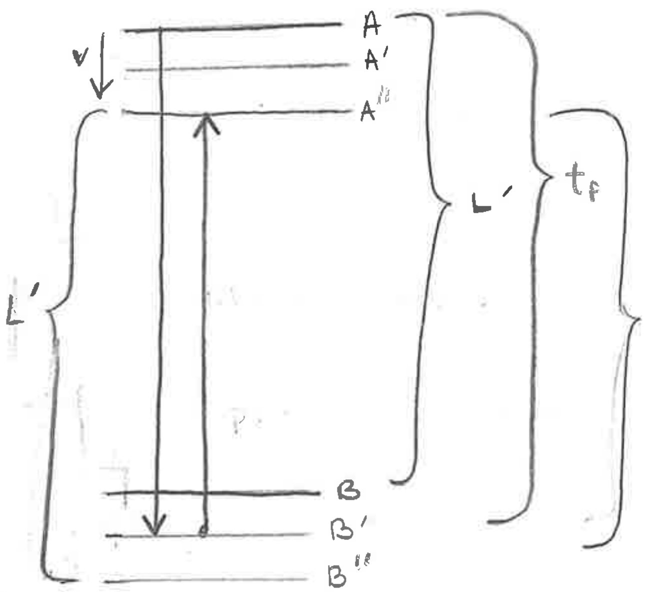
assume there are two mirrors and a photo moves from A → B then B → A

Then $2T = 2L/c$ but assume $c=1$

so $\boxed{2T = 2L}$ where T is the proper time from one mirror to the mirror for the photon's frame

L is the proper distance from one mirror to the other for the photon's refrence frame

Now, assume the mirrors are moving with a velocity of v such that it looks like these three snapshot from observer's perspective:



L' is the distance for an observer watching a proton go by.

(so L' = A → B, A' → B', A'' → B'')

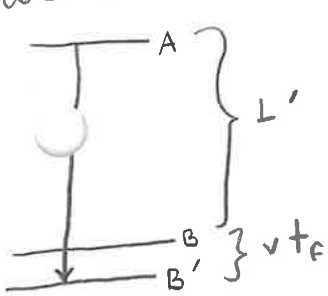
t_f is the time it takes for a photon to move forward from one mirror to the second mirror from the observer's perspectie (i.e A → B')

t_B is the time it takes for a photon to reflet backwards from a mirror to the other mirror from observer's perspecte (i.e B' → A'')

t is the time it takes from the photon to move from one mirror to the other mirror from the observer's perspective.

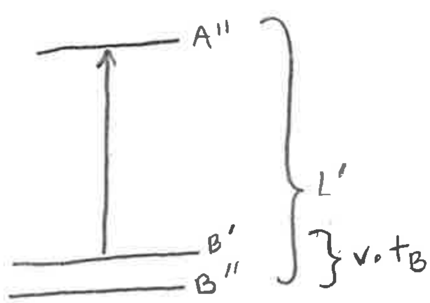
Therefor $\boxed{t = t_f + t_B}$

we can define t_f and t_B as such:



$t_f = L' + vt_f$

$\boxed{t_f = \frac{L'}{1-v}}$



$t_B = L' - vt_B$

$\boxed{t_B = \frac{L'}{1+v}}$

Find t:

$$t_F + t_B = \frac{L'}{1-v} + \frac{L'}{1+v} = L'\left(\frac{1}{1-v} + \frac{1}{1+v}\right) = L'\left(\frac{(1+v)(1-v)}{(1+v)(1-v)}\right) \cdot \left(\frac{1}{1-v} + \frac{1}{1+v}\right)$$

$$= L'\left(\frac{1+v}{(1-v)(1+v)} + \frac{1-v}{(1+v)(1-v)}\right) = L'\left(\frac{1+v+1-v}{(1-v)(1+v)}\right) = L' \cdot \frac{2}{1-v^2}$$

$$\boxed{t_F + t_B = \frac{2L'}{1-v^2}}$$

$$t = \frac{t_F + t_B}{2} = \frac{1}{2} \cdot \frac{2L'}{1-v^2} \qquad \boxed{t = \frac{L'}{1-v^2}}$$

Remember Time Dilation?

$$t = \gamma \cdot T$$

remember $T = L$

$$\frac{L'}{1-v^2} = \frac{1}{\sqrt{1-v^2}} \cdot L$$

$$L' = 1-v^2 \cdot \frac{1}{\sqrt{1-v^2}} \cdot L$$

$$= \sqrt{1-v^2} \cdot L$$

$$* \boxed{L' = \frac{1}{\gamma} \cdot L} *$$

To put it generally: $L' = \frac{L}{\gamma}$

where L' is the distance seen by the observer outside the refrence frame or "a person at rest"

where L is the distance by the "person moving with the clock" or the length measured inside the frame of refrence.