Cosmology and Black Holes

Spring 2023, Deep Springs College, Prof. **Brian Hill**

Overview

In two momentous leaps, Einstein discovered that space and time are not what we thought them to be. They are intertwined. I cannot explain how they are intertwined in this overview. If I could, there would be no need for a course. You have never experienced this intertwining, because it only becomes apparent when either (a) you are moving at near the speed of light, or (b) you are in the vicinity of an object even heavier and denser than our Sun. The intertwining is highly counterintuitive. We call the intertwined fabric of space and time, "spacetime."

More than anyone else since Einstein, John Archibald Wheeler has influenced our understanding of spacetime. He is an author or co-author on the best-known textbooks in the field and his students have gone on to advance the subject dramatically. Most notable of his students, perhaps, is Kip Thorne (one of the winners of the 2017 Nobel Prize)

We will work through Wheeler's, Thorne's and others' descriptions of three things: (1) special relativity, which is how space and time are related in the absence of gravity, (2) the behavior of spacetime around a black hole, and (3) the behavior of the spacetime of the entire universe, which is the subject of cosmology.

How can we do an honest treatment of a subject so advanced? We are not going to solve Einstein's equations. Instead, we will study the Schwarzschild solution for an uncharged, non-rotating black hole, and then the Friedman-Lemaître-Robertson-Walker solution for a spatially homogeneous and isotropic, expanding spacetime. By studying the solutions of Einstein's equations rather than the equations themselves, we can bypass a lot of advanced mathematics that even Einstein needed help (from Hilbert) to understand.

Unit Outline

Term 4 — Galilean Relativity, Special Relativity, Black Holes

I. Euclidean Space: Euclidean Metric, Euclidean Rotations, Galilean Relativity

II. Minkowski Space: Lorentz Metric, Lorentz Transformations, Special Relativity

III. Polar Coordinates in Flat and Curved Spacetime in 2+1 and 3+1 Dimensions

IV. Properties of the Schwarzschild Metric

Term 5 — The Big Bang, The Metric for the Universe

IV. Conservation of Map-Energy in the Schwarzschild Metric, Infalling Light, Infalling Stones, Rain

V. Homogeneity, Isotropy, Hubble Plots, the Cosmic Microwave Background

VI. The Friedman-Robertson-Walker (FRW) Metric

VII. Matter, Radiation, Dark Matter, and Dark Energy in FRW Cosmology

Expositions and Resources

We will use:

- ***Spacetime Physics, 2nd Edition***, by Edwin F. Taylor and John Archibald Wheeler ← Great! But it doesn't have enough general relativity for us. We will start our course with Chapters 1, 2, 3, and L.

- ***Exploring Black Holes: Introduction to General Relativity, 2nd Edition***, by Edwin F. Taylor, John Archibald Wheeler, and Edmund Bertschinger ← whether it was consciously or subliminally I don't know, this book is probably the inspiration for the course — It is closest in level to the level we will be working at, and there is now a second edition, which in addition to being free, has been rounded out to include much more modern cosmology.

Additionally, we will draw from:

- ***Black Holes and Time Warps***, by Kip Thorne ← a great survey, but at a mathematical level that is below where we want to be — I will be reading this for insights
- Selected historical papers that established Big Bang cosmology
- Selected modern papers containing the evidence for dark matter and dark energy ← Including some by the High-*z* Supernova Search Team and the Supernova Cosmology Project (whose leaders won the 2011 Nobel Prize)

Grading and Miscellaneous Policies

- 45% homework, 20% for midterm (near the end of term 4), 25% for final (near the end of term 5), 10% active preparation and participation in discussions

There will be a lot of handouts and our texts are PDFs that you will be printing out. To be organized, get a three-ring binder (or binders) to keep the texts, the handouts and problem sets organized. Assignments should be on 8 1/2 x 11 paper (and not torn out from a bound notebook). Multi-page assignments should be stapled. Corrections should be erased (if done in pencil) or recopied (if done in pen). To make nice diagrams and graphs, you will often need a ruler.

The College's general policies on absences and late work are applicable. There was an email from the Dean on this September 8, 2022. The policies below are consistent with that email:

Whereas missed coursework affects both your classmates and professors by lowering the thinking and understanding you bring to a given class, and interrupts the course schedule that has been set up and is adjusted on an ongoing basis with substantial care. The same is true for absences — whereas a handful of absences might be "normal" at colleges with large lectures or less serious academics, at Deep Springs we expect students to miss *no classes* save for legitimate health issues or emergencies requiring also missing labor and governance obligations. For a student wishing to submit a course assignment past its required deadline, the student may request an extension on the assignment directly from the professor 48 hours in advance. Within 48 hours of the due date, the student must request an extension directly from the Dean. Exceptions will be granted by the Dean only if the student faces unforeseen and unforeseeable circumstances. A student who misses the deadline will be penalized an amount that is roughly equivalent to a letter grade for each day the assignment is late. Assignments cannot be turned in after solutions and graded assignments have been passed back, which generally happens one to two classes after they were turned in.

Cosmology — Daily Schedule — Term 4

Week 1 — Free-Float Frames — Time Dilation — Tidal Forces

- Preparation for Monday, Jan. 9 — Read pp. 1-12 of Chapter 1 of ***Spacetime Physics, 2nd Edition*** (the link is to a PDF of Chapters 1, 2, 3, and L) — If pp. 1-12 seem straightforward, then you haven't yet read it carefully enough! — It took humanity from Galileo "Dialogue Concerning the Two Chief World Systems" in 1632 to Einstein in 1905 to progress from Galilean Relativity to Special Relativity (more accurately, Special Relativity was developed by Poincaré, Lorentz, Einstein, and Minkowski between 1900 and 1907), so read pp. 1-12 until you are bewildered
- Monday, Jan. 9 — Definition of velocity, units (the meter, the second, and the speed of light since the meter was re-defined in 1983), spacetime diagrams — **Various Values** used in examples and problems — Derivation of **Galilean Addition of Velocities** — What "the speed of light is constant" means in terms of Mary's rocket ship and John's lab — Discussion of pp. 1-12 of *Spacetime Physics*
- Preparation for Thursday, Jan. 12 — Read the rest of Chapter 1 and continue through the end of Section 2.3 of *Spacetime Physics* — I recommend you also get started on Problems Set 1 (not due until Monday, Jan. 16, see below) so that you can ask questions about the problems in Thursday's class — Print out all of Chapters 1, 2, 3, and L of ***Spacetime Physics***, three-hole punch them, and put them in a binder
- Thursday, Jan. 12 — We reviewed $F=ma$, $F=GmM\_E/r\_E^2$, the definition of $g = GM\_E/r\_E^2$, and the formula $d = ½\ at^2$ (some had not had these formulas - for most it was review - come see me if it was too fast) — We calculated the weakened acceleration of gravity 300km above Earth (the International Space Station is actually 408 km above Earth), and we understood why it seems to Chris Hadfield that "there is no gravity in space" when in fact it is only weakened by 12% — We discussed free-float frames (pp. 26-28), and even did the algebra for the ink trail on the wall of the falling house example on p. 28 — We discussed the small amount of calculus you will be using in Problem 2-8 and **a way of getting the needed result for Δ(1/r^2) without calculus** — We need to further discuss approximations (usually the leading non-zero term is sufficient), and today we only scratched the surface on what is a very general idea of power series expansions — We briefly discussed the "egg shape" referred to several times on p. 31 (such a shape is more accurately called an "ovoid," although if you look at the etymology and the lay definition, rather than the mathematician's definition, this just brings you right back to "egg-shaped") — The important thing is that unlike an actual egg, the mathematician's ovoid is not pointier at one end than the other — The mathematical ovoid is equally pointy at its two ends — Finally, I **derived the time dilation formula** (which gives you a shortcut for solving Problem 1-12)

Week 2 — Length Contraction — The Relativity of Simultaneity

- Preparation for Monday, Jan. 16 — Problem Set 1 due consisting of these six *Spacetime Physics* end-of-chapter Problems: 1-4, 1-8 (parts a, b, and c only), 1-10, 1-12, 2-6, and 2-8 — **Problem Set 1 Solution** — Continue reading to the end of Section 3.3 of *Spacetime Physics*
- Monday, Jan. 16 — Discussion of Problem 2-8 and **Falling Spherical Shell** — We approximated (1+x)^½ for small x by first recalling and appealing to Pascal's triangle and the expansion of (1+x)^n — We computed the volume of an ovoid — We got started on all three problems for Problem Set 2, due Tuesday — We contemplated: we say Mary's clock is ticking too slow. But Mary says our clock is ticking too slow. How can both be right?
- Preparation for Thursday, Jan. 19 — Read to the end of Section 3-5 (to p. 65) of Spacetime Physics* — Do **Problem Set 2** due — **Problem Set 2 Solution**
- Thursday, Jan. 19 — We went through the **Length Contraction Derivation** as outlined in Problem 3 on Assignment 2 — We did a lengthy and careful analysis of Einstein's lightning-strikes-the-train-car thought experiment and came to the conclusion that John says the clocks at the front of Mary's rocket are behind, and Mary says the clocks at the "front" of John's lab are behind (what does the "front" of the lab mean? It means the part of the lab that Mary encounters first) — So now we have derived Time Dilation, Length Contraction, and started to see that forced upon us is going to be the **Relativity of Simultaneity** — Disagreements about simultaneity are the escape route wherein we can reconcile that John says Mary's clocks run too slow, but Mary says John's clocks run too slow, and John says Mary's longitudinally-oriented meter sticks are too short, but Mary says John's longitudinally-oriented meter sticks are too short.

Week 3 — Exploding Universe — Red Shift — The Lorentz Transformation — Curved Surfaces — The Surface of a Cylinder

- Preparation for Monday, Jan. 23 — Read to the end of Chapter 3 — Do Problem Set 3 due (see additional problem added below, and now due at beginning of Thursday's class)
- Monday, Jan. 23 — We began with the **Exploding Universe Exercise** which is preparatory for Problem 3-10 — Then we launched into Problem 3-10 and derived the formula for z, the red shift — Then we did **A Rapid Review of Light Waves** (hopefully something you had in high school science), including the basic equations for T (the period), λ (the wavelength), f (the frequency), and c (the speed of light), and we used the Hydrogen-α line as an example, λ = 656.46nm — Then we went through Problem 3-10 (you still need to write it up) and by the time we were done, we had the formula for the relativistic Doppler shift, and we saw why it is commonly referred to as "red shift," and finally we found out what z refers to in the **latest z = 11-20 galaxy candidates** found using the James Webb Space Telescope's infrared cameras — These candidates were reported in Jan. 2023 and have undergone little confirmatory work, so they are "candidate discoveries," not yet widely agreed-upon, and many of the 87 signals may be spurious or otherwise unreproducible, or at least not have their z-value correctly estimated)
- Preparation for Thursday, Jan. 26 — Finish **Problem Set 3** — **Problem Set 3 Solution** — Continue reading into Chapter L of Taylor and Wheeler up to and including page 100 (including the form of the Lorentz transformation, Section L.4)
- Thursday, Jan. 26 — Derivation of **Relativistic Addition of Velocities** which was Problem 3-11 parts (a)-(f) — **Lorentz Transformation Derivation** — Pretty rushed at end: (a) Introducing the coordinates on a cylinder: z and ϕ, (b) Pythagorean theorem on a cylinder: $(\Delta s)^2=(\Delta z)^2+(R\Delta\phi)^2$, and (c) using Δs notation instead of ds in order to avoid using infinitesimals or any other calculus notation at this point in the course.

Week 4 — The Surface of the Sphere and the Torus — Energy and Momentum — The Principal of Extremal Aging

- Preparation for Monday, Jan. 30 — Finish reading Chapter L of Taylor and Wheeler and bring any remaining questions about the **Lorentz Transformation Derivation** — Print out **Taylor, Wheeler, and Bertschinger Exploring Black Holes, 2nd Edition - Chapters 1-7**, read Sections 2.1, 2.2, and 2.3, and do an extra-close reading of pp. 2-7 and 2-8 — Keep in mind the cylindrical coordinates we rushed through: z and ϕ — Be sure when you run into an equation that looks something like $ds^2=dz^2+R^2d\phi^2$ to translate it into the far more understandable notation $(\Delta s)^2=(\Delta z)^2+(R\Delta\phi)^2$ — Specifically, $ds^2=R^2\cos^2\lambda d\phi^2+R^2d\lambda^2$ on p. 2-7 should be translated into $(\Delta s)^2=(R\cos\lambda\Delta\phi)^2 + (R\Delta\lambda)^2$ and the $d\lambda$ and $d\phi$ notation that appears in Fig. 7 on p. 2-8 should be translated into $\Delta\lambda$ and $\Delta\phi$ — On Monday, Jan. 23rd, people expressed an interest in what happens to objects oriented or moving at intermediate angles, so the next problem set has three problems involving angles
- Monday, Jan. 30 — We quickly covered six topics to prepare you for the rest of Chapter 2 and for Problem Set 4. With these topics in hand, you should not need to refer back to Chapter 1, even though the authors occasionally do — (1) Simultaneity is relative, but causality is not: spacetime diagrams, the forward light-cone, and the backward light-cone — (2) The energy of a particle, $m\Delta t/\Delta\tau$ where m is the mass of the particle at rest (this formula does not work for photons); the momentum of a particle, the x-component of which is $m\Delta x/\Delta\tau$; recovering the non-relativistic formula for kinetic energy — (3) The Twin Paradox and the Principle of Extremal Aging — (4) Group exercise: shortest path between two faraway points as determined by local geometers who can each only straighten paths by consulting nearby points — (5) and (6): Get a start on two of the problems from the next problem set: Problem L-6(a) and The Torus

Getting into the mindset of local geometers is what Taylor, Wheeler, and Bertschinger were trying to get you to do in Section 2.2. This way of discovering the path that a particle trying to "go straight" across a curved surface will take is the direct analog of how we discover particle paths using the Principle of Extremal Aging in General Relativity.

- Preparation for Thursday, Feb. 2 — Finish your study of Chapter 2, which will hopefully go well now that we have spent 3 1/2 weeks studying all the prerequisite concepts — Bring serious questions about all the material so far, especially the last half of Chapter 2 and the most recent two problem sets.
- Thursday, Feb. 2 — One-third of class was devoted to answering questions about Problem Set 4 — Another third was devoted to understanding how we get metrics by determining distances on locally-

flat patches in a global coordinate system with the torus and the surface of the sphere as our primary examples — The final third was devoted (a) introducing the Schwarzschild metric and (b) making a start on Sample Problem 1 on p. 3-16, see **Stretching of Space-Part I** — This is the first time we have started to use any calculus — My goal is to make Sample Problem 1 understandable even if you haven't had any calculus by introducing all the concepts in the Riemann integral — Stay tuned for Stretching of Space-Part II

- Saturday, Feb. 4 — **Problem Set 4 due in my box by 6pm** — **Problem Set 4 - Solution**

Week 5 — Our Initial Encounter with Black Holes (The Schwarzschild Metric)

- Preparation for Monday, Feb. 6 — Study Taylor, Wheeler, and Bertschinger Sections 3.1, 3.2, and 3.3 to p. 3-17, but don't yet try to understand Sample Problem 1 unless your calculus is fresh — Remember that when you encounter equations like $d\tau^2=(1-2M/r)dt^2-dr^2/(1-2M/r)-r^2d\phi^2$ (which is Equation 5) or $d\sigma^2=-(1-2M/r)dt^2+dr^2/(1-2M/r)+r^2d\phi^2$ (which is Equation 6), that you should immediately translate them from differentials into the far more understandable notation of coordinate differences, *e.g.*, $(\Delta\sigma)^2=-(1-2M/r)(\Delta t)^2+(\Delta r)^2/(1-2M/r)+r^2(\Delta\phi)^2$ — Apply this procedure to any "infinitesimals" or "differentials" that the authors use (such as Equation 16)
- Monday, Feb. 6 — We need to review the Euclidean metric in spherical polar coordinates and the flat-Lorentzian spacetime version of the metric in spherical polar coordinates — Compare and contrast with the Schwarzschild metric — We need to discuss why the global coordinate *r* that we are using is known as the reduced circumference — Finally, I will continue unpacking Sample Problem 1 on p. 3-16 by deriving the integral the authors used to get a precise answer — See **Stretching of Space-Part I** and **Stretching of Space-Part II** — We need to start looking at Eqs. 24 to 26 on pp. 3-24 to 3-25
- Preparation for Thursday, Feb. 9 — Study Taylor, Wheeler, and Bertschinger Sections 3.4, 3.5, 3.6, and 3.7 to p. 3-26 — On p. 3-24, are Eqs. 24 and 25 that you need to rewrite in terms of Δt and Δr to make them understandable — I will need to further unpack the derivation of Eq. 26 on p. 3-25 for you, as I did with Sample Problem 1 on p. 3-16, but you already know enough about Eqs. 24 to 26 to use them — **Problem Set 5** due — **Problem Set 5 Solution**
- Thursday, Feb. 9 — Let's continue by working harder to unpack Eq. 26 on p. 3-25 — Let's look ahead to Section 3-8, which is mind-blowing (inside the event horizon, the global *r*-coordinate behaves like a time coordinate, accckkk!)

Week 6 — Finish our Initial Encounter with the Schwarzschild Metric

- Preparation for Tuesday, Feb. 9 — Finish Studying Taylor, Wheeler, and Bertschinger Chapter 3
- Monday, Feb. 13 — Discuss Remainder of Chapter 3 — Discuss Next Problem Set — Plan Reading for Thursday
- Preparation for Thursday, Feb. 16 — **Problem Set 6** due — **Problem Set 6 Solution** — Read *all* of TWB Chapter 5 — Max and Clara will present a pocket calculator solution to Problem 4 on Problem Set 6 which is a photon spiraling inward from r=1.5M to r=0
- Thursday, Feb. 16 — Presentation from Max and Clara — My Calculator Solution for **Photon Inside the Event Horizon** — We spent 2/3 of the class trying to understand Eqs. 7 to 11 on p. 5-15 — These equations are how local coordinate patches are used! — Most of the discussion in Chapter 5 leading up to these equations is just to help you understand them (and other equations you will encounter that are like them) — Finally, we looked ahead to Chapter 6 and I reminded you of how to read Equations such as Eq. 23 on p. 6-11: *Δτ_raindrop=-Δr/sqrt(2M/r)*

Week 7 — Term 4 Exam — Shakespeare Festival

- Monday, Feb. 20 — **Term 4 Exam** Covering TW Chapters, 1, 2, 3, and L, and TWB, Chapters 2 and 3 (open book but not open note, so I can re-assign, if I choose, some problems that you had on Problem Sets 1 to 6, without you being able to just consult the solutions!) — **Term 4 Exam Solution**
- Thursday, Feb. 23 — No class due to Shakespeare Festival

Cosmology — Daily Schedule — Term 5

Week 8 — Mid-Term Review — Map Energy Conservation

- Preparation for Monday, Mar. 12 — Study TWB Chapter 6 through Section 6.4, which is **pp. 6-1 to 6-12** — Don't attempt to understand Eqs. 3 to 7; understand Eq. 2 then jump to Eq. 8 and understand it — Reminder: as always, understand equations such as Eq. 23 on p. 6-11 as *Δτ_raindrop=-Δr/sqrt(2M/r)* — Ben will prepare a time-line of when Einstein's equations were solved for various important matter distributions (black hole, uniformly expanding spacetime, rotating black hole, and the charged-and-rotating black hole), and by whom, starting with Schwarzschild solving the black hole in 1916 — Schwarzschild's 1916 paper has been **translated** into English — A quick look at the translation will explain why we are studying the solutions of Einstein's equations as manna from Heaven, not trying to get those solutions!
- Monday, Mar. 12 — Ben's presentation — Extended discussion of how we can get through this material more effectively
- Preparation for Thursday, Mar. 15 — No new reading beyond what was already assigned for Monday (see above) — **Assignment 7**
- Thursday, Mar. 15 — Presentations of midterm exam problems — Barely started the detailed deconstruction of Eqs. 3 to 8 of Chapter 6 which is the derivation of the **Conservation of Map Energy** for an orbiting or diving stone

Week 9 — Diving Toward the Black Hole — Shell Energy — Stones — Rain

- Preparation for Monday, Mar. 19 — Continue TWB Chapter 6 through Section 6.6 (through p. 6-19)
- Monday, Mar. 19 — Now that we have conservation of map energy, what does it tell us about the **Shell Energy** of a stone — What extra relationships can we derive for the special case of **Rain**? — How **Feynman studies**
- Preparation for Thursday, Mar. 22 — **Problem Set 8** — **Problem Set 8 Solution** — Also, a very modest amount of cosmology reading (just Section 14.1 of TWB) — Here is the final installment of **TWB Chapters 14-16** on cosmology and gravitational waves (we will only get as far as Section 15.8)
- Thursday, Mar. 22 — A simple worksheet that helps to understand the expanding universe

Week 10 — Metric for a Uniform Expanding Universe — The Cosmological Redshift — The Cosmic Microwave Background Radiation

- Preparation for Monday, Mar. 26 — Continue TWB Chapter 14 by studying Sections 14.2 and 14.3 — Changes of variables suddenly become very important — What are the authors doing going from equation 7 to equation 8? — How about calculating $\Delta r=(\sin(\chi + \Delta\chi)-\sin(\chi))/K^{1/2}$? — You will need the formula for $\sin(A+B)$ with $A=\chi$ and $B=\Delta\chi$ — You will need to make the approximation that $B=\Delta\chi$ is small and use it judiciously — If you do this well, you will gain much insight into equation 8 — Equation 17 is exciting: the Friedman-Robertson-Walker metric first appears!
- Monday, Mar. 26 — **Coordinate Transformations**
- Preparation for Thursday, Mar. 29 — Before continuing, study my write-up of **Coordinate Transformations** (the hardest part of what I did in the last class) — Then continue in TWB Chapter 14 by studying Sections 14.4 and 14.5 only to p. 14-12 — Section 14.4 is the most intimidating-looking reading yet, but it isn't that bad — We have worked hard to numerically integrate equations like Eq. 22 — Set up Eq. 22 using Δt and $\Delta\chi$ instead of infinitesimals and integrals — Also, make sure you can get Eq. 31 from Eq. 17 (and write those equations using Δ's instead of infinitesimals) — The bottom of p. 14-12 (on which Eq. 31 appears) is a good place to stop — **Problem Set 9** — **Problem Set 9 Solution**
- Thursday, Mar. 29 — The residual radiation from t=370,000 years, when the temperature was about 3000K and protons and electrons cooled enough to combine into Hydrogen (mostly, some Helium too) and photons were no longer scattered by matter — **Penzias and Wilson 1965 - With Wilkinson Microwave Anisotropy Probe (WMAP) 2011 image** — Derivation of the **Cosmological Redshift** (yet again we translate into the ordinary and intuitive language of small changes and sums what TWB do with infinitesimals and integrals)

Week 11 — Black-Body Radiation — Conservation of Radial Momentum in the FRW

Metric — Luminosity and The Magnitude System

- Preparation for Monday, Apr. 3 — Go back and study everything you have in both of the textbooks we are using on the Principle of Maximal Aging and my handout where I derived the **Conservation of Map Energy** from the Principle of Maximal Aging — There is a small but significant mistake in Box 4 on TWB p. 14-13 — Find it and then understand the rest of the formulae in the box — Use everything you've learned to understand the rest of Section 14.5 (through p. 14-16) — The stones are now galaxies! — TWB are trying to find things that are true about the motion of galaxies through a uniform universe described by the FRW metric
- Monday, Apr. 3 — **Conservation of Radial Momentum** in the FRW Metric
- Preparation for Thursday, Apr. 6 — Study TWB Section 14.6 — Study the **Leavitt-Pickering Period-Luminosity paper** and **Hubble's 1929 paper** — Do **Problem Set 10** — **Problem Set 10 Solution**
- Thursday, Apr. 6 — Luminosity, Luminosity Distance, The Magnitude System

Week 12 — Relationship of Hubble's Law to the FRW Metric — Luminosity Distance — Friedman Equation

- Preparation for Monday, Apr. 10 — Finish studying Chapter 14 of TWB
- Monday, Apr. 10 — We re-derived the redshift formula from Special Relativity, which TWB have as Eq. 47 on p. 14-21 — I derived Eq. 60 on p. 14-24 which relates **Hubble's Law and the FRW Metric** — In short, H = dR/dt / R
- Preparation for Thursday, Apr. 13 — Complete your study of pp. 14-22 and 14-23 (especially Box 6 on the luminosity distance) — Study TWB Sections 15.1 and 15.2
- Thursday, Apr. 13 — Photons and Luminosity Distance (Box 6 on p. 14-22) — TWB Section 15.3 — **Problem Set 11** — **Problem Set 11 Solution**

Week 13 — Solving the Friedman Equation in the Presence of Matter, Energy, and Dark Energy

- Preparation for Monday, Apr. 17 — Study TWB Sections 15.4 and 15.5
- Monday, Apr. 17 — We only got as far as Eq. 26 at the beginning of 15.5 — A simplifying surprise is that the sum of the densities of all types of matter today is (within experimental error) equal to the critical density, which says that to the best we can observe it, $K=0$. That was important to deriving Eq. 26 at the beginning of Section 15.5 from Eq. 22 in Section 15.4
- Preparation for Thursday, Apr. 20 — Study TWB Sections 15.6 and 15.7 — However, much of the discussion in 15.7 is trying for another way of motivating the fact that Ω_Λ can cause accelerating expansion and that isn't really necessary if you accept that the dark energy is vacuum energy and is the energy of the vacuum (empty space) and therefore does not spread out as more vacuum (empty space) is created — The goal of the current reading is to understand how a matter-dominated, a radiation-dominated, and a dark-energy-dominated universe evolves — Because of the different powers of $a(t)$ in the Friedman equation, our universe goes or will go through all three phases
- Thursday, Apr. 20 — Derivation of the Integral for **Various Universes** — Matter domination, radiation domination, dark-energy domination

Week 14 — Term 5 Exam — The Principle Evidence for Dark Matter and Dark Energy

- Monday, Apr. 24 — **Term 5 Exam** covering the same material as this term's problem sets — **Term 5 Exam Solution**
- Preparation for Thursday, Apr. 27 — **Problem Set 12** — I will cover the critical parts of TWB sections 15.8, 15.9 and 15.10 in the last class
- Thursday, Apr. 27 — The observed contents of the universe and the evidence for dark matter and dark energy — These results are known as **Lambda CDM Cosmology** — **Problem Set 12 Solution**