SPACETIME PHYSICS

introduction to special relativity

Second Edition

Edwin F. Taylor
Massachusetts Institute of Technology

John Archibald Wheeler
Princeton University and
University of Texas at Austin

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Both males and females make competent observers. We ordinarily treat the laboratory observer as male and the rocket observer as female. Beyond this, to avoid alternating "his" and "her" in a single chapter, we use female pronouns for an otherwise undesignated observer in odd-numbered chapters and male pronouns in even-numbered chapters.

Epigram, facing page: Einstein remark to his assistant Ernst Straus, quoted in Mainsprings of Scientific Discovery by Gerald Holton in The Nature of Scientific Discovery, Owen Gingerich, Editor (Smithsonian Institution Press, Washington, 1975).

What I'm really interested in is whether God could have made the world in a different way; that is, whether the necessity of logical simplicity leaves any freedom at all.

—Albert Einstein

Edwin F. Taylor and John Archibald Wheeler have written a general relativity sequel to *Spacetime Physics,* namely: *Exploring Black Holes: Introduction to General Relativity* Addison Wesley Longman, San Francisco, 2000

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CHAPTER 1

SPACETIME: OVERVIEW

Our imagination is stretched to the utmost, not, as in fiction, to imagine things which are not really there, but just to comprehend those things which are there.

Richard P. Feynman

1.1 PARABLE OF THE SURVEYORS

disagree on northward and eastward separations; agree on *distance*

Daytime surveyor uses magnetic north

Once upon a time there was a Daytime surveyor who measured off the king's lands. He took his directions of north and east from a magnetic compass needle. Eastward separations from the center of the town square he measured in meters. The northward direction was sacred. He measured northward separations from the town square in a different unit, in miles. His records were complete and accurate and were often consulted by other Daytimers.

Nighttime surveyor uses North-Star north

A second group, the Nighttimers, used the services of another surveyor. Her north and east directions were based on a different standard of north: the direction of the North Star. She too measured separations eastward from the center of the town square in meters and sacred separations northward in miles. The records of the Nighttime surveyor were complete and accurate. Marked by a steel stake, every corner of a plot appeared in her book, along with its eastward and northward separations from the town square.

Daytimers and Nighttimers did not mix but lived mostly in peace with one another. However, the two groups often disputed the location of property boundaries. Why? Because a given corner of the typical plot of land showed up with different numbers in the two record books for its eastward separation from the town center, measured in meters (Figure 1-1). Northward measurements in miles also did not agree between the two record books. The differences were small, but the most careful surveying did not succeed in eliminating them. No one knew what to do about this single source of friction between Daytimers and Nighttimers.

One fall a student of surveying turned up with novel open-mindedness. Unlike all previous students at the rival schools, he attended both. At Day School he learned

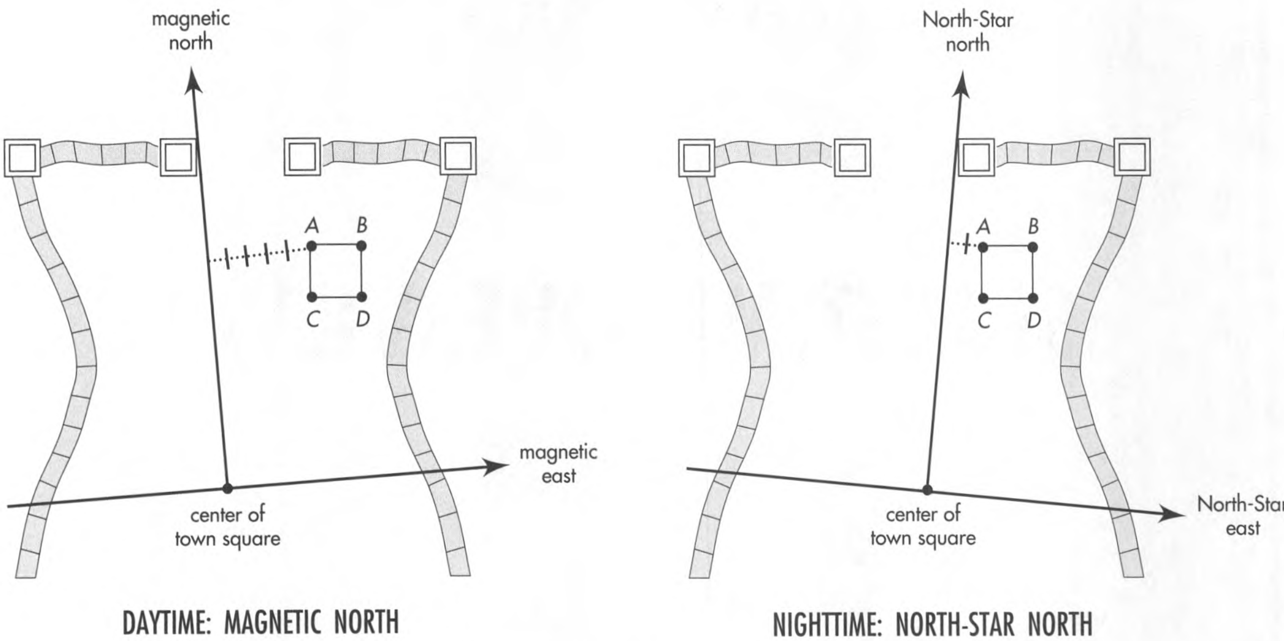


FIGURE 1-1. ***The town as plotted by Daytime and Nighttime surveyors.*** *Notice that the line of Daytime magnetic north just grazes the* left *side of the north gate, while the line of Nighttime North-Star north just grazes the* right *side of the same gate. Steel stakes* A, B, C, D *driven into the ground mark the corners of a disputed plot of land. As shown, the eastward separation of stake* A *from the north–south line measured by the Daytime surveyor is different from that measured by the Nighttime surveyor.*

from one expert his method of recording locations of gates of the town and corners of plots of land based on magnetic north. At Night School he learned the other method, based on North-Star north.

As days and nights passed, the student puzzled more and more in an attempt to find some harmonious relationship between rival ways of recording location. His attention was attracted to a particular plot of land, the subject of dispute between Daytimers and Nighttimers, and to the steel stakes driven into the ground to mark corners of this disputed plot. He carefully compared records of the two surveyors (Figure 1-1, Table 1-1).

Student converts miles to meters

In defiance of tradition, the student took the daring and heretical step of converting northward measurements, previously expressed always in miles, into meters by multiplying with a constant conversion factor k. He found the value of this conversion factor to be $k = 1609.344$ meters/mile. So, for example, a northward separation of 3 miles could be converted to $k \times 3$ miles $= 1609.344$ meters/mile $\times$ 3 miles $= 4828.032$ meters. "At last we are treating both directions the same!" he exclaimed.

Next the student compared Daytime and Nighttime measurements by trying various combinations of eastward and northward separation between a given stake and the center of the town square. Somewhere the student heard of the Pythagorean Theorem, that the sum of squares of the lengths of two perpendicular legs of a right triangle equals the square of the length of the hypotenuse. Applying this theorem, he discovered that the expression

$$\overset{\textbf{Daytime}}{\left[k \times \begin{pmatrix}\text{northward}\\ \text{separation}\\ \text{(miles)}\end{pmatrix}\right]^2} + \overset{\textbf{Daytime}}{\begin{bmatrix}\text{eastward}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2} \qquad (1\text{-}1)$$

TABLE 1-1

TWO DIFFERENT SETS OF RECORDS; SAME PLOT OF LAND

	Daytime surveyor's axes oriented to magnetic north		*Nighttime surveyor's axes oriented to North-Star north*	
	Eastward (meters)	Northward (miles)	Eastward (meters)	Northward (miles)
Town square	0	0	0	0
Corner stakes:				
Stake *A*	4010.1	1.8330	3950.0	1.8827
Stake *B*	5010.0	1.8268	4950.0	1.8890
Stake *C*	4000.0	1.2117	3960.0	1.2614
Stake *D*	5000.0	1.2054	4960.0	1.2676

based on Daytime measurements of the position of steel stake *C* had exactly the same numerical value as the quantity

$$\overset{\textbf{Nighttime}}{\left[k \times \begin{pmatrix}\text{northward}\\ \text{separation}\\ \text{(miles)}\end{pmatrix}\right]^2} + \overset{\textbf{Nighttime}}{\begin{bmatrix}\text{eastward}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2} \tag{1-2}$$

computed from the readings of the Nighttime surveyor for stake *C* (Table 1-2). He

TABLE 1-2

"INVARIANT DISTANCE" FROM CENTER OF TOWN SQUARE TO STAKE *C*

(Data from Table 1-1)

Daytime measurements		Nighttime measurements	
Northward separation 1.2117 miles		Northward separation 1.2614 miles	
Multiply by $k = 1609.344$ meters/mile to convert to meters: 1950.0 meters		Multiply by $k = 1609.344$ meters/mile to convert to meters: 2030.0 meters	
Square the value	3,802,500 (meters)^2	Square the value	4,120,900 (meters)^2
Eastward separation 4000.0 meters		Eastward separation 3960.0 meters	
Square the value and add	+ 16,000,000 (meters)^2	Square the value and add	+ 15,681,600 (meters)^2
Sum of squares	= 19,802,500 (meters)^2	Sum of squares	= 19,802,500 (meters)^2
Expressed as a number squared	= (4450 meters)^2	Expressed as a number squared	= (4450 meters)^2
This is the square of what measurement?	**4450 meters**	This is the square of what measurement?	**4450 meters**

SAME
DISTANCE
from center of Town Square

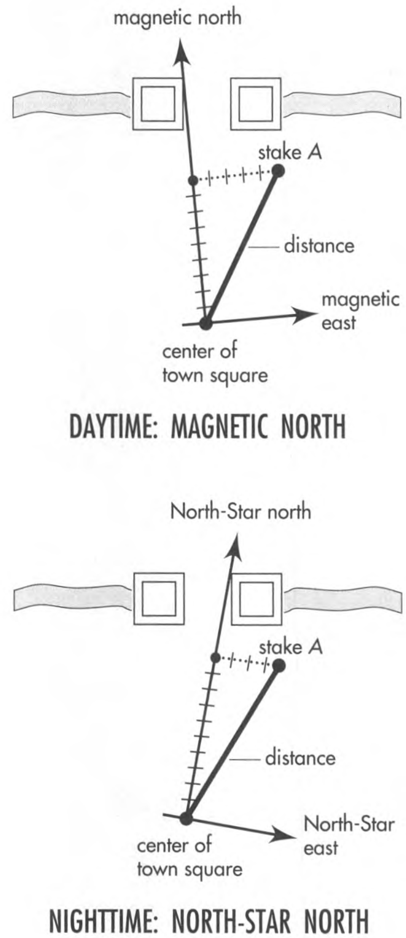


FIGURE 1-2. *The distance between stake* A *and the center of the town square has the same value for Daytime and Nighttime surveyors, even though the northward and eastward separations, respectively, are not the same for the two surveyors.*

tried the same comparison on recorded positions of stakes *A, B,* and *D* and found agreement here too. The student's excitement grew as he checked his scheme of comparison for all stakes at the corners of disputed plots—and found everywhere agreement.

Discovery: Invariance of distance

Flushed with success, the student methodically converted all northward measurements to units of meters. Then the student realized that the quantity he had calculated, the numerical value of the above expressions, was not only the same for Daytime and Nighttime measurements. It was also the square of a length: (meters)^2. He decided to give this length a name. He called it the **distance** from the center of town.

$$(\text{distance})^2 = \begin{bmatrix} \text{northward} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 + \begin{bmatrix} \text{eastward} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 \qquad (1\text{-}3)$$

He said he had discovered the **principle of invariance of distance;** he reckoned exactly the same value for distance from Daytime measurements as from Nighttime measurements, despite the fact that the two sets of surveyors' numbers differed significantly (Figure 1-2).

After some initial confusion and resistance, Daytimers and Nighttimers welcomed the student's new idea. The invariance of distance, along with further results, made it possible to harmonize Daytime and Nighttime surveys, so everyone could agree on the location of each plot of land. In this way the last source of friction between Daytimers and Nighttimers was removed.

1.2 SURVEYING SPACETIME

disagree on separations in space and time; agree on spacetime *interval*

The Parable of the Surveyors illustrates the naive state of physics before the discovery of **special relativity** by Einstein of Bern, Lorentz of Leiden, and Poincaré of Paris. Naive in what way? Three central points compare physics at the turn of the twentieth century with surveying before the student arrived to help Daytimers and Nighttimers.

The second: A sacred unit

First, surveyors in the mythical kingdom measured northward separations in a sacred unit, the mile, different from the unit used in measuring eastward separations. Similarly, people studying physics measured time in a sacred unit, called the second, different from the unit used to measure space. No one suspected the powerful results of using the same unit for both, or of squaring and combining space and time separations when both were measured in meters. Time in meters is just the time it takes a light flash to go that number of meters. The conversion factor between seconds and meters is the speed of light, $c = 299{,}792{,}458$ meters/second. The velocity of light c (in meters/second) multiplied by time t (in seconds) yields ct (in meters).

Speed of light converts seconds to meters

The speed of light is the only natural constant that has the necessary units to convert a time to a length. Historically the value of the speed of light was regarded as a sacred number. It was not recognized as a mere conversion factor, like the factor of conversion between miles and meters — a factor that arose out of historical accident in humankind's choice of units for space and time, with no deeper physical significance.

Second, in the parable northward readings as recorded by two surveyors did not differ much because the two directions of north were inclined to one another by only the small angle of 1.15 degrees. At first our mythical student thought that small differences between Daytime and Nighttime northward measurements were due to surveying error alone. Analogously, we used to think of the separation in time between two electric sparks as the same, regardless of the motion of the observer. Only with the publication of Einstein's relativity paper in 1905 did we learn that the separation in time between two sparks really has different values for observers in different states of motion — in different **frames.**

Time between events: Different for different frames

Think of John standing quietly in the front doorway of his laboratory building. Suddenly a rocket carrying Mary flashes through the front door past John, zooms down the middle of the long corridor, and shoots out the back door. An antenna projects from the side of Mary's rocket. As the rocket passes John, a spark jumps across the 1-millimeter gap between the antenna and a pen in John's shirt pocket. The rocket continues down the corridor. A second spark jumps 1 millimeter between the antenna and the fire extinguisher mounted on the wall 2 meters farther down the corridor. Still later other metal objects nearer the rear receive additional sparks from the passing rocket before it finally exits through the rear door.

One observer uses laboratory frame

John and Mary each measure the lapse of time between "pen spark" and "fire-extinguisher spark." They use accurate and fast electronic clocks. John measures this time lapse as 33.6900 thousand-millionths of a second (0.0000000336900 second $= 33.6900 \times 10^{-9}$ second). This equals 33.6900 **nanoseconds** in the terminology of high-speed electronic circuitry. (One nanosecond $= 10^{-9}$ second.) Mary measures a slightly different value for the time lapse between the two sparks, 33.0228 nanoseconds. For John the fire-extinguisher spark is separated in space by 2.0000 meters from the pen spark. For Mary in the rocket the pen spark and fire-extinguisher spark occur at the same place, namely at the end of her antenna. Thus for her their space separation equals zero.

Another observer uses rocket frame

Later, laboratory and rocket observers compare their space and time measurements between the various sparks (Table 1-3). Space locations and time lapses in both frames are measured from the pen spark.

TABLE 1-3

SPACE AND TIME LOCATIONS OF THE SAME SPARKS AS SEEN BY TWO OBSERVERS

	Distance and time between sparks as measured by observer who is standing in laboratory (John)		*moving by in rocket (Mary)*	
	Distance (meters)	Time (nanoseconds)	Distance (meters)	Time (nanoseconds)
Reference spark (pen spark)	0	0	0	0
Spark *A* (fire-extinguisher spark)	2.0000	33.6900	0	33.0228
Spark *B*	3.0000	50.5350	0	49.5343
Spark *C*	5.0000	84.2250	0	82.5572
Spark *D*	8.0000	134.7600	0	132.0915

Discovery: Invariance of spacetime interval

The third point of comparison between the Parable of the Surveyors and the state of physics before special relativity is this: The mythical student's discovery of the concept of distance is matched by the Einstein–Poincaré discovery in 1905 of the **invariant spacetime interval** (formal name **Lorentz interval,** but we often say just **interval**), a central theme of this book. Let each time measurement in seconds be converted to meters by multiplying it by the "conversion factor c," the speed of light:

$$\begin{aligned} c &= 299{,}792{,}458 \text{ meters/second} = 2.99792458 \times 10^8 \text{ meters/second} \\ &= 0.299792458 \times 10^9 \text{ meters/second} = 0.299792458 \text{ meters/nanosecond} \end{aligned}$$

Then the square of the spacetime interval is calculated from the laboratory observer's measurements by *subtracting* the square of the space separation from the square of the time separation. Note the minus sign in equation (1-4).

$$(\text{interval})^2 = \left[c \times \underset{\textbf{Laboratory}}{\begin{pmatrix} \text{time} \\ \text{separation} \\ \text{(seconds)} \end{pmatrix}} \right]^2 - \underset{\textbf{Laboratory}}{\begin{bmatrix} \text{space} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}}^2 \qquad (1\text{-}4)$$

The rocket calculation gives exactly the same value of the interval as the laboratory calculation,

$$(\text{interval})^2 = \left[c \times \underset{\textbf{Rocket}}{\begin{pmatrix} \text{time} \\ \text{separation} \\ \text{(seconds)} \end{pmatrix}} \right]^2 - \underset{\textbf{Rocket}}{\begin{bmatrix} \text{space} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}}^2 \qquad (1\text{-}5)$$

even though the respective space and time separations are not the same. Two observers find different space and time separations, respectively, between pen spark and fire-extinguisher spark, but when they calculate the spacetime interval between these sparks their results agree (Table 1-4).

The student surveyor found that invariance of distance was most simply written with both northward and eastward separations expressed in the same unit, the meter. Likewise, invariance of the spacetime interval is most simply written with space and

TABLE 1-4

"INVARIANT SPACETIME INTERVAL" FROM REFERENCE SPARK TO SPARK A

(Data from Table 1-3)

Laboratory measurements		Rocket measurements	
Time lapse 33.6900×10^{-9} seconds = 33.6900 nanoseconds		Time lapse 33.0228×10^{-9} seconds = 33.0228 nanoseconds	
Multiply by $c = 0.299792458$ meters per nanosecond to convert to meters: 10.1000 meters		Multiply by $c = 0.299792458$ meters per nanosecond to convert to meters: 9.9000 meters	
Square the value	102.010 (meters)^2	Square the value	98.010 (meters)^2
Spatial separation 2.000 meters		Spatial separation zero	
Square the value and subtract	− 4.000 (meters)^2	Square the value and subtract	− 0
Result of subtaction expressed as a number squared	= 98.010 (meters)^2 = (9.900 meters)^2	Result of subtaction expressed as a number squared	= 98.010 (meters)^2 = (9.900 meters)^2
This is the square of what measurement?	**9.900 meters**	This is the square of what measurement?	**9.900 meters**

SAME SPACETIME
INTERVAL
from the reference event

time separations expressed in the same unit. Time is converted to meters: t (meters) = $c \times t$ (seconds). Then the interval appears in simplified form:

$$(\text{interval})^2 = \begin{bmatrix} \text{time} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 - \begin{bmatrix} \text{space} \\ \text{separation} \\ \text{(meters)} \end{bmatrix}^2 \qquad (1\text{-}6)$$

The **invariance of the spacetime interval**—its independence of the state of motion of the observer—forces us to recognize that time cannot be separated from space. Space and time are part of a single entity, **spacetime.** Space has three dimensions: northward, eastward, and upward. Time has one dimension: onward! The interval combines all four dimensions in a single expression. The geometry of spacetime is truly four-dimensional.

Space and time are part of spacetime

To recognize the unity of spacetime we follow the procedure that makes a landscape take on depth—we look at it from several angles. That is why we compare space and time separations between events A and B as recorded by two different observers in relative motion.

Why the minus sign in the equation for the interval? Pythagoras tells us to ADD the squares of northward and eastward separations to get the square of the distance. Who tells us to SUBTRACT the square of the space separation between events from the square of their time separation in order to get the square of the spacetime interval?

Shocked? Then you're well on the way to understanding the new world of very fast motion! This world goes beyond the three-dimensional textbook geometry of Euclid, in which distance is reckoned from a sum of squares. In this book we use another kind of geometry, called **Lorentz geometry,** more real, more powerful than Euclid for the world of the very fast. In Lorentz geometry the squared space separation is combined with the squared time separation in a new way—by *subtraction.* The result is the square of a new unity called the *spacetime interval* between events. The numerical value of this interval is *invariant,* the same for all observers, no matter how fast they are moving past one another. Proof? Every minute of every day an experiment somewhere in the world demonstrates it. In Chapter 3 we derive the invariance of the spacetime interval—with its minus sign—from experiments. They show the finding that no experiment conducted in a closed room will reveal whether that room is "at rest" or "in motion" (Einstein's Principle of Relativity). We won't wait until then to cash in on the idea of interval. We can begin to enjoy the payoff right now.

SAMPLE PROBLEM 1-1

SPARKING AT A FASTER RATE

Another, even faster rocket follows the first, entering the front door, zipping down the long corridor, and exiting through the back doorway. Each time the rocket clock ticks it emits a spark. As before, the first spark jumps the 1 millimeter from the passing rocket antenna to the pen in the pocket of John, the laboratory observer. The second flash jumps when the rocket antenna reaches a doorknob 4.00000000 meters farther along the hall as measured by the laboratory observer, who records the time between these two sparks as 16.6782048 nanoseconds.

a. What is the time between sparks, measured in meters by John, the laboratory observer?

b. What is the value of the spacetime interval between the two events, calculated from John's laboratory measurements?

c. Predict: What is the value of the interval calculated from measurements in the new rocket frame?

d. What is the distance between sparks as measured in this rocket frame?

e. What is the time (in meters) between sparks as measured in this rocket frame? Compare with the time between the same sparks as measured by John in the laboratory frame.

f. What is the speed of this rocket as measured by John in the laboratory?

SOLUTION

a. Time in meters equals time in nanoseconds multiplied by the conversion factor, the speed of light in meters per nanosecond. For John, the laboratory observer,

$$16.6782048 \text{ nanoseconds} \times 0.299792458 \text{ meters/nanosecond} = 5.00000000 \text{ meters}$$

b. The square of the interval between two flashes is reckoned by subtracting the square of the space separation from the square of the time separation. Using laboratory figures:

$$\begin{aligned}(\text{interval})^2 &= (\text{laboratory time})^2 - (\text{laboratory distance})^2 \\ &= (5 \text{ meters})^2 - (4 \text{ meters})^2 = 25 \text{ (meters)}^2 - 16 \text{ (meters)}^2 \\ &= 9 \text{ (meters)}^2 = (3 \text{ meters})^2\end{aligned}$$

Therefore the interval between the two sparks has the value 3 meters (to nine significant figures).

c. We strongly assert in this chapter that the **spacetime interval is invariant**—has the same value by whomever calculated. Accordingly, the interval between the two sparks calculated from rocket observations has the same value as the interval (3 meters) calculated from laboratory measurements.

d. From the rocket rider's viewpoint, both sparks jump from the same place, namely the end of her antenna, and so distance between the sparks equals zero for the rocket rider.

e. We know the value of the spacetime interval between two sparks as computed in the rocket frame (**c**). And we know that the interval is computed by subtracting the square of the space separation from the square of the time separation in the rocket frame. Finally we know that the space separation in the rocket frame equals zero (**d**). Therefore the rocket time lapse between the two sparks equals the interval between them:

$$\begin{aligned}(\text{interval})^2 &= (\text{rocket time})^2 - (\text{rocket distance})^2 \\ (3 \text{ meters})^2 &= (\text{rocket time})^2 - (\text{zero})^2\end{aligned}$$

from which 3 meters equals the rocket time between sparks. Compare this with 5 meters of light-travel time between sparks as measured in the laboratory frame.

f. Measured in the laboratory frame, the rocket moves 4 meters of distance (statement of the problem) in 5 meters of light-travel time (**a**). Therefore its speed in the laboratory is 4/5 light speed. Why? Well, light moves 4 meters of distance in 4 meters of time. The rocket takes longer to cover this distance: 5 meters of time. Suppose that instead of 5 meters of time, the rocket had taken 8 meters of time, twice as long as light, to cover the 4 meters of distance. In that case it would be moving at 4/8—or half—the speed of light. In the present case the rocket travels the 4 meters of distance in 5 meters of time, so it moves at 4/5 light speed. Therefore its speed equals

$$(4/5) \times 2.99792458 \times 10^8 \text{ meters/second} = 2.3983397 \times 10^8 \text{ meters/second}$$

1.3 EVENTS AND INTERVALS ALONE!

tools enough to chart matter and motion without any reference frame

Surveying locates a place

In surveying, the fundamental concept is **place.** The surveyor drives a steel stake to mark the corner of a plot of land—to mark a place. A second stake marks another corner of the same plot—another place. Every surveyor—no matter what his or her standard of north—can agree on the value of the distance between the two stakes, between the two places.

Every stake has its own reality. Likewise the *distance* between every pair of stakes also has its own reality, which we can experience directly by pacing off the straight line from one stake to the other stake. The reading on our pedometer—the distance between stakes — is independent of all surveyors' systems, with their arbitrary choice of north.

More: Suppose we have a table of distances between every pair of stakes. That is all we need! From this table and the laws of Euclidean geometry, we can construct the map of every surveyor (see the exercises for this chapter). Distances between stakes: That is all we need to locate every stake, every place on the map.

Physics locates an event

In physics, the fundamental concept is **event.** The collision between one particle and another is an event, with its own location in spacetime. Another event is the emission of a flash of light from an atom. A third is the impact of the pebble that chips the windshield of a speeding car. A fourth event, likewise fixing in and by itself a location in spacetime, is the strike of a lightning bolt on the rudder of an airplane. An event marks a location in spacetime; it is like a steel stake driven into spacetime.

Every laboratory and rocket observer — no matter what his or her relative velocity — can agree on the spacetime interval between any pair of events.

Wristwatch measures interval directly

Every event has its own reality. Likewise the *interval* between every pair of events also has its own reality, which we can experience directly. We carry our wristwatch at constant velocity from one event to the other one. It is not enough just to pass through the two physical locations — we must pass through the actual *events;* we must be at each event precisely when it occurs. Then the space separation between the two events is zero for us — they both occur at our location. As a result, our wristwatch reads directly the spacetime interval between the pair of events:

$$(\text{interval})^2 = \begin{bmatrix}\text{time}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2 - \begin{bmatrix}\text{space}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2$$

$$= \begin{bmatrix}\text{time}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2 - [\text{zero}]^2 = \begin{bmatrix}\text{time}\\ \text{separation}\\ \text{(meters)}\end{bmatrix}^2 \quad \text{[wristwatch time]}$$

The time read on a wristwatch carried between two events — the interval between those events — is independent of all laboratory and rocket reference frames.

More: To chart all happenings, we need no more than a table of spacetime intervals between every pair of events. That is all we need! From this table and the laws of Lorentz geometry, it turns out, we can construct the space and time locations of events as observed by every laboratory and rocket observer. Intervals between events: That is all we need to specify the location of every event in spacetime.

"Do science" with intervals alone

In brief, we can completely describe and locate events entirely without a reference frame. We can analyze the physical world — we can "do science" — simply by cataloging every event and listing the interval between it and every other event. The unity of spacetime is reflected in the simplicity of entries in our table: intervals only.

Of course, if we want to use a reference frame, we can do so. We then list in our table the individual northward, eastward, upward, and time separations between pairs of events. However, these laboratory-frame listings for a given pair of events will be different from the corresponding listings that our rocket-frame colleague puts in her table. Nevertheless, we can come to agreement if we use the individual separations to reckon the interval between each pair of events:

$$(\text{interval})^2 = (\text{time separation})^2 - (\text{space separation})^2$$

That returns us to a universal, frame-independent description of the physical world.

When two events both occur at the position of a certain clock, that special clock measures directly the interval between these two events. The interval is called the **proper time** (or sometimes the **local time**). The special clock that records the proper time directly has the name **proper clock** for this pair of events. In this book we often call the proper time the **wristwatch time** and the proper clock the **wristwatch** to emphasize that the proper clock is carried so that it is "present" at each of the two events as the events occur.

In Einstein's German, the word for proper time is *Eigenzeit,* or "own-time," implying "one's very own time." The German word provides a more accurate description than the English. In English, the word "proper" has come to mean "following conventional rules." Proper time certainly does not do that!

Hey! I just thought of something: Suppose two events occur at the same time in my frame but very far apart, for example two handclaps, one in New York City and one in San Francisco. Since they are simultaneous in my frame, the time separation between handclaps is zero. But the space separation is not zero — they are separated by the width of a continent. Therefore the square of the interval is a negative number:

$$
\begin{aligned}
(\textit{interval})^2 &= (\textit{time separation})^2 - (\textit{space separation})^2 \\
&= (\textit{zero})^2 - (\textit{space separation})^2 = -(\textit{space separation})^2
\end{aligned}
$$

How can the square of the spacetime interval be negative?

In most of the situations described in the present chapter, there exists a reference frame in which two events occur at the same place. In these cases time separation predominates in all frames, and the interval squared will always be positive. We call these intervals **timelike intervals.**

Euclidean geometry adds squares in reckoning distance. Hence the result of the calculation, distance squared, is always positive, regardless of the relative magnitudes of north and east separations. Lorentz geometry, however, is richer. For your simultaneous handclaps in New York City and San Francisco, space separation between handclaps predominates. In such cases, the interval is called a **spacelike interval** and its form is altered to

$$(\text{interval})^2 = (\text{space separation})^2 - (\text{time separation})^2 \qquad \text{[when spacelike]}$$

This way, the squared interval is never negative.

The *timelike* interval is measured directly using a wristwatch carried from one event to the other in a special frame in which they occur at the *same place.* In contrast, a *spacelike* interval is measured directly using a rod laid between the events in a special frame in which they occur at the *same time.* This is the frame you describe in your example.

Spacelike interval or timelike interval: In either case the interval is invariant — has the same value when reckoned using rocket measurements as when reckoned using laboratory measurements. You may want to skim through Chapter 6 where timelike and spacelike intervals are described more fully.

1.4 SAME UNIT FOR SPACE AND TIME: METER, SECOND, MINUTE, OR YEAR

meter for particle accelerators; *minute* for planets; *year* for the cosmos

The parable of the surveyors cautions us to use the same unit to measure both space and time. So we use meter for both. Time can be measured in meters. Let a flash of light bounce back and forth between parallel mirrors separated by 0.5 meter of

Measure time in meters

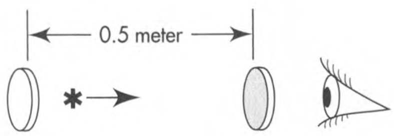


FIGURE 1-3. *This two-mirror "clock" sends to the eye flash after flash, each separated from the next by 1 meter of light-travel time. A light flash (represented by an asterisk) bounces back and forth between parallel mirrors separated from one another by 0.5 meter of distance. The silver coating of the right-hand mirror does not reflect perfectly: It lets 1 percent of the light pass through to the eye each time the light pulse hits it. Hence the eye receives a pulse of light every meter of light-travel time.*

distance (Figure 1-3). Such a device is a "clock" that "ticks" each time the light flash arrives back at a given mirror. Between ticks the light flash has traveled a round-trip distance of 1 meter. Therefore we call the stretch of time between ticks **1 meter of light-travel time** or more simply **1 meter of time.**

One meter of light-travel time is quite small compared to typical time lapses in our everyday experience. Light travels nearly 300 million meters per second (300,000,000 meters/second $= 3 \times 10^8$ meters/second, four fifths of the way to Moon in one second). Therefore one second equals 300 million meters of light-travel time. So 1 meter of light-travel time has the small value of one three-hundred-millionth of a second. [How come? Because (1) light goes 300 million meters in one second, and (2) one three-hundred-millionth of that distance (one meter!) is covered in one three-hundred-millionth of that time.] Nevertheless this unit of time is very useful when dealing with light and with high-speed particles. A proton early in its travel through a particle accelerator may be jogging along at "only" one half the speed of light. Then it travels 0.5 meter of distance in 1 meter of light-travel time.

We, our cars, even our jet planes, creep along at the pace of a snail compared with light. We call a deed quick when we've done it in a second. But a second for light means a distance covered of 300 million meters, seven trips around Earth. As we dance around the room to the fastest music, oh, how slow we look to light! Not zooming. Not dancing. Not creeping. Oozing! That long slow ooze racks up an enormous number of meters of light-travel time. That number is so huge that, by the end of one step of our frantic dance, the light that carries the image of the step's beginning is well on its way to Moon.

Meter officially defined using light speed

In 1983 the General Conference on Weights and Measures officially redefined the meter in terms of the speed of light. **The meter is now defined as the distance that light travels in a vacuum in the fraction 1/299,792,458 of a second.** (For the definition of the second, see Box 3-2.) Since 1983 the speed of light is, *by definition,* equal to $c = 299{,}792{,}458$ meters/second. This makes official the central position of the speed of light as a conversion factor between time and space.

This official action defines distance (meter) in terms of time (second). Every day we use time to measure distance. "My home is only ten minutes (by car) from work." "The business district is a five-minute walk." Each statement implies a speed — the speed of driving or walking — that converts distance to time. But these speeds can vary — for example, when we get caught in traffic or walk on crutches. In contrast, the speed of light in a vacuum does not vary. It always has the same value when measured over time and the same value as measured by every observer.

Measure distance in light-years

We often describe distances to stars and galaxies using a unit of time. These distances we measure in light-years. One light-year equals the distance that light travels in one year. Along with the light-year of space goes the year of time. Here again, space and time are measured in the same units — years. Here again the speed of light is the conversion factor between measures of time and space. From our everyday perspective one light-year of space is quite large, almost 10,000 million million meters: 1 light-year $= 9{,}460{,}000{,}000{,}000{,}000$ meters $= 0.946 \times 10^{16}$ meters. Nevertheless it is a convenient unit for measuring distance between stars. For example, the nearest star to our Sun, Proxima Centauri, lies 4.28 light-years away.

Any common unit of space or time may be used as the same unit for both space *and* time. For example, Table 1-5 gives us another convenient measure of time, seconds, compared with time in meters. We can also measure space in the same units, light-seconds. Our Sun is 499 light-seconds — or, more simply, 499 seconds — of distance from Earth. Seconds are convenient for describing distances and times among events that span the solar system. Alternatively we could use minutes of time and light-minutes of distance: Our Sun is 8.32 light-minutes from Earth. We can also use hours of time and light-hours of distance. In all cases, the speed of light is the conversion factor between units of space and time.

TABLE 1-5

SOME LIGHT-TRAVEL TIMES

	Time in seconds of light-travel time	*Time in meters*
Telephone call one way: New York City to San Francisco via surface microwave link	0.0138	4,139,000
Telephone call one way: New York City to San Francisco via Earth satellite	0.197	59,000,000
Telephone call one way: New York City to San Francisco bounced off Moon	2.51	752,000,000
Flash of light: Emitted by Sun, received on Earth	499.0	149,600,000,000

Expressing time and space in the same unit **meter** is convenient for describing motion of high-speed particles in the confines of the laboratory. Time and space in the same unit **second** (or **minute** or **hour**) is convenient for describing relations among events in our solar system. Time and space in the same unit **year** is convenient for describing relations among stars and among galaxies. In all three arenas spacetime is the stage and special relativity is the spotlight that illuminates the inner workings of Nature.

Use convenient units, the same for space and time

We are not accustomed to measuring time in meters. So as a reminder to ourselves we add a descriptor: meters *of light-travel time*. But the unit of time is still the meter. Similarly, the added words "seconds *of distance*" and "*light*-years" help to remind us that distance is measured in seconds or years, units we usually associate with time. But this unit of distance is really just second or year. The modifying descriptors are for our convenience only. In Nature, space and time form a unity: spacetime!

The words sound OK. The mathematics appears straightforward. The Sample Problems seem logical. But the ideas are so strange! *Why should I believe them? How can invariance of the interval be proved?*

No wonder these ideas seem strange. Particles zooming by at nearly the speed of light—how far this is from our everyday experience! Even the soaring jet plane crawls along at less than one-millionth light speed. Is it so surprising that the world appears different at speeds a million times faster than those at which we ordinarily move with respect to Earth?

The notion of *spacetime interval* distills a wealth of real experience. We begin with interval because it endures: It illuminates observations that range from the core of a nucleus to the center of a black hole. Understand the spacetime interval and you vault, in a single bound, to the heart of spacetime.

Chapter 3 presents a logical proof of the invariance of the interval. Chapter 4 reports a knock-down argument about it. Chapters that follow describe many experiments whose outcomes are totally incomprehensible unless the interval is invariant. Real verification comes daily and hourly in the on-going enterprise of experimental physics.

SAMPLE PROBLEM 1-2
PROTON, ROCK, AND STARSHIP

a. A proton moving at 3/4 light speed (with respect to the laboratory) passes through two detectors 2 meters apart. Events 1 and 2 are the transits through the two detectors. What are the laboratory space and time separations between the two events, in meters? What are the space and time separations between the events in the proton frame?

b. A speeding rock from space streaks through Earth's outer atmosphere, creating a short fiery trail (Event 1) and continues on its way to crash into Sun (Event 2) 10 minutes later as observed in the Earth frame. Take Sun to be 1.4960×10^{11} meters from Earth. In the Earth frame, what are space and time separations between Event 1 and Event 2 in minutes? What are space and time separations between the events in the frame of the rock?

c. In the twenty-third century a starship leaves Earth (Event 1) and travels at 95 percent light speed, later arriving at Proxima Centauri (Event 2), which lies 4.3 light-years from Earth. What are space and time separations between Event 1 and Event 2 as measured in the Earth frame, in years? What are space and time separations between these events in the frame of the starship?

SOLUTION

a. The space separation measured in the laboratory equals 2 meters, as given in the problem. A flash of light would take 2 meters of light-travel time to travel between the two detectors. Something moving at 1/4 light speed would take four times as long: 2 meters/(1/4) = 8 meters of light-travel time to travel from one detector to the other. The proton, moving at 3/4 light speed, takes 2 meters/(3/4) = 8/3 meters = 2.66667 meters of light-travel time between events as measured in the laboratory.

Event 1 and Event 2 both occur at the position of the proton. Therefore the space separation between the two events equals zero in the proton frame. This means that the spacetime interval—the proper time—equals the time between events in the proton frame.

$$\begin{aligned}
(\text{proton time})^2 - (\text{proton distance})^2 &= (\text{interval})^2 = (\text{lab time})^2 - (\text{lab distance})^2 \\
(\text{proton time})^2 - (\text{zero})^2 &= (2.66667 \text{ meters})^2 - (2 \text{ meters})^2 \\
&= (7.1111 - 4)\,(\text{meters})^2 \\
(\text{proton time})^2 &= 3.1111\,(\text{meters})^2
\end{aligned}$$

So time between events in the proton frame equals the square root of this, or 1.764 meters of time.

b. Light travels 60 times as far in one minute as it does in one second. Its speed in meters per minute is therefore:

$$\begin{aligned}
&2.99792458 \times 10^{8} \text{ meters/second} \times 60 \text{ seconds/minute} \\
&\qquad = 1.798754748 \times 10^{10} \text{ meters/minute}
\end{aligned}$$

So the distance from Earth to Sun is

$$\frac{1.4960 \times 10^{11} \text{ meters}}{1.798754748 \times 10^{10} \text{ meters/minute}} = 8.3169 \text{ light-minutes}$$

This is the distance between the two events in the Earth frame, measured in light-minutes. The Earth-frame time between the two events is 10 minutes, as stated in the problem.

In the frame traveling with the rock, the two events occur at the same place; the time between the two events in this frame equals the spacetime interval—the proper time—between these events:

$$\begin{aligned}(\text{interval})^2 &= (10 \text{ minutes})^2 - (8.3169 \text{ minutes})^2 \\ &= (100 - 69.1708)\ (\text{minutes})^2 \\ &= 30.8292\ (\text{minutes})^2\end{aligned}$$

The time between events in the rest frame of the rock equals the square root of this, or 5.5524 minutes.

c. The distance between departure from Earth and arrival at Proxima Centauri is 4.3 light-years, as given in the problem. The starship moves at 95 percent light speed, or 0.95 light-years/year. Therefore it takes a time 4.3 light-years/(0.95 light-years/year) = 4.53 years to arrive at Proxima Centauri, as measured in the Earth frame.

Starship time between departure from Earth and arrival at Proxima Centauri equals the interval:

$$\begin{aligned}(\text{interval})^2 &= (4.53 \text{ years})^2 - (4.3 \text{ years})^2 \\ &= (20.52 - 18.49)\ (\text{years})^2 \\ &= 2.03\ (\text{years})^2\end{aligned}$$

The time between events in the rest frame of the starship equals the square root of this, or 1.42 years. Compare with the value 4.53 years as measured in the Earth frame. This example illustrates the famous idea that astronaut wristwatch time —proper time—between two events is less than the time between these events measured by any other observer in relative motion. Travel to stay young! This result comes simply and naturally from the invariance of the interval.

1.5 UNITY OF SPACETIME

time and space: equal footing but distinct nature

When time and space are measured in the same unit—whether meter or second or year—the expression for the square of the spacetime interval between two events takes on a particularly simple form:

$$\begin{aligned}(\text{interval})^2 &= (\text{time separation})^2 - (\text{space separation})^2 \\ &= t^2 - x^2\end{aligned}$$

[same units for time and space]

Spacetime is a unity

This formula shows forth the unity of space and time. Impressed by this unity, Einstein's teacher Hermann Minkowski (1864–1909) wrote his famous words, "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a union of the two will preserve an independent reality." Today this union of space and time is called spacetime. Spacetime provides the true theater for

BOX 1-1

PAYOFF OF THE PARABLE

from distance in space to interval in spacetime

DISCUSSION	SURVEYING TOWNSHIP	ANALYZING NATURE
Location marker	Steel stake driven in ground	Collision between two particles Emission of flash from atom Spark jumping from antenna to pen
General name for such a location marker	**Point** or **place**	**Event**
Can its location be staked out for all to see, independent of any scheme of measurement, and independent of all numbers?	Yes	Yes
Simple descriptor of separation between two location markers	**Distance**	**Spacetime interval**
Are there ways directly to measure this separation?	Yes	Yes
With enough markers already staked out, how can we tell someone where we want the next one?	Specify **distances** from other points.	Specify **spacetime intervals** from other events.
Instead of boldly staking out the new marker, or instead of positioning it relative to existing markers, how else can we place the new marker?	By locating point relative to a **reference frame**	By locating event relative to a **reference frame**
Nature of this reference frame?	**Surveyor's grid** yields northward and eastward readings of point (Chapter 1).	**Lattice frame** of rods and clocks yields space and time readings of event (Chapter 2).
Is such a reference frame unique?	No	No
How do two such reference frames differ from one another?	**Tilt** of one surveyor's grid relative to the other	**Uniform velocity** of one frame relative to the other
What are names of two such possible reference frames?	**Daytime grid:** oriented to magnetic north **Nighttime grid:** oriented to North-Star north	**Laboratory frame** **Rocket frame**
What common unit simplifies analysis of the results?	The unit **meter** for both northward and eastward readings	The unit **meter** for both space and time readings
What is the conversion factor from conventional units to meters?	Converting miles to meters: $k = 1609.344$ meters/mile	Converting seconds to meters using the speed of light: $c = 299{,}792{,}458$ meters/second

DISCUSSION	SURVEYING TOWNSHIP	ANALYZING NATURE
For convenience, all measurements are referred to what location?	A common **origin** (center of town)	A common **event** (reference spark)
How do readings for a single marker differ between two reference frames?	Individual northward and eastward readings for one point—for one steel stake—do not have the same values respectively for two surveyors' grids that are tilted relative to one another.	Individual space and time readings for one event—for one spark—do not have the same values respectively for two frames that are in motion relative to one another.
When we change from one marker to two, how do we specify the offset between them in reference-frame language?	**Subtract:** Figure the difference between eastward readings of the two points; also the difference in northward readings.	**Subtract:** Figure the difference between space readings of the two events; also the difference in time readings.
How to figure from offset readings a measure of separation that has the same value whatever the choice of reference frame?	Figure the **distance** between the two points.	Figure the **spacetime interval** between the two events.
Figure how?	$(\text{distance})^2 = \left(\begin{matrix}\text{difference in}\\ \text{northward readings}\end{matrix}\right)^2 + \left(\begin{matrix}\text{difference in}\\ \text{eastward readings}\end{matrix}\right)^2$	$(\text{interval})^2 = \left(\begin{matrix}\text{difference in}\\ \text{time readings}\end{matrix}\right)^2 - \left(\begin{matrix}\text{difference in}\\ \text{space readings}\end{matrix}\right)^2$
Result of this reckoning?	Distance between points as figured from readings using one surveyor's grid is the **same** as figured from readings using a second surveyor's grid tilted with respect to first grid.	Interval between events as figured from readings using one latticework frame is the **same** as figured from readings using a second frame in steady straight-line motion relative to first frame.
Phrase to summarize this identity of separation as figured in two reference frames?	**Invariance of the distance** between points	**Invariance of the spacetime interval** between events.
Conclusions from this analysis?	(1) Northward and eastward dimensions are part of a single entity: **space.** (2) **Distance** is the simple measure of separation between two **points,** natural because invariant: the same for different surveyor grids.	(1) Space and time dimensions are part of a single entity: **spacetime.** (2) **Spacetime interval** is the simple measure of separation between two **events,** natural because invariant: the same for different reference frames.

every event in the lives of stars, atoms, and people. Space is different for different observers. Time is different for different observers. Spacetime is the same for everyone.

Minkowski's insight is central to the understanding of the physical world. It focuses attention on those quantities, such as spacetime interval, electrical charge, and particle mass, that are the same for all observers in relative motion. It brings out the merely relative character of quantities such as velocity, momentum, energy, separation in time, and separation in space that depend on relative motion of observers.

Today we have learned not to overstate Minkowski's argument. It is right to say that time and space are inseparable parts of a larger unity. It is wrong to say that time is identical in quality with space.

Difference between time and space

Why is it wrong? Is not time measured in meters, just as space is? In relating the positions of two steel stakes driven into the ground, does not the surveyor measure northward and eastward separations, quantities of identical physical character? By analogy, in locating two events is not the observer measuring quantities of the same nature: space and time separations? How else could it be legitimate to treat these quantities on an equal footing, as in the formula for the interval?

Equal footing, yes; same nature, no. There is a minus sign in the formula for the interval squared = (time separation)^2 − (space separation)^2 that no sleight of hand can ever conjure away. This minus sign distinguishes between space and time. No twisting or turning can ever give the same sign to real space and time separations in the expression for the interval.

The invariance of the spacetime interval evidences the unity of space and time while also preserving—in the formula's minus sign—the distinction between the two.

The principles of special relativity are remarkably simple—simpler than the axioms of Euclidean geometry or the principles of operating an automobile. Yet both Euclid and the automobile have been mastered—perhaps with insufficient surprise—by generations of ordinary people. Some of the best minds of the twentieth century struggled with the concepts of relativity, not because nature is obscure, but because (1) people find it difficult to outgrow established ways of looking at nature, and (2) the world of the very fast described by relativity is so far from common experience that everyday happenings are of limited help in developing an intuition for its descriptions.

By now we have won the battle to put relativity in understandable form. The concepts of relativity can now be expressed simply enough to make it easy to think correctly—"to make the bad difficult and the good easy." This leaves only the second difficulty, that of developing intuition—a practiced way of seeing. We understand distance intuitively from everyday experience. Box 1.1 applies our intuition for **distance in space** to help our intuition for **interval in spacetime.**

To put so much into so little, to subsume all of Einstein's teaching on light and motion in the single word *spacetime*, is to cram a wealth of ideas into a small picnic basket that we shall be unpacking throughout the remainder of this book.

REFERENCES

Introductory quote: Richard P. Feynman, *The Character of Physical Law* (MIT Press, Cambridge, Mass., 1967), page 127.

Quote from Minkowski in Section 1.5: H. A. Minkowski, "Space and Time," in H. A. Lorentz *et al.*, *The Principle of Relativity* (Dover Publications, New York, 1952), page 75.

Quote at end of Section 1.5: "to make the bad difficult and the good easy," *"rend le mal difficile et le bien facile."* Einstein, in a similar connection, in a letter to the architect Le Corbusier. Private communication from Le Corbusier.

For an appreciation of Albert Einstein, see John Archibald Wheeler, "Albert Einstein," in *The World Treasury of Physics, Astronomy, and Mathematics,* Timothy Ferris, ed. (Little, Brown, New York, 1991), pages 563–576.

ACKNOWLEDGMENTS

Many students in many classes have read through sequential versions of this text, shared with us their detailed difficulties, and given us advice. We asked students to write down comments, perplexities, and questions as they read and turn in these reading memos for personal response by the teacher. Italicized objections in the text come, in part, from these commentators. Both we who write and you who read are in their debt. Some readers not in classes have also been immensely helpful; among these we especially acknowledge Steven Bartlett. No one could have read the chapters more meticulously than Eric Sheldon, whose wide knowledge has enriched and clarified the presentation. William A. Shurcliff has been immensely inventive in devising new ways of viewing the consequences of relativity; a few of these are specifically acknowledged in later chapters. Electronic-mail courses using this text brought a flood of comments and reading memos from teachers and students around the world. Richard C. Smith originated, organized, and administered these courses, for which we are very grateful. The clarity and simplicity of both the English and the physics were improved by Penny Hull.

Some passages in this text, both brief and extended, have been adapted from the book *A Journey into Gravity and Spacetime* by John Archibald Wheeler (W.H. Freeman, New York, 1990). In turn, certain passages in that book were adapted from earlier drafts of the present text. We have also used passages, logical arguments, and figures from the book *Gravitation* by Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler (W. H. Freeman, New York, 1973).

INTRODUCTION TO THE EXERCISES

Important areas of current research can be analyzed very simply using the theory of relativity. This analysis depends heavily on a physical intuition, which develops with experience. Wide experience is not easy to obtain in the laboratory—simple experiments in relativity are difficult and expensive because the speed of light is so great. As alternatives to experiments, the exercises and problems in this text evoke a wide range of physical consequences of the properties of spacetime. These properties of spacetime recur here over and over again in different contexts:

- paradoxes
- puzzles

- derivations
- technical applications
- experimental results
- estimates
- precise calculations
- philosophical difficulties

The text presents all formal tools necessary to solve these exercises and problems, but intuition — a practiced way of seeing — is best developed without hurry. For this reason we suggest continuing to do more and more of these exercises in relativity after you have moved on to material outside this book. The mathematical manipulations in the exercises and problems are very brief: only a few answers take more than five lines to write down. On the other hand, the exercises require some "rumination time."

In some chapters, exercises are divided into two categories, Practice and Problems. The Practice exercises help you to get used to ideas in the text. The Problems apply these ideas to physical systems, thought experiments, and paradoxes.

WHEELER'S FIRST MORAL PRINCIPLE: *Never make a calculation until you know the answer.* Make an estimate before every calculation, try a simple physical argument (symmetry! invariance! conservation!) before every derivation, guess the answer to every paradox and puzzle. Courage: No one else needs to know what the guess is. Therefore make it quickly, by instinct. A right guess reinforces this instinct. A wrong guess brings the refreshment of surprise. In either case life as a spacetime expert, however long, is more fun!

CHAPTER 1 EXERCISES

PRACTICE

1-1 comparing speeds

Compare the speeds of an automobile, a jet plane, an Earth satellite, Earth in its orbit around Sun, and a pulse of light. Do this by comparing the relative distance each travels in a fixed time. Arbitrarily choose the fixed time to give convenient distances. A car driving at the USA speed limit of 65 miles/hour (105 kilometers/hour) covers 1 meter of distance in about 35 milliseconds = 35×10^{-3} second.

a How far does a commercial jetliner go in 35 milliseconds? (speed: 650 miles/hour = 1046 kilometers/hour)

b How far does an Earth satellite go in 35 milliseconds? (speed: 17,000 miles/hour ≈ 27,350 kilometers/hour)

c How far does Earth travel in its orbit around Sun in 35 milliseconds? (speed: 30 kilometers/second)

d How far does a light pulse go in a vacuum in 35 milliseconds? (speed: 3×10^8 meters /second). This distance is roughly how many times the distance from Boston to San Francisco (5000 kilometers)?

1-2 images from Neptune

At 9:00 P.M. Pacific Daylight Time on August 24, 1989, the planetary probe *Voyager II* passed by the planet Neptune. Images of the planet were coded and transmitted to Earth by microwave relay.

It took 4 hours and 6 minutes for this microwave signal to travel from Neptune to Earth. Microwaves (electromagnetic radiation, like light, but of frequency lower than that of visible light), when propagating through interplanetary space, move at the "standard" light speed of one meter of distance in one meter of light-travel time, or 299,792,458 meters/second. In the following, neglect any relative motion among Earth, Neptune, and *Voyager II*.

a Calculate the distance between Earth and Neptune at fly-by in units of minutes, seconds, years, meters, and kilometers.

b Calculate the time the microwave signal takes to reach Earth. Use the same units as in part **a**.

1-3 units of spacetime

Light moves at a speed of 3.0×10^8 meters/second. One mile is approximately equal to 1600 meters. One furlong is approximately equal to 200 meters.

a How many meters of time in one day?
b How many seconds of distance in one mile?
c How many hours of distance in one furlong?
d How many weeks of distance in one light-year?
e How many furlongs of time in one hour?

1-4 time stretching and the spacetime interval

A rocket clock emits two flashes of light and the rocket observer records the time lapse (in seconds) between these two flashes. The laboratory observer records the time separation (in seconds) and space separation (in light-seconds) between the same pair of flashes. The results for both laboratory and rocket observers are recorded in the first line of the table.

Now a clock in a different rocket, moving at a different speed with respect to the laboratory, emits a different pair of flashes. The set of laboratory and rocket space and time separations are recorded on the second line of the table. And so on. Complete the table.

EXERCISE 1-4

SPACE AND TIME SEPARATIONS

	Rocket time lapse (seconds)	Laboratory time lapse (seconds)	Laboratory distance (light-seconds)
Example	20	29	21
a	?	10.72	5.95
b	20	?	99
c	66.8	72.9	?
d	?	8.34	6.58
e	21	22	?

1-5 where and when?

Two firecrackers explode at the same place in the laboratory and are separated by a time of 3 years as measured on a laboratory clock.

a What is the spatial distance between these two events in a rocket in which the events are separated in time by 5 years as measured on rocket clocks?

b What is the relative speed of the rocket and laboratory frames?

1-6 mapmaking in space

The table shows distances between cities. The units are kilometers. Assume all cities lie on the same flat plane.

a Use a ruler and a compass (the kind of compass that makes circles) to construct a map of these cities. Choose a convenient scale, such as one centimeter on the map corresponds to ten kilometers on Earth.

Discussion: How to start? With three arbitrary decisions! (1) Choose any city to be at the center of the map. (2) Choose any second city to be "due north" — that is, along any arbitrary direction you select. (3) Even with these choices, there are two places you can locate the third city; choose either of these two places arbitrarily.

b If you rotate the completed map in its own plane — for example, turning it while keeping it flat on the table — does the resulting map also satisfy the distance entries above?

c Hold up your map between you and a light, with the marks on the side of the paper facing the

EXERCISE 1-6

DISTANCES BETWEEN CITIES

Distance to city / from city	A	B	C	D	E	F	G	H
A	0	20.0	28.3	28.3	28.3	20.0	28.3	44.7
B		0	20.0	20.0	44.7	40.0	44.7	40.0
C			0	40.0	40.0	44.7	56.6	60.0
D				0	56.6	44.7	40.0	20.0
E					0	20.0	40.0	72.1
F						0	20.0	56.6
G							0	44.7
H								0

light. Does the map you see from the back also satisfy the table entries?

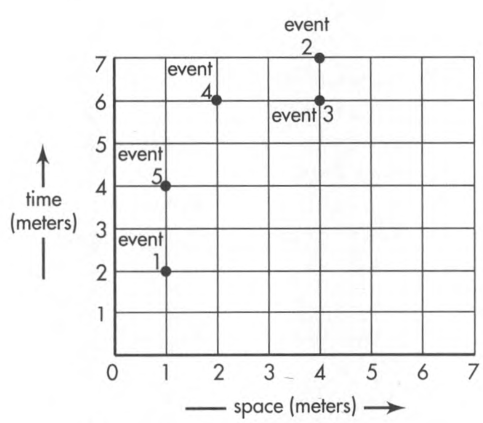
Discussion: In this exercise you use a table consisting only of distances between pairs of cities to construct a map of these cities from the point of view of a surveyor using a given direction for north. In Exercise 5-3 you use a table consisting only of spacetime intervals between pairs of events to draw a "spacetime map" of these events from the point of view of one free-float observer. Exercise 1-7 previews this kind of spacetime map.

1-7 spacetime map

The laboratory space and time measurements of events 1 through 5 are plotted in the figure. Compute the value of the spacetime interval

a between event 1 and event 2.
b between event 1 and event 3.
c between event 1 and event 4.
d between event 1 and event 5.

e A rocket moves with constant velocity from event 1 to event 2. That is, events 1 and 2 occur at the same place in this rocket frame. What time lapse is recorded on the rocket clock between these two events?



EXERCISE 1-7. *Spacetime map of some events.*

PROBLEMS

1-8 size of a computer

In one second some desktop computers can carry out one million instructions in sequence. One instruction might be, for instance, multiplying two numbers together. In technical jargon, such a computer operates at "one megaflop." Assume that carrying out one instruction requires transmission of data from the memory (where data is stored) to the processor (where the computation is carried out) and transmission of the result back to the memory for storage.

a What is the maximum average distance between memory and processor in a "one-megaflop" computer? Is this maximum distance increased or decreased if the signal travels through conductors at one half the speed of light in a vacuum?

b Computers are now becoming available that operate at "one gigaflop," that is, they carry out 10^9 sequential instructions per second. What is the maximum average distance between memory and processor in a "one-gigaflop" machine?

c Estimate the overall maximum size of a "one-teraflop" machine, that is, a computer that can carry out 10^{12} sequential instructions per second.

d **Discussion question:** In contrast with most current personal computers, a "parallel processing" computer contains several or many processors that work together on a computing task. One might think that a machine with 10,000 processors would complete a given computation task in 1/10,000 the time. However, many computational problems cannot be divided up in this way, and in any case some fraction of the computing capacity must be devoted to coordinating the team of processors. What limits on physical size does the speed of light impose on a parallel processing computer?

1-9 trips to Andromeda by rocket

The Andromeda galaxy is approximately two million light-years distant from Earth as measured in the Earth-linked frame. Is it possible for you to travel from Earth to Andromeda in your lifetime? Sneak up on the answer to this question by considering a series of trips from Earth to Andromeda, each one faster than the one before. For simplicity, assume the Earth-Andromeda distance to be exactly two million light-years in the Earth frame, treat Earth and Andromeda as points, and neglect any relative motion between Earth and Andromeda.

a TRIP 1. Your one-way trip takes a time 2.01×10^6 years (measured in the Earth-linked frame) to cover the distance of 2.00×10^6 light-years. How long does the trip last as measured in your rocket frame?

b What is your rocket speed on Trip 1 as measured in the Earth-linked frame? Express this speed as a decimal fraction of the speed of light. Call this fraction, $v = v_{conv}/c$, where v_{conv} is speed in conventional units, such as meters/second. **Discussion:** If your rocket moves at half the speed of light, it takes

4×10^6 years to cover the distance 2×10^6 light-years. In this case

$$v = \frac{2 \times 10^6 \text{ light-years}}{4 \times 10^6 \text{ years}} = \frac{1}{2}$$

Therefore . . .

c TRIP 2. Your one-way Earth-Andromeda trip takes 2.001×10^6 years as measured in the Earth-linked frame. How long does the trip last as measured in your rocket frame? What is your rocket speed for Trip 2, expressed as a decimal fraction of the speed of light?

d TRIP 3. Now set the rocket time for the one-way trip to 20 years, which is all the time you want to spend getting to Andromeda. In this case, what is your speed as a decimal fraction of the speed of light?

Discussion: Solutions to many exercises in this text are simplified by using the following approximation, which is the first two terms in the binomial expansion

$$(1 + z)^n \approx 1 + nz \qquad \text{if} \qquad |z| << 1$$

Here n can be positive or negative, a fraction or an integer; z can be positive or negative, as long as its magnitude is very much smaller than unity. This approximation can be used twice in the solution to part **d**.

1-10 trip to Andromeda by Transporter

In the *Star Trek* series a so-called Transporter is used to "beam" people and their equipment from a starship to the surface of nearby planets and back. The Transporter mechanism is not explained, but it appears to work only locally. (If it could transport to remote locations, why bother with the starship at all?) Assume that one thousand years from now a Transporter exists that reduces people and things to data (elementary bits of information) and transmits the data by light or radio signal to remote locations. There a Receiver uses the data to reassemble travelers and their equipment out of local raw materials.

One of your descendants, named Samantha, is the first "transporternaut" to be beamed from Earth to the planet Zircon orbiting a star in the Andromeda Nebula, two million light-years from Earth. Neglect any relative motion between Earth and Zircon, and assume: (1) transmission produces a Samantha identical to the original in every respect (except that she is 2 million light-years from home!), and (2) the time required for disassembling Samantha on Earth and reassembling her on Zircon is negligible as measured in the common rest frame of Transporter and Receiver.

a How much does Samantha age during her outward trip to Zircon?

b Samantha collects samples and makes observations of the Zirconian civilization for one Earth-year, then beams back to Earth. How much has Samantha aged during her entire trip?

c How much older is Earth and its civilization when Samantha returns?

d Earth has been taken over by a tyrant, who wishes to invade Zircon. He sends one warrior and has him duplicated into attack battalions at the Receiver end. How long will the Earth tyrant have to wait to discover whether his ambition has been satisfied?

e A second transporternaut is beamed to a much more remote galaxy that is moving away from Earth at 87 percent of the speed of light. This time, too, the traveler stays in the remote galaxy for one year *as measured by clocks moving with the galaxy* before returning to Earth by Transporter. How much has the transporternaut aged when she arrives back at Earth? (Careful!)

1-11 time stretching with muons

At heights of 10 to 60 kilometers above Earth, cosmic rays continually strike nuclei of oxygen and nitrogen atoms and produce muons (muons: elementary particles of mass equal to 207 electron masses produced in some nuclear reactions). Some of the muons move vertically downward with a speed nearly that of light. Follow one of the muons on its way down. In a given sample of muons, half of them decay to other elementary particles in 1.5 microseconds (1.5×10^{-6} seconds), measured with respect to a reference frame in which they are at rest. Half of the remainder decay in the next 1.5 microseconds, and so on. Analyze the results of this decay as observed in two different frames. Idealize the rather complicated actual experiment to the following roughly equivalent situation: All the muons are produced at the same height (60 kilometers); all have the same speed; all travel straight down; none are lost to collisions with air molecules on the way down.

a Approximately how long a time will it take these muons to reach the surface of Earth, as measured in the Earth frame?

b If the decay time were the same for Earth observers as for an observer traveling with the muons, approximately how many half-lives would have passed? Therefore what fraction of those created at a height of 60 kilometers would remain when they reached sea level on Earth? You may express your answer as a power of the fraction 1/2.

c An experiment determines that the fraction 1/8 of the muons reaches sea level. Call the rest frame of the muons the rocket frame. In this rocket frame, how many half-lives have passed between creation of a given muon and its arrival as a survivor at sea level?

d *In the rocket frame,* what is the space separation between birth of a survivor muon and its arrival at the surface of Earth? (Careful!)

e From the rocket space and time separations, find the value of the spacetime interval between the birth event and the arrival event for a single surviving muon.

Reference: Nalini Easwar and Douglas A. MacIntire, *American Journal of Physics,* Volume 59, pages 589–592 (July 1991).

1-12 time stretching with π^+-mesons

Laboratory experiments on particle decay are much more conveniently done with π^+-mesons (pi-plus mesons) than with μ-mesons, as is seen in the table.

In a given sample of π^+-mesons half will decay to other elementary particles in 18 nanoseconds (18×10^{-9} seconds) measured in a reference frame in which the π^+-mesons are at rest. Half of the remainder will decay in the next 18 nanoseconds, and so on.

a In a particle accelerator π^+-mesons are produced when a proton beam strikes an aluminum target inside the accelerator. Mesons leave this target with nearly the speed of light. If there were no time stretching and if no mesons were removed from the resulting beam by collisions, what would be the greatest distance from the target at which half of the mesons would remain undecayed?

b The π^+-mesons of interest in a particular experiment have a speed 0.9978 that of light. By what factor is the predicted distance from the target for half-decay increased by time dilation over the previous prediction — that is, by what factor does this dilation effect allow one to increase the separation between the detecting equipment and target?

EXERCISE 1-12

TIME STRETCHING WITH π^+-MESONS

Particle	Time for half to decay (measured in rest frame)	"Characteristic distance" (speed of light multiplied by foregoing time)
muon (207 times electron mass)	1.5×10^{-6} second	450 meters
π^+-meson (273 times electron mass)	18×10^{-9} second	5.4 meters

CHAPTER 2

FLOATING FREE

At that moment there came to me the happiest thought of my life . . . *for an observer falling freely from the roof of a house no gravitational field exists during his fall* . . .

Albert Einstein

2.1 FLOATING TO MOON

will the astronaut stand on the floor — or float?

Less than a month after the surrender at Appomattox ended the American Civil War (1861 – 1865), the French author Jules Verne began writing *A Trip From the Earth to the Moon* and *A Trip Around the Moon*. Eminent American cannon designers, so the story goes, cast a great cannon in a pit, with cannon muzzle pointing skyward. From this cannon they fire a ten-ton projectile containing three men and several animals (Figure 2-1).

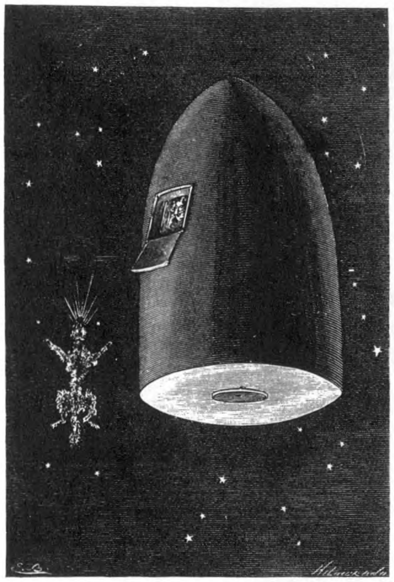
Jules Verne: Passenger stands on floor

As the projectile coasts outward in unpowered flight toward Moon, Verne says, its passengers walk normally inside the projectile on the end nearer Earth (Figure 2-2). As the trip continues, passengers find themselves pressed less and less against the floor of the spaceship until finally, at the point where Earth and Moon exert equal but opposite gravitational attraction, passengers float free of the floor. Later, as the ship nears Moon, they walk around once again — according to Verne — but now against the end of the spaceship nearer Moon.

Early in the coasting portion of the trip a dog on the ship dies from injuries sustained at takeoff. Passengers dispose of its remains through a door in the spaceship, only to find the body floating outside the window during the entire trip (Figure 2-1).

Paradox of passenger and dog

This story leads to a paradox whose resolution is of crucial importance to relativity. Verne thought it reasonable that Earth's gravitational attraction would keep a passenger pressed against the Earth end of the spaceship during the early part of the trip. He also thought it reasonable that the dog should remain next to the ship, since both ship and dog independently follow the same path through space. But since the dog floats outside the spaceship during the entire trip, why doesn't the passenger float around inside the spaceship? If the ship were sawed in half would the passenger, now "outside," float free of the floor?



IT WAS THE BODY OF SATELLITE.

FIGURE 2-1. ***Illustration from an early edition of* A Trip Around the Moon.** *Satellite is the name of the unfortunate dog.*

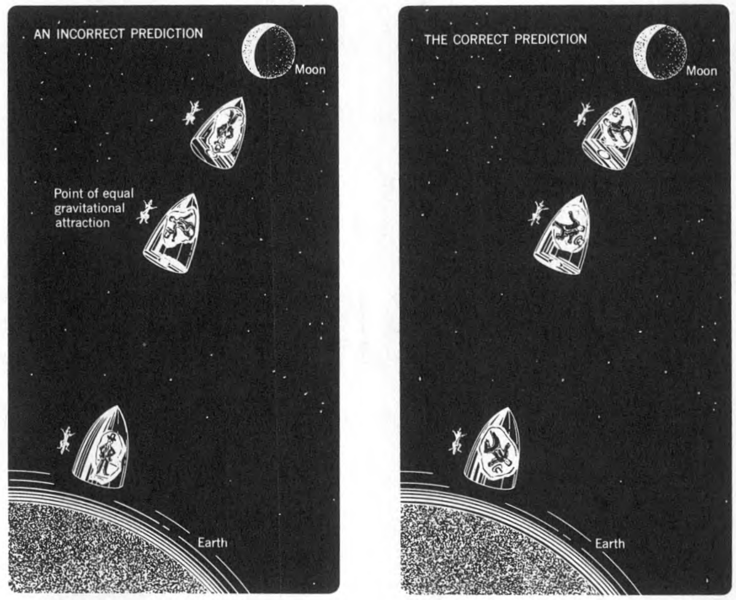


FIGURE 2-2. ***Incorrect prediction:*** *Jules Verne believed that a passenger inside a free projectile would stand against the end of the projectile nearest Earth or Moon, whichever had greater gravitational attraction — but that the dog would float along beside the projectile for the entire trip.* ***Correct prediction:*** *Verne was right about the dog, but a passenger also floats with respect to the free projectile during the entire trip.*

Reality:
Passenger floats in spaceship

Our experience with actual space flights enables us to resolve this paradox (Figure 2-2). Jules Verne was wrong about the passenger's motion inside the unpowered spaceship. Like the dog outside, the passenger inside independently follows the same path through space as the spaceship itself. Therefore he floats freely relative to the ship during the entire trip (after the initial boost inside the cannon barrel). True: Earth's gravity acts on the passenger. But it also acts on the spaceship. In fact, with respect to Earth, gravitational acceleration of the spaceship just equals gravitational acceleration of the passenger. Because of this equality, there is no *relative* acceleration between passenger and spaceship. Thus the spaceship serves as a **reference frame** relative to which the passenger does not experience any acceleration.

To say that acceleration of the passenger relative to the unpowered spaceship equals zero is *not* to say that his velocity relative to it necessarily also equals zero. He may jump from the floor or spring from the side — in which case he hurtles across the spaceship and strikes the opposite wall. However, when he floats with zero initial velocity relative to the ship the situation is particularly interesting, for he will also float with zero velocity relative to it at all later times. He and the ship follow identical paths through space. How remarkable that the passenger, who cannot see outside, nevertheless moves on this deterministic orbit! Without a way to control his motion and even with his eyes closed he will not touch the wall. How could one do better at eliminating detectable gravitational influences?

2.2 THE INERTIAL (FREE-FLOAT) FRAME

goodbye to the "force of gravity"

It is easy to talk about the simplicity of motion in a spaceship. It is hard to think of conditions being equally simple on the surface of Earth (Figure 2-3). The reason for

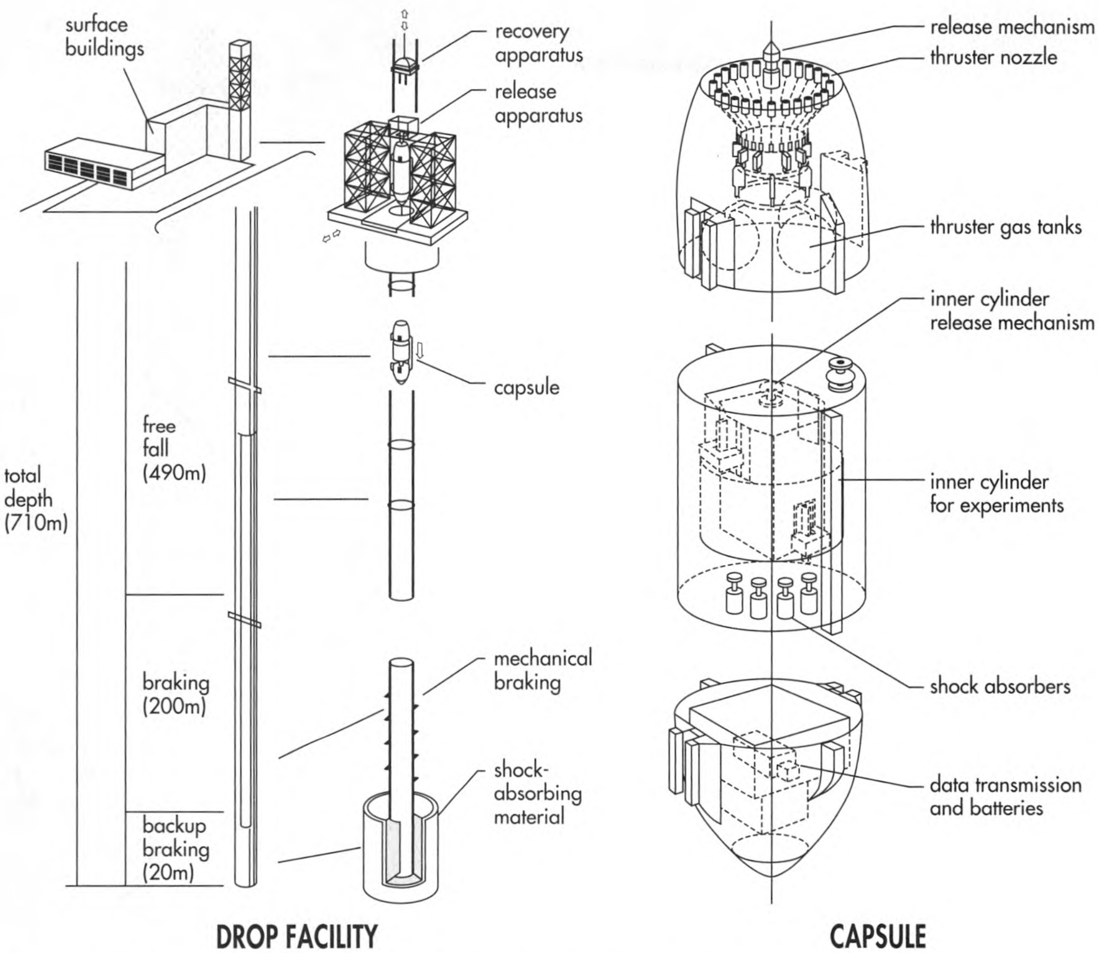


FIGURE 2-3. ***The Japan Microgravity Center (JAMIC) installed in an abandoned coal mine 710 meters deep in the small town of Kamisunagawa on the northern island of Hokkaido, Japan.*** *The capsule carrying the experimental apparatus provides a free-float frame for 10 seconds as it falls 490 meters through a vertical tube, achieving a maximum velocity of nearly 100 meters/second. It is guided by two contact-free magnetic suspensions along the tube. The vertical tube is not evacuated; downward-thrusting gas jets on the capsule compensate for air drag as the capsule drops. The capsule is slowed down in an additional distance of 200 meters near the bottom of the tube by air resistance after thrusters are turned off, followed by mechanical braking. Twenty meters of cushioning material at the very bottom of the tube provide emergency stopping. The falling capsule is nearly 8 meters long and nearly 2 meters in diameter with a mass of 5000 kilograms, including 1000 kilograms of experimental equipment contained in an inner cylinder 1.3 meters in diameter and 1.8 meters long. The space between capsule and experimental cylinder is evacuated. The inner experimental cylinder is released just before the outer capsule itself. Optical monitoring of the vertical position of the inner cylinder triggers downward-pushing thrusters as needed to overcome air resistance. Thus the experimental cylinder itself acts as an internal "conscience," ensuring that the capsule takes the same course that it would have taken had both resistance and thrust been absent. The result? A nearly free-float frame, with a maximum acceleration of* 1.0×10^{-4} g *in the experimental capsule, where* g *is the acceleration of gravity at Earth's surface. Experiments carried out in this facility benefit from conditions of "no air pressure, no heat convection, no floating or sinking buoyancy, no resistance to motion," as well as much lower cost and less environmental damage than those involved in launching and monitoring an Earth satellite. The facility is designed to carry out 400 drops per year, with experiments such as forming large superconducting crystals, creating alloys of materials that do not normally mix, studying transitions between gas and liquid phases, and burning under zero-*g. *(See also Figure 9-2.)*

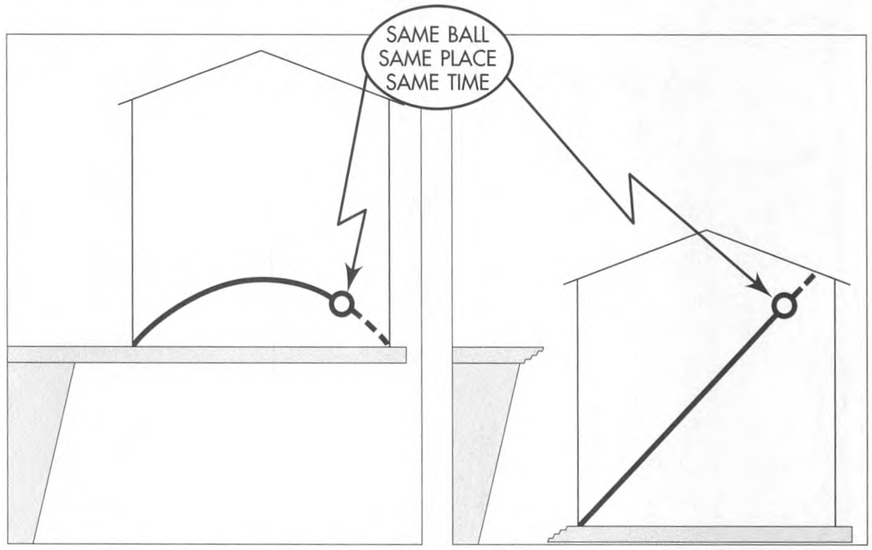


FIGURE 2-4. ***Illusion and Reality.*** *The same ball thrown from the same corner of the same room in the same direction with the same speed is seen to undergo very different motions depending on whether it is recorded by an observer with a floor pushing up against his feet or by an observer in "free fall" ("free float") in a house sawed free from the cliff. In both descriptions the ball arrives at the same place—relative to Mother Earth—at the same instant. Let each ball squirt a jet of ink on the wall we are looking at. The resulting record is as crisp for the arc as for the straight line. Is the arc real and the straight line illusion? Or is the straight line real and the arc illusion? Einstein tells us that the two ink trails are equally valid. We have only to be honest and say whether the house, the wall, and the describer of the motion are in free float or whether the describer is continually being driven away from a condition of free float by a push against his feet. Einstein also tells us that physics always looks simplest in a free-float frame. Finally, he tells us that every truly local manifestation of "gravity" can be eliminated by observing motion from a frame of reference that is in free float.*

concern is not far to seek. We experience it every day, every minute, every second. We call it gravity. It shows in the arc of a ball tossed across the room (Figure 2-4, left). How can anyone confront a mathematical curve like that arc and not be trapped again in that tortuous trail of thought that led from ancient Greeks to Galileo to Newton? They thought of gravity as a force acting through space, as something mysterious, as something that had to be "explained."

Einstein put forward a revolutionary new idea. Eliminate gravity!

Concept of free-float frame

Where lies the cause of the curved path of the ball? Is it the ball? Is it some mysterious "force of gravity"? Neither, Einstein tells us. It is the fault of the viewers—and the fault of the floor that forces us away from the natural state of motion: the state of **free fall,** or better put, **free float.** Remove the floor and our motion immediately becomes natural, effortless, free from gravitational effects.

Let the room be cut loose at the moment we throw the ball slantwise upward from the west side at floor level (Figure 2-4, right). The ball has the same motion as it did before. However, the motion looks different. It looks different because we who look at it are in a different frame of reference. We are in a **free-float frame.** In this free-float frame the ball has straight-line motion. What could be simpler?

Even when the room was not cut away from the cliff, the floor did not affect the midair flight of the ball. But the floor did affect us who watched the flight. The floor forced us away from our natural motion, the motion of free fall (free float). We blamed the curved path of the ball on the "force of gravity" acting on the *ball.* Instead we should have blamed the floor for its force acting on *us.* Better yet, get rid of the floor by cutting the house away from the cliff. Then our point of view becomes the natural one: We enter a free-float frame. In our free-float frame the ball flies straight.

What's the fault of the force on my feet?
What pushes my feet down on the floor?
Says Newton, the fault's at Earth's core.
Einstein says, the fault's with the floor;
Remove that and gravity's beat!

—Frances Towne Ruml

How could humankind have lived so many centuries without realizing that the "arc" is an unnecessary distraction, that the idea of local "gravity" is superfluous—the fault of the observer for not arranging to look at matters from a condition of free float?

Even today we recoil instinctively from the experience of free float. We and a companion ride in the falling room, which does not crash on the ground but drops into a long vertical tunnel dug for that purpose along the north–south axis of Earth. Our companion is so filled with consternation that he takes no interest in our experimental findings about free float. He grips the door jamb in terror. "We're falling!" he cries out. His fear turns to astonishment when we tell him not to worry.

"A shaft has been sunk through Earth," we tell him. "It's not the fall that hurts anyone but what stops the fall. All obstacles have been removed from our way, including air. Free fall," we assure him, "is the safest condition there is. That's why we call it free float."

"You may call it float," he says, "but I still call it fall."

Free-float through Earth

"Right now that way of speaking may seem reasonable," we reply, "but after we pass the center of Earth and start approaching the opposite surface, won't the word 'fall' seem rather out of place? Might you not then prefer the word 'float'?" And with "float" our companion at last is happy.

What do we both see? Weightlessness. Free float. Motion in a straight line and at uniform speed for marbles, pennies, keys, and balls in free motion in any direction within our traveling home. No jolts. No shudders. No shakes at any point in all the long journey from one side of Earth to the other.

For our ancestors, travel into space was a dream beyond realization. Equally beyond our reach today is the dream of a house floating along a tunnel through Earth, but this dream nonetheless illuminates the simplicity of motion in a free-float frame. Given the necessary conditions, nothing that we observe inside our traveling room gives us the slightest possibility of discriminating among different free-float frames: one just above Earth's surface, a second passing through Earth's interior, a third in the uttermost reaches of space. Floating inside any of them we find no evidence whatever for the presence of "gravity."

Wait a minute! If the idea of local "gravity" is unnecessary, why does my pencil begin to fall when I hold it in the air and let go? If there is no gravity, my pencil should remain at rest.

And so it does remain at rest—as observed from a free-float frame! The natural motion of your pencil is to remain at rest or to move with constant velocity in a free-float frame. So it is not helpful to ask: "Why does the pencil begin to fall when I let go?" A more helpful question: "Before I let go, why must I apply an upward force to keep the pencil at rest?" Answer: Because you are making observations from an unnatural frame: one held fixed at the surface of Earth. Remove that fixed hold by dropping your room off a cliff. Then for you "gravity" disappears. For you, no force is required to keep the pencil at rest in your free-float frame.

2.3 LOCAL CHARACTER OF FREE-FLOAT FRAME

tidal effects intrude in larger domains

First to strike us about the concept free float has been its paradoxical character. As a first step to explaining gravity Einstein got rid of gravity. There is no evidence of gravity in the freely falling house.

Earth's pull nonuniform: Large spaceship not a free-float frame

Well, *almost* no evidence. The second feature of free float is its local character. Riding in a very small spaceship (Figure 2-5, left) we find no evidence of gravity. But the enclosure in which we ride—falling near Earth or plunging through Earth—cannot be too large or fall for too long a time without some unavoidable relative changes in motion being detected between particles in the enclosure. Why? Because widely separated particles within a large enclosed space are differently affected by the nonuniform gravitational field of Earth, to use the Newtonian way of speaking. For example, two particles released side by side are both attracted toward the center of Earth, so they move closer together as measured inside a falling long narrow *horizontal* railway coach (Figure 2-5, center). This has nothing to do with "gravitational attraction" between the particles, which is entirely negligible.

As another example, think of two particles released far apart vertically but directly above one another in a long narrow *vertical* falling railway coach (Figure 2-5, right). This time their gravitational accelerations toward Earth are in the same direction,

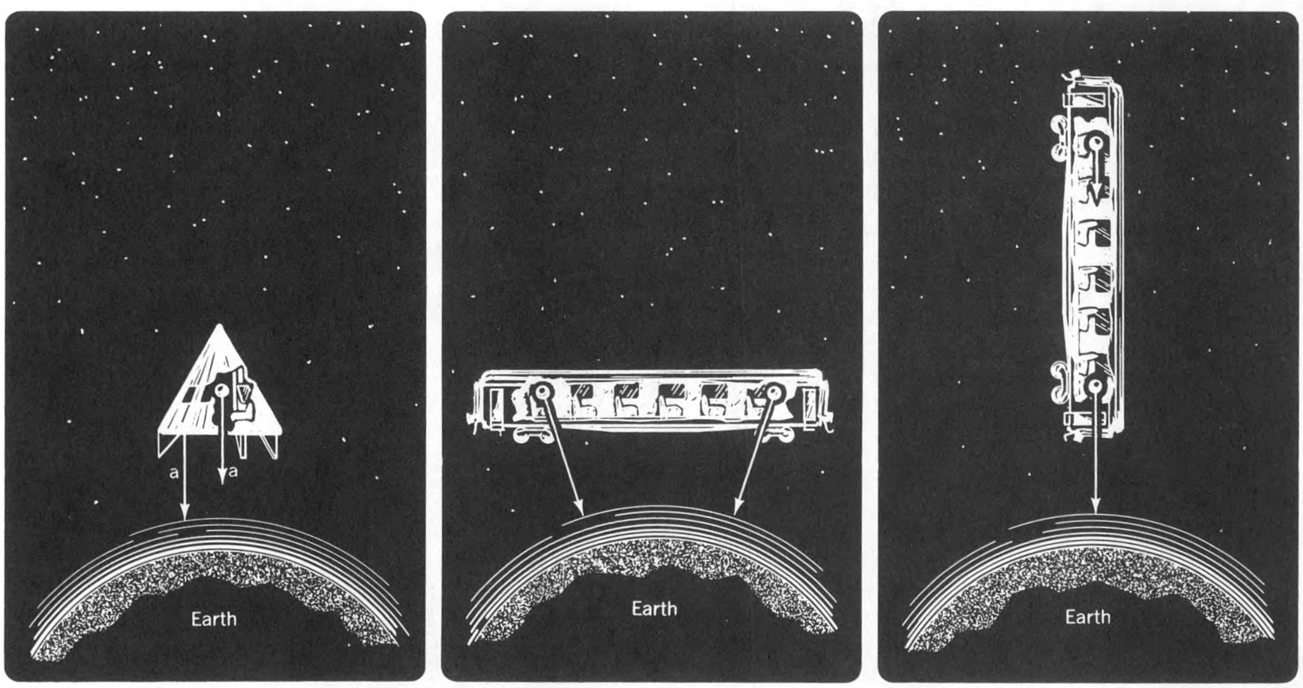


FIGURE 2-5. *Three vehicles in free fall near Earth: small space capsule, Einstein's old-fashioned railway coach in free fall in a* horizontal *orientation, and another railway coach in* vertical *orientation.*

according to the Newtonian analysis. However, the particle nearer Earth is more strongly attracted to Earth and slowly leaves the other behind: the two particles move farther apart as the coach falls. Conclusion: the large enclosure is not a free-float frame.

Free-float frame is local

Even a small room fails to qualify as free-float when we sample it over a long enough time. In the 42 minutes it takes our small room to fall through the tunnel from North Pole to South Pole, we notice relative motion between test particles released initially from rest at opposite sides of the room.

Now, we want the laws of motion to look simple in our floating room. Therefore we want to eliminate all relative accelerations produced by external causes. "Eliminate" means to reduce these accelerations below the limit of detection so that they do not interfere with more important accelerations we wish to study, such as those produced when two particles collide. We eliminate the problem by choosing a room that is sufficiently small. Smaller room? Smaller relative accelerations of objects at different points in the room!

Let someone have instruments for detection of relative accelerations with any given degree of sensitivity. No matter how fine that sensitivity, the room can always be made so small that these perturbing relative accelerations are too small to be detectable. Within these limits of sensitivity our room is a free-float frame. "Official" names for such a frame are the **inertial reference frame** and the **Lorentz reference frame.** Here, however, we often use the name **free-float frame,** which we find more descriptive. These are all names for the same thing.

Free-float (inertial) frame formally defined

> **A reference frame is said to be an "inertial" or "free-float" or "Lorentz" reference frame in a certain region of space and time when, throughout that region of spacetime — and within some specified accuracy — every free test particle initially at rest with respect to that frame remains at rest, and every free test particle initially in motion with respect to that frame continues its motion without change in speed or in direction.**

Wonder of wonders! This test can be carried out entirely within the free-float frame. The observer need not look out of the room or refer to any measurements made external to the room. A free-float frame is "local" in the sense that it is limited in space and time — and also "local" in the sense that its free-float character can be determined from within, locally.

Sir Isaac Newton stated his First Law of Motion this way: "Every body perseveres in its state of rest, or of uniform motion in a right [straight] line, unless it is compelled to change that state by forces impressed upon it." For Newton, **inertia** was a property of objects that described their tendency to maintain their state of motion, whether of rest or constant velocity. For him, objects obeyed the "Law of Inertia." Here we have turned the "Law of Inertia" around: Before we certify a reference frame to be inertial, we *require* observers in that frame to demonstrate that every free particle maintains its initial state of motion or rest. Then Newton's First Law of Motion *defines* a reference frame — an arena or playing field — in which one can study the motion of objects and draft the laws of their motion.

When is the room, the spaceship, or any other vehicle small enough to be called a local free-float frame? Or when is the relative acceleration of two free particles placed at opposite ends of the vehicle too slight to be detected?

"Local" is a tricky word. For example, drop the old-fashioned 20-meter-long railway coach in a horizontal orientation from rest at a height of 315 meters onto the surface of Earth (Figure 2-5, center). Time from release to impact equals 8 seconds, or 2400 million meters of light-travel time. At the same instant you drop the coach, release tiny ball bearings from rest — and in midair — at opposite ends of the coach.

BOX 2-1

THE TIDE-DRIVING POWER OF MOON AND SUN

Note: *Neither astronomers nor newspapers say "the Venus" or "the Mars." All say simply "Venus" or "Mars." Astronomers follow the same snappy practice for Earth, Moon, and Sun. More and more of the rest of the world now follows — as do we in this book — the recommendations of the International Astronomical Union.*

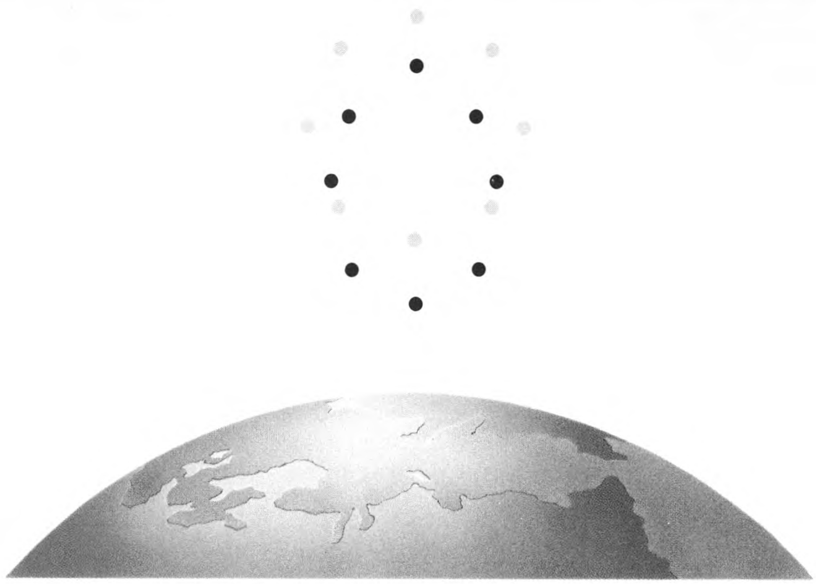
The ocean's rise and fall in a never-ending rhythmic cycle bears witness to the tide-driving power of Moon and Sun. In principle those influences are no different from those that cause relative motion of free particles in the vicinity of Earth. In a free-float frame near Earth, particles separated vertically *increase* their separation with time; particles separated horizontally *decrease* their separation with time (Figure 2-5). More generally, a thin spattering of free-float test masses, spherical in pattern, gradually becomes egg-shaped, with the long axis vertical. Test masses nearer Earth, more strongly attracted than the average, move downward to form the lower bulge. Similarly, test masses farther from Earth, less strongly attracted than the average, lag behind to form the upper bulge.

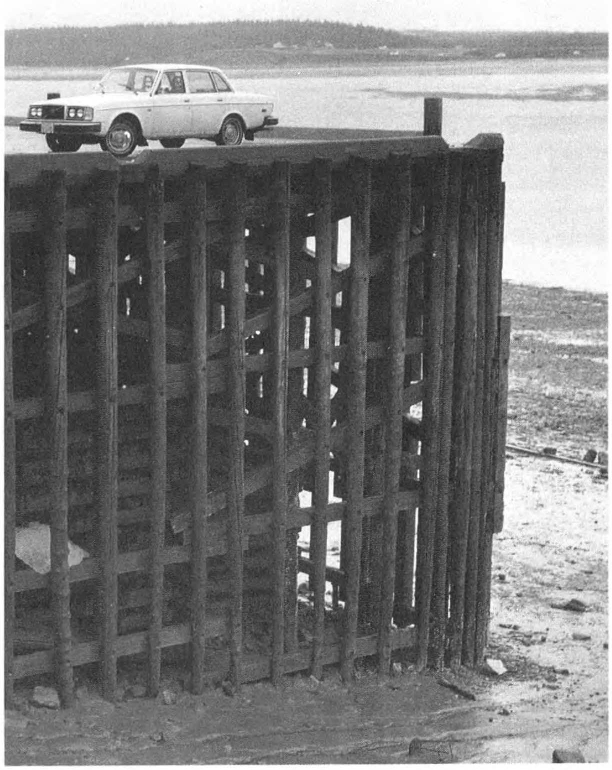
By like action Moon, acting on the waters of Earth — floating free in space — would draw them out into an egg-shaped pattern if there were water everywhere, water of uniform depth. There isn't. The narrow Straits of Gibraltar almost cut off the Mediterranean from the open ocean, and almost kill all tides in it. Therefore it is no wonder that Galileo Galilei, although a great pioneer in the study of gravity, did not take the tides as seriously as the more widely traveled Johannes Kepler, an expert on the motion of Moon and the planets. Of Kepler, Galileo even said, "More than other people he was a person of independent genius . . . [but he] later pricked up his ears and became interested in the action of the moon on the water, and in other occult phenomena, and similar childishness."

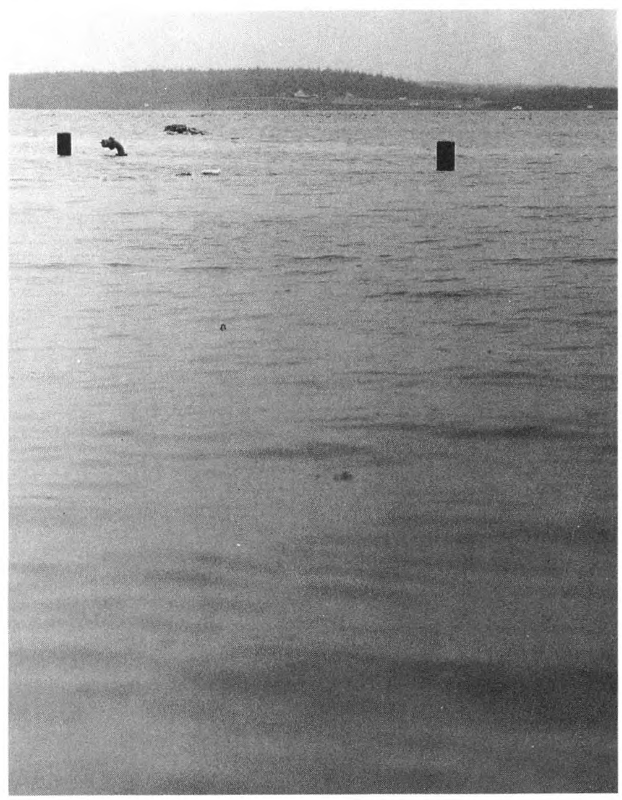
Foolishness indeed, it must have seemed, to assign to the tiny tides of the Mediterranean an explanation so cosmic as Moon. But mariners in northern waters face destruction unless they track the tides. For good reason they remember that Moon reaches its summit overhead an average 50.47 minutes later each day. Their own bitter experience tells them that, of the two high tides a day — *two* because there are two projections on an egg — each also comes about 50 minutes later than it did the day before.

Geography makes Mediterranean tides minuscule. Geography also makes tides in the Gulf of Maine and Bay of Fundy the highest in the world. How come? Resonance! The Bay of Fundy and the Gulf of Maine make together a great bathtub in which water sloshes back and forth with a natural period of 13 hours, near to the 12.4-hour timing of Moon's tide-driving power — and to the 12-hour timing of Sun's influence. Build a big power-producing dam in the upper reaches of the Bay of Fundy? Shorten the length of the bathtub? Decrease the slosh time from 13 hours to exact resonance with Moon? Then get one-foot higher tides along the Maine coast!

Want to see the highest tides in the Bay of Fundy? Then choose your visit according to these rules: (1) Come in summer, when this northern body of water tilts most strongly toward Moon. (2) Come when Moon, in its elliptic orbit, is closest to Earth — roughly 10 percent closer than its most distant point, yielding roughly 35 percent greater tide-producing power. (3) Take into account the tide-producing power of Sun, about 45 percent as great as that of Moon. Sun's effect reinforces Moon's influence when Moon is dark, dark because interposed, or almost interposed, between Earth and Sun, so Sun and Moon pull from the same side. But an egg has two projections, so Sun and Moon also assist each other in producing tides when they are on opposite sides of Earth; in this case we see a full Moon.







The result? Burncoat Head in the Minas Basin, Nova Scotia, has the greatest mean range of 14.5 meters (47.5 feet) between low and high tide when Sun and Moon line up. At nearby Leaf Basin, a unique value of 16.6 meters (54.5 feet) was recorded in 1953.

High and low tides witness to the relative accelerations of portions of the ocean separated by the diameter of Earth. High tides show the "stretching" relative acceleration at different radial distances from Moon or Sun. Low tides witness to the "squeezing" relative accelerations at the same radial distance from Moon or Sun but at opposite sides of Earth.

During the time of fall, they move toward each other a distance of 1 millimeter — a thousandth of a meter, the thickness of 16 pages of this book. Why do they move toward one another? Not because of the gravitational attraction between the ball bearings; this is far too minute to bring about any "coming together." Rather, according to Newton's nonlocal view, they are both attracted toward the center of Earth. Their relative motion results from the difference in direction of Earth's gravitational pull on them, says Newton.

As another example, drop the same antique railway coach from rest in a *vertical* orientation, with the lower end of the coach initially 315 meters from the surface of Earth (Figure 2-5, right). Again release tiny ball bearings from rest at opposite ends of the coach. In this case, during the time of fall, the ball bearings move *apart* by a distance of 2 millimeters because of the greater gravitational acceleration of the one nearer Earth, as Newton would put it. This is twice the change that occurs for horizontal separation.

In either of these examples let the measuring equipment in use in the coach be just short of the sensitivity required to detect this relative motion of the ball bearings. Then, with a limited time of observation of 8 seconds, the railway coach — or, to use the earlier example, the freely falling room — serves as a free-float frame.

When the sensitivity of measuring equipment is increased, the railway coach may no longer serve as a local free-float frame unless we make additional changes. Either shorten the 20-meter domain in which observations are made, or decrease the time given to the observations. Or better, cut down some appropriate combination of space and time dimensions of the region under observation. Or as a final alternative, shoot the whole apparatus by rocket up to a region of space where one cannot detect locally the "differential gravitational acceleration" between one side of the coach and another — to use Newton's way of speaking. In another way of speaking, relative accelerations of particles in different parts of the coach must be too small to perceive. Only when these relative accelerations are too small to detect do we have a reference frame with respect to which laws of motion are simple. That's why "local" is a tricky word!

Hold on! You just finished saying that the idea of local gravity is unnecessary. Yet here you use the "differential gravitational acceleration" to account for relative accelerations of test particles and ocean tides near Earth. Is local gravity necessary or not?

Near Earth, two explanations of projectile paths or ocean flow give essentially the same numerical results. Newton says there is a force of gravity, to be treated like any other force in analyzing motion. Einstein says gravity differs from all other forces: Get rid of gravity locally by climbing into a free-float frame. Near the surface of Earth both explanations accurately predict relative accelerations of falling particles toward or away from one another and motions of the tides. In this chapter we use the more familiar Newtonian analysis to predict relative accelerations.

When tests of gravity are very sensitive, or when gravitational effects are large, such as near white dwarfs or neutron stars, then Einstein's predictions are not the same as Newton's. In such cases Einstein's battle-tested 1915 theory of gravity (general relativity) predicts results that are observed; Newton's theory makes incorrect predictions. This justifies Einstein's insistence on getting rid of gravity locally using free-float frames. All that remains of gravity is the relative accelerations of nearby particles — tidal accelerations.

2.4 REGIONS OF SPACETIME

special relativity is limited to free-float frames

"Region of spacetime." What is the precise meaning of this term? The long narrow railway coach in Figure 2-5 probes spacetime for a limited stretch of time and in one or another single direction in space. It can be oriented north–south or east–west or up–down. Whatever its orientation, relative acceleration of the tiny ball bearings released at the two ends can be measured. For all three directions—and for all intermediate directions—let it be found by calculation that the relative drift of two test particles equals half the minimum detectable amount or less. Then throughout a cube of space 20 meters on an edge and for a lapse of time of 8 seconds (2400 million meters of light-travel time), test particles moving every which way depart from straight-line motion by undetectable amounts. In other words, the reference frame is free-float in a local region of spacetime with dimensions

(20 meters $\times$ 20 meters $\times$ 20 meters of space) $\times$ 2400 million meters of time

"Region of spacetime" is four-dimensional

Notice that this "region of spacetime" is four-dimensional: three dimensions of space and one of time.

Why pay so much attention to the small relative accelerations described above? Why not from the beginning consider as reference frames only spaceships very far from Earth, far from our Sun, and far from any other gravitating body? At these distances we need not worry at all about any relative acceleration due to a nonuniform gravitational field, and a free-float frame can be huge without worrying about relative accelerations of particles at the extremities of the frame. Why not study special relativity in these remote regions of space?

Most of our experiments are carried out near Earth and almost all in our part of the solar system. Near Earth or Sun we cannot eliminate relative accelerations of test particles due to nonuniformity of gravitational fields. So we need to know how large a region of spacetime our experiment can occupy and still follow the simple laws that apply in free-float frames.

When is general relativity required?

For some experiments local free-float frames are not adequate. For example, a comet sweeps in from remote distances, swings close to Sun, and returns to deep space. (Consider only the head of the comet, not its 100-million-kilometer-long tail.) Particles traveling near the comet during all those years move closer together or farther apart due to tidal forces from Sun (assuming we can neglect effects of the gravitational field of the comet itself). These relative forces are called **tidal,** because similar differential forces from Sun and Moon act on the ocean on opposite sides of Earth to cause tides (Box 2-1). A frame large enough to include these particles is not free-float. So reduce spatial size until relative motion of encompassed particles is undetectable during that time. The resulting frame is very much smaller than the head of the comet! You cannot analyze the motion of a comet in a frame smaller than the comet. So instead think of a larger free-float frame that surrounds the comet for a limited time during its orbit, so that the comet passes through a series of such frames. Or think of a whole collection of free-float frames plunging radially toward Sun, through which the comet passes in sequence. In either case, motion of the comet over a small portion of its trajectory can be analyzed rigorously with respect to one of these local free-float frames using special relativity. However, questions about the entire trajectory cannot be answered using only one free-float frame; for this we require a series of frames. General relativity—the theory of gravitation—tells how to describe and predict orbits that traverse a string of adjacent free-float frames. Only general relativity can describe motion in unlimited regions of spacetime.

Please stop beating around the bush! In defining a free-float frame, you say that every test particle at rest in such a frame remains at rest "within some specified accuracy." What accuracy? Can't you be more specific? Why do these definitions depend on whether or not we are able to perceive the tiny motion of some test particle? My eyesight gets worse. Or I take my glasses off. Does the world suddenly change, along with the standards for "inertial frame"? Surely science is more exact, more objective than that!

Science can be "exact" only when we agree on acceptable accuracy. A 1000-ton rocket streaks 1 kilometer in 3 seconds; do you want to measure the sequence of its positions during that time with an accuracy of 10 centimeters? An astronaut in an orbiting space station releases a pencil that floats at rest in front of her; do you want to track its position to 1-millimeter accuracy for 2 hours? Each case places different demands on the inertial frame from which the observations are made. Specific figures imply specific requirements for inertial frames, requirements that must be verified by test particles. The astronaut takes off her glasses; then she can determine the position of the pencil with only 3-millimeter accuracy. Suddenly—yes!—requirements on the inertial frame have become less stringent—unless she is willing to observe the pencil over a longer period of time.

2.5 TEST PARTICLE

ideal tool to probe spacetime without affecting it

Test particle defined

"Test particle." How small must a particle be to qualify as a **test particle?** It must have so little mass that, within some specified accuracy, its presence does not affect the motion of other nearby particles. In terms of Newtonian mechanics, gravitational attraction of the test particle for other particles must be negligible within the accuracy specified.

As an example, consider a particle of mass 10 kilograms. A second and less massive particle placed 10 centimeters from it and initially at rest will, in less than 3 minutes, be drawn toward it by 1 millimeter (see the exercises for this chapter). For measurements of this sensitivity or greater sensitivity, the 10-kilogram object is not a test particle. A particle counts as a test particle only when it accelerates as a result of gravitational forces without itself causing measurable gravitational acceleration in other objects—according to the Newtonian way of speaking.

Free-float frame definable because every substance falls with same acceleration

It would be impossible to define a free-float frame were it not for a remarkable feature of nature. Test particles of different size, shape, and material in the same location all fall with the same acceleration toward Earth. If this were not so, an observer inside a falling room would notice that an aluminum object and a gold object accelerate relative to one another, even when placed side by side. At least one of these test particles, initially at rest, would not remain at rest within the falling room. That is, the room would not be a free-float frame according to definition.

How sure are we that particles in the same location but of different substances all fall toward Earth with equal acceleration? John Philoponus of Alexandria argued, in 517 A.D., that when two bodies "differing greatly in weight" are released simultaneously to fall, "the difference in their time [of fall] is a very small one." According to legend Galileo dropped balls made of different materials from the Leaning Tower of Pisa in order to verify this assumption. In 1905 Baron Roland von Eötvös checked that the gravitational acceleration of wood toward Earth is equal to that of platinum within 1 part in 100 million. In the 1960s R. H. Dicke, Peter G. Roll, and Robert V. Krotkov reduced this upper limit on difference in accelerations—for aluminum and gold responding to the gravitational field of Sun—to less than 1 part in 100,000 million (less than 1 in 10^{11}). This—and a subsequent experiment by Vladimir Braginsky and colleagues—is one of the most sensitive checks of fundamental physical principles in all of science: the equality of acceleration produced by gravity on test particles of every kind.

It follows that a particle made of any material can be used as a test particle to determine whether a given reference frame is free-float. A frame that is free-float for a test particle of one kind is free-float for test particles of all kinds.

2.6 LOCATING EVENTS WITH A LATTICEWORK OF CLOCKS

only the nearest clock records an event

The fundamental concept in physics is **event.** An event is specified not only by a place but also by a time of happening. Some examples of events are emission of a particle or a flash of light (from, say, an explosion), reflection or absorption of a particle or light flash, a collision.

Latticework of rods and clocks

How can we determine the place and time at which an event occurs in a given free-float frame? Think of constructing a frame by assembling meter sticks into a cubical latticework similar to the jungle gym seen on playgrounds (Figure 2-6). At every intersection of this latticework fix a clock. These clocks are identical. They can be constructed in any manner, but their readings are in meters of light-travel time (Section 1.4).

How are the clocks to be set? We want them all to read the "same time" as one another for observers in this frame. When one clock reads midnight (00.00 hours = 0 meters), all clocks in the same frame should read midnight (zero). That is, we want the clocks to be **synchronized** in this frame.

Synchronizing clocks in lattice

How are the several clocks in the lattice to be synchronized? As follows: Pick one clock in the lattice as the standard and call it the **reference clock.** Start this reference

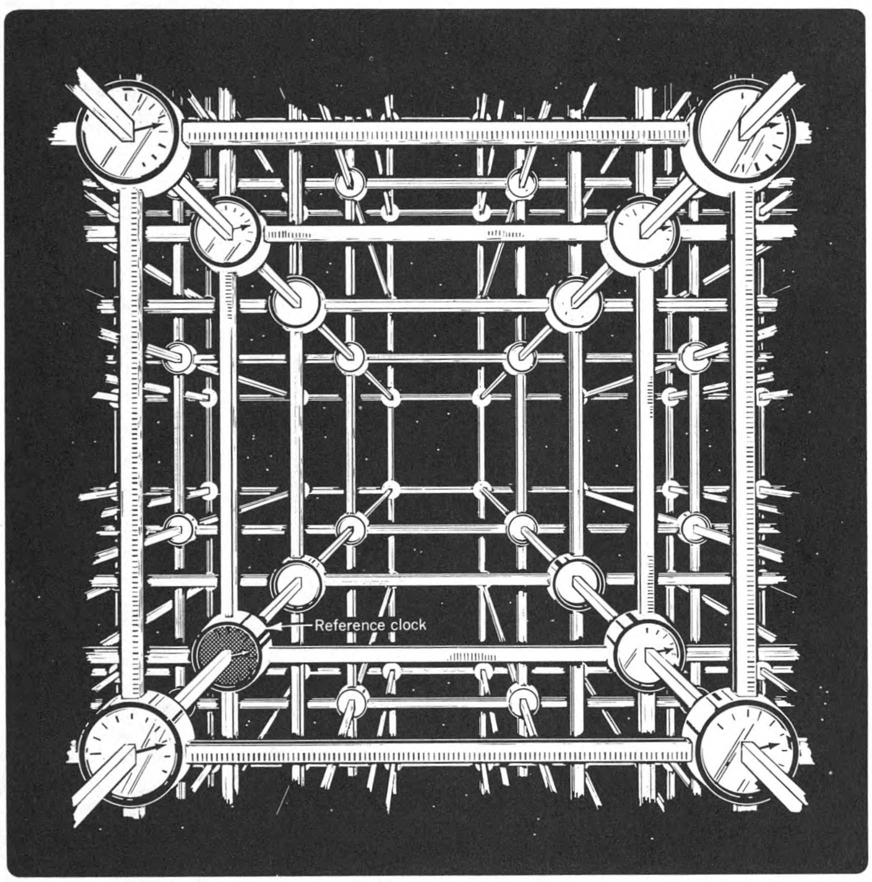


FIGURE 2-6. *Latticework of meter sticks and clocks.*

Reference event defined

clock with its pointer set initially at zero time. At this instant let it send out a flash of light that spreads out as a spherical wave in all directions. Call the flash emission the **reference event** and the spreading spherical wave the **reference flash.**

When the reference flash gets to a slave clock 5 meters away, we want that clock to read 5 meters of light-travel time. Why? Because it takes light 5 meters of light-travel time to travel the 5 meters of distance from reference clock to slave clock. So an assistant sets the slave clock to 5 meters of time long before the experiment begins, holds it at 5 meters, and releases it only when the reference flash arrives. (The assistant has zero reaction time or the slave clock is set ahead an additional time equal to the reaction time.) When assistants at all slave clocks in the lattice follow this prearranged procedure (each setting his slave clock to a time in meters equal to his own distance from the reference clock and starting it when the reference light flash arrives), the lattice clocks are said to be **synchronized.**

This is an awkward way to synchronize lattice clocks with one another. Is there some simpler and more conventional way to carry out this synchronization?

There are other possible ways to synchronize clocks. For example, an extra portable clock could be set to the reference clock at the origin and carried around the lattice in order to set the rest of the clocks. However, this procedure involves a moving clock. We saw in Chapter 1 that the time between two events is not necessarily the same as recorded by clocks in relative motion. The portable clock will not even agree with the reference clock when it is brought back next to it! (This idea is explored more fully in Section 4.6.) However, when we use a moving clock traveling at a speed that is a very small fraction of light speed, its reading is only slightly different from that of clocks fixed in the lattice. In this case the second method of synchronization gives a result nearly equal to the first — and standard — method. Moreover, the error can be made as small as desired by carrying the portable clock around sufficiently slowly.

Locate event with latticework

Use the latticework of synchronized clocks to determine location and time at which any given event occurs. The space position of the event is taken to be the location of the clock nearest the event. The location of this clock is measured along three lattice directions from the reference clock: northward, eastward, and upward. The time of the event is taken to be the time recorded on the same lattice clock nearest the event. The spacetime location of an event then consists of four numbers, three numbers that specify the space position of the clock nearest the event and one number that specifies the time the event occurs as recorded by that clock.

The clocks, when installed by a foresighted experimenter, will be *recording* clocks. Each clock is able to detect the occurrence of an event (collision, passage of light-flash or particle). Each reads into its memory the nature of the event, the time of the event, and the location of the clock. The memory of all clocks can then be read and analyzed, perhaps by automatic equipment.

Why a latticework built of rods that are 1 meter long? What is special about 1 meter? Why not a lattice separation of 100 meters between recording clocks? Or 1 millimeter?

When a clock in the 1-meter lattice records an event, we will not know whether the event so recorded is 0.4 meters to the left of the clock, for instance, or 0.2 meters to the right. The location of the event will be uncertain to some substantial fraction of a meter. The time of the event will also be uncertain with some appreciable fraction of a meter of light-travel time, because it may take that long for a light signal from the event to reach the nearest clock. However, this accuracy of a meter or less is quite adequate for observing the passage of a rocket. It is extravagantly good for measurements on planetary orbits — for a planet it would even be reasonable to increase the lattice spacing from 1 meter to hundreds of meters.

Neither 100 meters nor 1 meter is a lattice spacing suitable for studying the tracks of particles in a high-energy accelerator. There a centimeter or a millimeter would be more appropriate. The location and time of an event can be determined to whatever accuracy is desired by constructing a latticework with sufficiently small spacing.

2.7 OBSERVER

ten thousand local witnesses

In relativity we often speak about the **observer.** Where is this observer? At one place, or all over the place? Answer: **The word "observer" is a shorthand way of speaking about the whole collection of recording clocks associated with one free-float frame.** No one real observer could easily do what we ask of the "ideal observer" in our analysis of relativity. So it is best to think of the observer as a person who goes around reading out the memories of all recording clocks under his control. This is the sophisticated sense in which we hereafter use the phrase "the observer measures such-and-such."

Observer defined

Location and time of each event is recorded by the clock nearest that event. We intentionally limit the observer's report on events to a summary of data collected from clocks. We do not permit the observer to report on widely separated events that he himself views by eye. The reason: travel time of light! It can take a long time for light from a distant event to reach the observer's eye. Even the order in which events are seen by eye may be wrong: Light from an event that occurred a million years ago and a million light-years distant in our frame is just entering our eyes now, after light from an event that occurred on Moon a few seconds ago. We see these two events in the "wrong order" compared with observations recorded by our far-flung latticework of recording clocks. For this reason, we limit the observer to collecting and reporting data from the recording clocks.

Observer limited to clock readings

The wise observer pays attention only to clock records. Even so, light speed still places limits on how soon he can analyze events after they occur. Suppose that events in a given experiment are widely separated from one another in interstellar space, where a single free-float frame can cover a large region of spacetime. Let remote events be recorded instantly on local clocks and transmitted by radio to the observer's central control room. This information transfer cannot take place faster than the speed of light — the same speed at which radio waves travel. Information on dispersed events is available for analysis at a central location only after light-speed transmission. This information will be full and accurate and in no need of correction — but it will be late. Thus all analysis of events must take place after — sometimes long after! — events are over as recorded in that frame. The same difficulty occurs, in principle, for a free-float frame of any size.

Nature puts an unbreakable speed limit on signals. This limit has profound consequences for decision making and control. A space probe descends onto Triton, a moon of the planet Neptune. The probe adjusts its rocket thrust to provide a slow-speed "soft" landing. This probe must carry equipment to detect its distance from Triton's surface and use this information to regulate rocket thrust on the spot, without help from Earth. Earth is never less than 242 light-minutes away from Neptune, a round-trip radio-signal time of 484 minutes — more than eight hours. Therefore the probe would crash long before probe-to-surface distance data could be sent to Earth and commands for rocket thrust returned. This time delay of information transmission does not prevent a detailed retrospective analysis on Earth of the probe's descent onto Triton — but this analysis cannot take place until at least 242 minutes

Speed limit: c
It's the law!

SAMPLE PROBLEM 2-1

METEOR ALERT!

Interstellar Command Center receives word by radio that a meteor has just whizzed past an outpost situated 100 light-seconds distant (a fifth of Earth-Sun distance). The report warns that the meteor is headed directly toward Command Center at one quarter light speed. Assume radio signals travel with light speed. How long do Command Center personnel have to take evasive action?

SOLUTION

The warning radio signal and the meteor leave the outpost at the same time. The radio signal moves with light speed from outpost to Command Center, covering the 100 light-seconds of distance in 100 seconds of time. During this 100 seconds the meteor also travels toward Command Center. The meteor moves at one quarter light speed, so in 100 seconds it covers one quarter of 100 light-seconds, or 25 light-seconds of distance. Therefore, when the warning arrives at Command Center, the meteor is $100 - 25 = 75$ light-seconds away.

The meteor takes an additional 100 seconds of time to move each additional 25 light-seconds of distance. So it covers the remaining 75 light-seconds of distance in an additional time of 300 seconds.

In brief, after receiving the radio warning, Command Center personnel have a relaxed 300 seconds — or five minutes — to stroll to their meteor-proof shelter.

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after the event. Could we gather last-minute information, make a decision, and send back control instructions? No. Nature rules out micromanagement of the far-away (Sample Problem 2-1).

2.8 MEASURING PARTICLE SPEED

reference frame clocks and rods put to use

The recording clocks reveal particle motion through the lattice: Each clock that the particle passes records the time of passage as well as the space location of this event. How can the path of the particle be described in terms of numbers? By recording locations of these events along the path. Distances between locations of successive events and time lapse between them reveal the particle speed — speed being space separation divided by time taken to traverse this separation.

Speed in meters per meter

The conventional unit of speed is meters per second. However, when time is measured in meters of light-travel time, speed is expressed in meters of distance covered per meter of time. A flash of light moves one meter of distance in one meter of light-travel time: its speed has the value unity in units of meters per meter. In contrast, a particle loping along at half light speed moves one half meter of distance per meter of time; its speed equals one half in units of meter per meter. More generally, particle speed in meters per meter is the ratio of its speed to light speed:

$$\text{(particle speed)} = \frac{\text{(meters of distance covered by particle)}}{\text{(meters of time required to cover that distance)}}$$
$$= \frac{\text{(particle speed in meters/second)}}{\text{(speed of light in meters/second)}}$$

In this book we use the letter v to symbolize the speed of a particle in meters of distance per meter of time, or simply meters per meter. Some authors use the lowercase Greek letter beta: β. Let v_{conv} stand for velocity in conventional units (such as meters per second) and c stand for light speed in the same conventional units. Then

$$v = \frac{v_{conv}}{c} \tag{2-1}$$

Test for free-float frame

From the motion of test particles through a latticework of clocks — or rather from records of coincidences of these particles with clocks — we determine whether the latticework constitutes a free-float frame. IF records show (a) that — within some specified accuracy — a test particle moves consecutively past clocks that lie in a straight line, (b) that test-particle speed calculated from the same records is constant — again, within some specified accuracy — and, (c) that the same results are true for as many test-particle paths as the most industrious observer cares to trace throughout the given region of space and time, THEN the lattice constitutes a free-float (inertial) frame throughout that region of spacetime.

Particle speed as a fraction of light speed is certainly an unconventional unit of measure. What advantages does it have that justify the work needed to become familiar with it?

The big advantage is that it is a measure of speed independent of units of space and time. Suppose that a particle moves with respect to Earth at half light speed. Then it travels — with respect to Earth — one half meter of distance in one meter of light travel time. It travels one half light-year of distance in a period of one year. It travels one half light-second of distance in a time of one second, one half light-minute in one minute. Units do not matter as long as we use the same units to measure distance and time; the result always equals the same number: 1/2. Another way to say this is that speed is a fraction; same units on top and bottom of the fraction cancel one another. Fundamentally, v is unit-free. Of course, if we wish we can speak of "meters per meter."

2.9 ROCKET FRAME

does it move? or is it the one at rest?

Rocket frame defined

Let two reference frames be two different latticeworks of meter sticks and clocks, one moving uniformly relative to the other, and in such a way that one row of clocks in each frame coincides along the direction of relative motion of the two frames (Figure 2-7). Call one of these frames **laboratory frame** and the other — moving to the right relative to the laboratory frame — **rocket frame.** The rocket is *unpowered* and coasts along with constant velocity relative to the laboratory. Let rocket and laboratory latticeworks be overlapping in the sense that a region of spacetime exists common to both frames. Test particles move through this common region of spacetime. From motion of these test particles as recorded by his own clocks, the laboratory observer verifies that his frame is free-float (inertial). From motion of the same test particles as recorded by her own clocks, the rocket observer verifies that her frame is also free-float (inertial).

Now we can describe the motion of any particle with respect to the laboratory frame. The same particles and — if they collide — the same collisions may be measured and described with respect to the free-float rocket frame as well. These particles, their paths through spacetime, and events of their collisions have an existence inde-

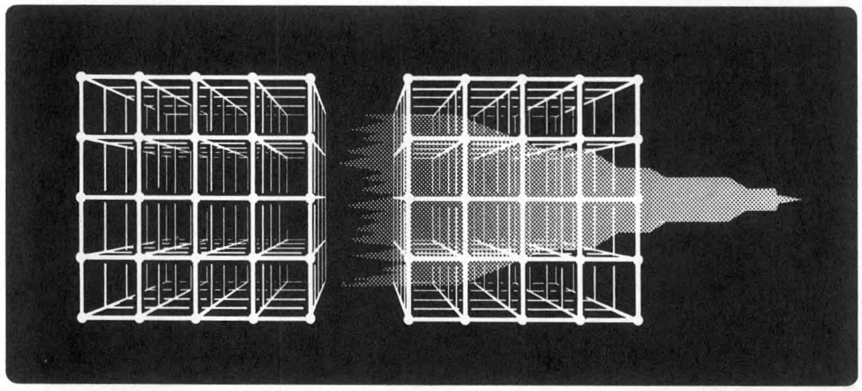

FIGURE 2-7. *Laboratory and rocket frames.* *A second ago the two latticeworks were intermeshed.*

Different frames lead to different descriptions

pendent of any free-float frames in which they are observed, recorded, and described. However, descriptions of these common paths and events are typically different for different free-float frames. For example, laboratory and rocket observers may not agree on the direction of motion of a given test particle (Figure 2-8). Every track that is straight as plotted with respect to one reference frame is straight also with respect to the other frame, because both are free-float frames. This straightness in both frames is possible only because *one free-float frame has uniform velocity relative to any other*

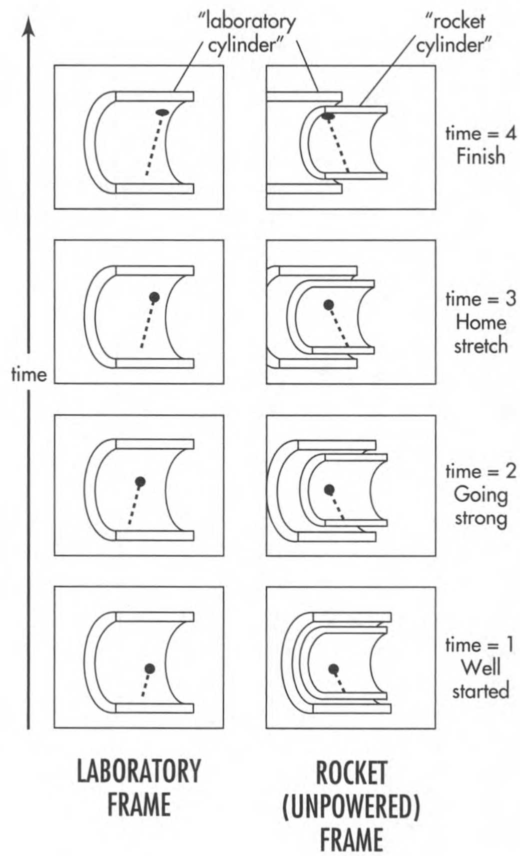

FIGURE 2-8. *A series of "snapshots" of a typical test particle as measured from laboratory and rocket free-float frames, represented by cutaway cylinders.* *Start at the bottom and read upward (time progresses from bottom to top).*

overlapping free-float frame. However, the direction of this path differs from laboratory to rocket frame, except in the special case in which the particle moves along the line of relative motion of two frames.

Many possible free-float frames

How many different free-float rocket frames can there be in a given region of spacetime? An unlimited number! Any unpowered rocket moving through that region in any direction is an acceptable free-float frame from which to make observations. More: There is nothing unique about any of these frames as long as each of them is free-float. All "rocket" frames are unpowered, all are equivalent for carrying out experiments. Even the so-called "laboratory frame" is not unique; you can rename it "Rocket Frame Six" and no one will ever know the difference! All free-float (inertial) frames are equivalent arenas in which to carry out physics experiment. That is the logical basis for special relativity, as described more fully in Chapter 3.

No unique free-float frame

A rocket carries a firecracker. The firecracker explodes. Does this event — the explosion — take place in the rocket frame or in the laboratory frame? Which is the "home" frame for the event? A second firecracker, originally at rest in the laboratory frame, explodes. Does this second event occur in the laboratory frame or in the rocket frame?

Events are primary, the essential stuff of Nature. Reference frames are secondary, devised by humans for locating and comparing events. A given event occurs in both frames — and in all possible frames moving in all possible directions and with all possible constant relative speeds through the region of spacetime in which the event occurs. The apparatus that "causes" the event may be at rest in one free-float frame; another apparatus that "causes" a second event may be at rest in a second free-float frame in motion relative to the first. No matter. Each event has its own unique existence. Neither is "owned" by any frame at all.

A spark jumps 1 millimeter from the antenna of Mary's passing spaceship to a pen in the pocket of John who lounges in the laboratory doorway (Section 1.2). The "apparatus" that makes the spark has parts riding in different reference frames — pen in laboratory frame, antenna in rocket frame. The spark jump — in which frame does this event occur? It is not the property of Mary, not the property of John — not the property of any other observer in the vicinity, no matter what his or her state of motion. The spark-jump event provides data for every observer.

Drive a steel surveying stake into the ground to mark the corner of a plot of land. Is this a "Daytime stake" or a "Nighttime stake"? Neither! It is just a *stake,* marking a location in *space,* the arena of surveying. Similarly an event is neither a "laboratory event" nor a "rocket event." It is just an *event,* marking a location in *spacetime,* the arena of science.

Laboratory frame or rocket frame: Which one is the "primary" free-float frame, the one "really" at rest? There is no way to tell! We apply the names "laboratory" and "rocket" to two free-float enclosures in interstellar space. Someone switches the nameplates while we sleep. When we wake up, there is no way to decide which is which. This realization leads to Einstein's Principle of Relativity and proof of the invariance of the interval, as described in Chapter 3.

2.10 SUMMARY

what a free-float frame is and what it's good for

The **free-float frame** (also called the **inertial frame** and the **Lorentz frame**) provides a setting in which to carry out experiments without the presence of so-called "gravitational forces." In such a frame, a particle released from rest remains at rest and a particle in motion continues that motion without change in speed or in direction (Section 2.2), as Newton declared in his First Law of Motion.

Where does that frame of reference sit? Where do the east-west, north-south, up-down lines run? We might as well ask where on the flat landscape in the state of Iowa we see the lines that mark the boundaries of the townships. A concrete marker, to be sure, may show itself as a corner marker at a place where a north-south line meets an east-west line. Apart from such on-the-spot evidence, those lines are largely invisible. Nevertheless, they serve their purpose: They define boundaries, settle lawsuits, and fix taxes. Likewise imaginary for the most part are the clock and rod paraphernalia of the idealized inertial reference frame. Work of the imagination though they are, they provide the conceptual framework for everything that goes on in the world of particles and radiation, of masses and motions, of annihilations and creations, of fissions and fusions in every context where tidal effects of gravity are negligible.

Our ability to define a free-float frame depends on the fact that a **test particle** made of any material whatsoever experiences the same acceleration in a given gravitational field (Section 2.5).

Near a massive ("gravitating") body, we can still define a free-float frame. However, in such a frame, free test particles typically accelerate toward or away from one another because of the nonuniform field of the gravitating body (Section 2.3). This limits — in both space and time — the size of a free-float frame, the domain in which the laws of motion are simple. The frame will continue to qualify as free-float and special relativity will continue to apply, provided we reduce the spatial extent, or the time duration of our experiment, or both, until these relative, or **tidal,** motions of test particles cannot be detected in our circumscribed region of spacetime. This is what makes special relativity "special" or limited (French: *relativité restreinte:* "restricted relativity"). General relativity (the theory of gravitation) removes this limitation (Chapter 9).

So there are three central characteristics of a free-float frame. (1) We can "get rid of gravity" by climbing onto (getting into) a free-float frame. (2) The existence of a free-float frame depends on the equal acceleration of all particles at a given location in a gravitational field — in Newton's way of speaking. (3) Every free-float frame is of limited extent in spacetime. All three characteristics appear in a fuller version of the quotation by Albert Einstein that began this chapter:

> At that moment there came to me the happiest thought of my life . . . *for an observer falling freely from the roof of a house no gravitational field exists during his fall* — at least not in his immediate vicinity. That is, if the observer releases any objects, they remain in a state of rest or uniform motion relative to him, respectively, independent of their unique chemical and physical nature. Therefore the observer is entitled to interpret his state as that of "rest."

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Figure 2-1 and Jules Verne story: Jules Verne, *A Trip From the Earth to the Moon* and *A Trip Around the Moon,* paperback edition published by Dover Publications, New York. Hardcover edition published in the Great Illustrated Classics Series by Dodd, Mead and Company, New York, 1962.

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Relative acceleration of different materials, Section 2-5: P. G. Roll, R. Krotkov, and R. H. Dicke, "The equivalence of inertial and passive gravitational mass," *Annals of Physics (USA),* Volume 26, pages 442–517 (1964); V. B. Braginsky and V. I. Panov, *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki,* Volume 61, page 873 (1972) [*Soviet Physics JETP,* Volume 34, page 463 (1972)].

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CHAPTER 2 EXERCISES

PRACTICE

2-1 human cannonball

A person rides in an elevator that is shot upward out of a cannon. Think of the elevator after it leaves the cannon and is moving freely in the gravitational field of Earth. Neglect air resistance.

a While the elevator is still on the way up, the person inside jumps from the "floor" of the elevator. Will the person (1) fall back to the "floor" of the elevator? (2) hit the "ceiling" of the elevator? (3) do something else? If so, what?

b The person waits to jump until after the elevator has passed the top if its trajectory and is falling back toward Earth. Will your answers to part **a** be different in this case?

c How can the person riding in the elevator tell when the elevator reaches the top of its trajectory?

2-2 free-float bounce

Test your skill as an acrobat and contortionist! Fasten a weight-measuring bathroom scale under your feet and bounce up and down on a trampoline while reading the scale. Describe readings on the scale at different times during the bounces. During what part of each jump will the scale have zero reading? Neglecting air resistance, what is the longest part of the cycle during which you might consider yourself to be in a free-float frame?

2-3 practical synchronization of clocks

You are an observer in the laboratory frame stationed near a clock with spatial coordinates $x = 6$ light-seconds, $y = 8$ light-seconds, and $z = 0$ light-seconds. You wish to synchronize your clock with the one at the origin. Describe in detail and with numbers how to proceed.

2-4 synchronization by a traveling clock

Mr. Engelsberg does not approve of our method of synchronizing clocks by light flashes (Section 2.6).

a "I can synchronize my clocks in any way I choose!" he exclaims. Is he right?

Mr. Engelsberg wishes to synchronize two identical clocks, named Big Ben and Little Ben, which are relatively at rest and separated by one million kilometers, which is 10^9 meters or approximately three times the distance between Earth and Moon. He uses a third clock, identical in construction with the first two, that travels with constant velocity between them. As his moving clock passes Big Ben, it is set to read the same time as Big Ben. When the moving clock passes Little Ben, that outpost clock is set to read the same time as the traveling clock.

b "Now Big Ben and Little Ben are synchronized," says Mr. Engelsberg. Is he right?

c How much out of synchronism are Big Ben and Little Ben as measured by a latticework of clocks — at rest relative to them both — that has been synchronized in the conventional manner using light flashes? Evaluate this lack of synchronism in milliseconds when the traveling clock that Mr. Engelsberg uses moves at 360,000 kilometers/hour, or 10^5 meters/second.

d Evaluate the lack of synchronism when the traveling clock moves 100 times as fast.

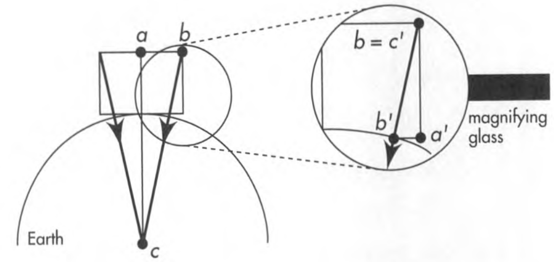
e Is there any earthly reason — aside from matters of personal preference — why we all should not adopt the method of synchronization used by Mr. Engelsberg?

2-5 Earth's surface as a free-float frame

Many experiments involving fast-moving particles and light itself are observed in earthbound laboratories. Typically these laboratories are not in free fall! Nevertheless, under many circumstances laboratories fixed to the surface of Earth can satisfy the conditions required to be called free-float frames. An example:

a In an earthbound laboratory, an elementary particle with speed $v = 0.96$ passes from side to side through a cubical spark chamber one meter wide. For what length of laboratory time is this particle in transit through the spark chamber? Therefore for how long a time is the experiment "in progress"? How far will a separate test particle, released from rest, fall in this time? [Distance of fall from rest $= (1/2)gt_{sec}^2$, where $g =$ acceleration of gravity ≈ 10 meters/second^2 and t_{sec} is the time of free fall in seconds.] Compare your answer with the diameter of an atomic nucleus (a few times 10^{-15} meter).

b How wide *can* the spark chamber be and still be considered a free-float frame for this experiment? Suppose that by using sensitive optical equipment (an **interferometer**) you can detect a test particle change of position as small as one wavelength of visible light, say 500 nanometers $= 5 \times 10^{-7}$ meter. How long will it take the test particle to fall this distance from rest? How far does the fast elementary particle of part **a** move in that time? Therefore how long can an earthbound spark chamber be and still be considered free-float for this sensitivity of detection?



EXERCISE 2-6. *Schematic diagram of two ball bearings falling onto Earth's surface. Not to scale.*

2-6 horizontal extent of free-float frame near Earth

Consider two ball bearings near the surface of Earth and originally separated horizontally by 20 meters (Section 2.3). Demonstrate that when released from rest (relative to Earth) the particles move closer together by 1 millimeter as they fall 315 meters, using the following method of similar triangles or some other method.

Each particle falls from rest toward the center of Earth, as indicated by arrows in the figure. Solve the problem using the ratio of sides of similar triangles abc and $a'b'c'$. These triangles are upside down with respect to each other. However, they are similar because their respective sides are parallel: Sides ac and $a'c'$ are parallel to each other, as are sides bc and $b'c'$ and sides ab and $a'b'$. We know the lengths of some of these sides. Side $a'c' = 315$ meters is the height of fall (greatly exaggerated in the diagram); side ac is effectively equal to the radius of Earth, 6,371,000 meters. Side $ab = (1/2)$ (20 meters) equals half the original separation of the particles. Side $a'b'$ equals HALF their CHANGE in separation as they fall onto Earth's surface. Use the ratio of sides of similar triangles to find this "half-change" and therefore the entire change in separation as two particles initially 20 meters apart horizontally fall from rest 315 meters onto the surface of Earth.

2-7 limit on free-float frame near Earth's Moon

Release two ball bearings from rest a horizontal distance 20 meters apart near the surface of Earth's Moon. By how much does the separation between them decrease as they fall 315 meters? How many seconds elapse during this 315-meter fall? Assume that an initial vertical separation of 20 meters is increased by twice the change in horizontal separation in a fall through the same height. State clearly and completely the dimensions of the region of spacetime in which such a freely falling frame constitutes an inertial frame (to the given accuracy). Moon radius equals

1738 kilometers. Gravitational acceleration at Moon's surface: $g = 1.62$ meters/second^2.

2-8 vertical extent of free-float frame near Earth

Note: This exercise makes use of elementary calculus and the Newtonian theory of gravitation.

A paragraph in Section 2.3 says:

> As another example, drop the same antique [20-meter-long] railway coach from rest in a *vertical* orientation, with the lower end of the coach initially 315 meters from the surface of Earth (Figure 2-5, right). Again release two tiny ball bearings from rest at opposite ends of the coach. In this case, during the time of fall [8 seconds], the ball bearings move *apart* by a distance of two millimeters because of the greater gravitational acceleration of the one nearer Earth, as Newton would put it. This is twice the change that occurs for horizontal separation.

Demonstrate this 2-millimeter increase in separation. The following outline may be useful. Take the gravitational acceleration at the surface of Earth to be $g_o = 9.8$ meters/second^2 and the radius of Earth to be $r_o = 6.37 \times 10^6$ meters. More generally, the gravitational acceleration g of a particle of mass m a distance r from the center of Earth (mass M) is given by the expression

$$g = \frac{F}{m} = \frac{GM}{r^2} = \frac{GM}{r_o^2}\frac{r_o^2}{r^2} = \frac{g_o r_o^2}{r^2}$$

a Take the differential of this equation for g to obtain an approximate algebraic expression for Δg, the change in g, for a small change Δr in height.

b Now use $\Delta y = \frac{1}{2}\Delta g t^2$ to find an algebraic expression for increase in distance Δy between ball bearings in a fall that lasts for time t.

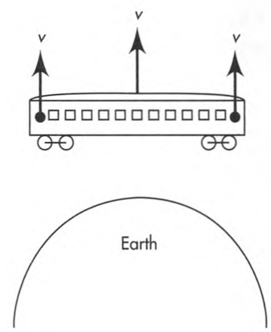
c Substitute numbers given in the quotation above to verify the 2-millimeter change in separation during fall.

2-9 the rising railway coach

You are launched upward inside a railway coach in a horizontal position with respect to the surface of Earth, as shown in the figure. After the launch, but while the coach is still rising, you release two ball bearings at opposite ends of the train and at rest with respect to the train.

a Riding inside the coach, will you observe the distance between the ball bearings to increase or decrease with time?

b Now you ride in a second railway coach launched upward in a *vertical* position with respect to the surface of Earth (not shown). Again you release two ball bearings at opposite ends of the coach and at rest with respect to the coach. Will you observe these ball bearings to move together or apart?



EXERCISE 2-9. *Free-float railway coach rising from Earth's surface, as observed in Earth frame. Two ball bearings were just released from rest with respect to the coach. What will be their subsequent motion as observed from inside the coach? Figure not to scale.*

c In either of the cases described above, can you, the rider in the railway coach, distinguish whether the coach is rising or falling with respect to the surface of Earth solely by observing the ball bearings from inside the coach? What do you observe at the moment the coach stops rising with respect to Earth and begins to fall?

2-10 test particle?

a Verify the statement in Section 2.5 that a candidate test particle of mass 10 kilograms placed 0.1 meter from a less massive particle (initially stationary with respect to it), draws the second toward it by 1 millimeter in less than 3 minutes. If this relative motion is detectable by equipment in use at the test site, the result disqualifies the 10-kilogram particle as a "test particle." Assume that both particles are spherically symmetric. Use Newton's Law of Gravitation:

$$F = \frac{GMm}{r^2}$$

where the gravitation constant G has the value $G = 6.673 \times 10^{-11}$ meter^3/(kilogram-second^2). Assume that this force does not change appreciably as the particles decrease separation by one millimeter.

b Section 2.3 describes two ball bearings released 20 meters apart horizontally in a freely falling railway coach. They move 1 millimeter closer together during 8 seconds of free fall, showing the limitations on this inertial frame. Verify that these ball bearings qualify as test particles by estimating the distance that one will move from rest in 8 seconds under the gravitational attraction of the other, if both were initially at rest in interstellar space far from Earth. Make your own estimate of the mass of each ball bearing.

PROBLEMS

2-11 communications storm!

Sun emits a tremendous burst of particles that travels toward Earth. An astronomer on Earth sees the emission through a solar telescope and issues a warning. The astronomer knows that when the particles arrive, they will wreak havoc with broadcast radio transmission. Communications systems require three minutes to switch from broadcast to underground cable. What is the maximum speed of the particle pulse emitted by Sun such that the switch can occur in time, between warning and arrival of the pulse? Take Sun to be 500 light-seconds from Earth.

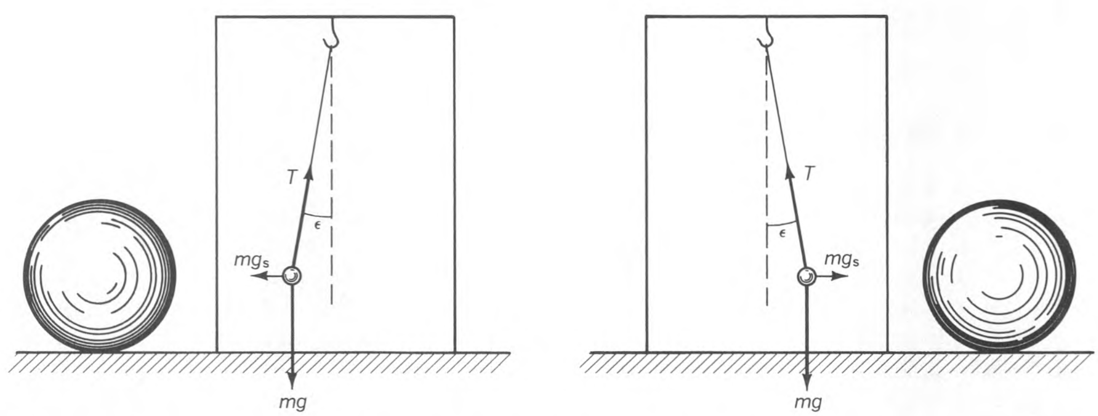
2-12 the Dicke experiment

a The Leaning Tower of Pisa is about 55 meters high. Galileo says, "The variation of speed in air between balls of gold, lead, copper, porphyry, and other heavy materials is so slight that in a fall of 100 cubits [about 46 meters] a ball of gold would surely not outstrip one of copper by as much as four fingers. Having observed this I came to the conclusion that in a medium totally devoid of resistance all bodies would fall with the same speed."

Taking four fingers to be equal to 7 centimeters, find the maximum fractional difference in the acceleration of gravity $\Delta g/g$ between balls of gold and copper that would be consistent with Galileo's experimental result.

b The result of the more modern Dicke experiment is that the fraction $\Delta g/g$ is not greater than 3×10^{-11}. Assume that the fraction has this more recently determined maximum value. Reckon how far behind the first ball the second one will be when the first reaches the ground if they are dropped simultaneously from the top of a 46-meter vacuum chamber. Under these same circumstances, how far would balls of different materials have to fall in a vacuum in a uniform gravitational field of 10 meters/second/second for one ball to lag behind the other one by a distance of 1 millimeter? Compare this distance with the Earth–Moon separation (3.8×10^8 meters). Clearly the Dicke experiment was not carried out using falling balls!

c A plumb bob of mass m hangs on the end of a long line from the ceiling of a closed room, as shown in the first figure (left). A very massive sphere at one side of the closed room exerts a horizontal gravitational force mg_s on the plumb bob, where $g_s = GM/R^2$, M is the mass of the large sphere, and R the distance between plumb bob and the center of the sphere. This horizontal force causes a static deflection of the plumb line from the vertical by the small angle ε. (Similar practical example: In northern India the mass of the Himalaya Mountains results in a slight sideways deflection of plumb lines, causing difficulties in precise surveying.) The sphere is now rolled around to a corresponding position on the other side of the room (right), causing a static deflection of the plumb by an angle ε of the same magnitude but in the opposite direction.



EXERCISE 2-12, first figure. *Left: Nearby massive sphere results in static deflection of plumb line from vertical.* ***Right:*** *Rolling the sphere to the other side results in static deflection of plumb line in the opposite direction.*

Now the angle ε is very small. (Deflection due to the Himalayas is about 5 seconds of arc, which equals 0.0014 degrees.) However, as the sphere is rolled around and around outside the closed room, an observer inside the room can measure the gravitational field g_s due to the sphere by measuring with greater and greater precision the total deflection angle $2\varepsilon \approx 2 \sin \varepsilon$ of the plumb line, where ε is measured in radians. Derive the equation that we will need in the calculation of g_s.

d We on Earth have a large sphere effectively rolling around us once every day. It is the most massive sphere in the solar system: Sun itself! What is the value of the gravitational acceleration $g_s = GM/R^2$ due to Sun at the position of Earth? (Some constants useful in this calculation appear inside the back cover of this book.)

e One additional acceleration must be considered that, however, will not enter our final comparison of gravitational acceleration g_s for different materials. This additional acceleration is the centrifugal acceleration due to the motion of Earth around Sun. When you round a corner in a car you are pressed against the side of the car on the outward side of the turn. This outward force—called the centrifugal pseudoforce or the centrifugal inertial force—is due to the acceleration of your reference frame (the car) toward the center of the circular turn. This centrifugal inertial force has the value mv^2_{conv}/r, where v_{conv} is the speed of the car in conventional units and r is the radius of the turn. Now Earth moves around Sun in a path that is nearly circular. Sun's gravitational force mg_s acts on a plumb bob in a direction toward Sun; the centrifugal inertial force mv^2_{conv}/R acts in a direction away from Sun. Compare the "centrifugal acceleration" v^2_{conv}/R at the position of Earth with the oppositely directed gravitational acceleration g_s calculated in part **d**. What is the net acceleration toward or away from Sun of a particle riding on Earth as observed in the (accelerated) frame of Earth?

f Of what use is the discussion thus far? A plumb bob hung near the surface of Earth experiences a gravitational acceleration g_s toward Sun—and an equal but opposite centrifugal acceleration mv^2_{conv}/R away from Sun. Therefore—in the accelerating reference frame of Earth—the bob experiences no net force at all due to the presence of Sun. Indeed this is the method by which we constructed an inertial frame in the first place (Section 2.2): Let the frame be in free fall about the center of gravitational attraction. A particle at rest on Earth's surface is in free fall about Sun and therefore experiences no net force due to Sun. What then does all this have to do with measuring the equality of gravitational acceleration for particles made of different substances—the subject of the Dicke experiment? Answer: Our purpose is to detect the difference—if any—in the gravitational acceleration g_s toward Sun for different materials. The centrifugal acceleration v^2/R away from Sun is presumably the same for all materials and therefore need not enter any comparison of different materials.

Consider a torsion pendulum suspended from its center by a thin quartz fiber (second figure). A light rod of length L supports at its ends two bobs of equal mass made of different materials—say aluminum and gold. Suppose that the gravitational acceleration g_1 of the gold due to Sun is slightly greater than the acceleration g_2 of the aluminum due to Sun. Then there will be a slight net torque on the torsion pendulum due to Sun. For the position of Sun shown at left in the figure, show that the net torque is counterclockwise when viewed from above. Show also that the magnitude of this net torque is given by the expression

$$\begin{aligned}\text{torque} &= mg_1\, L/2 - mg_2\, L/2 = m(g_1 - g_2)\, L/2 \\ &= mg_s(\Delta g/g_s)\, L/2\end{aligned}$$

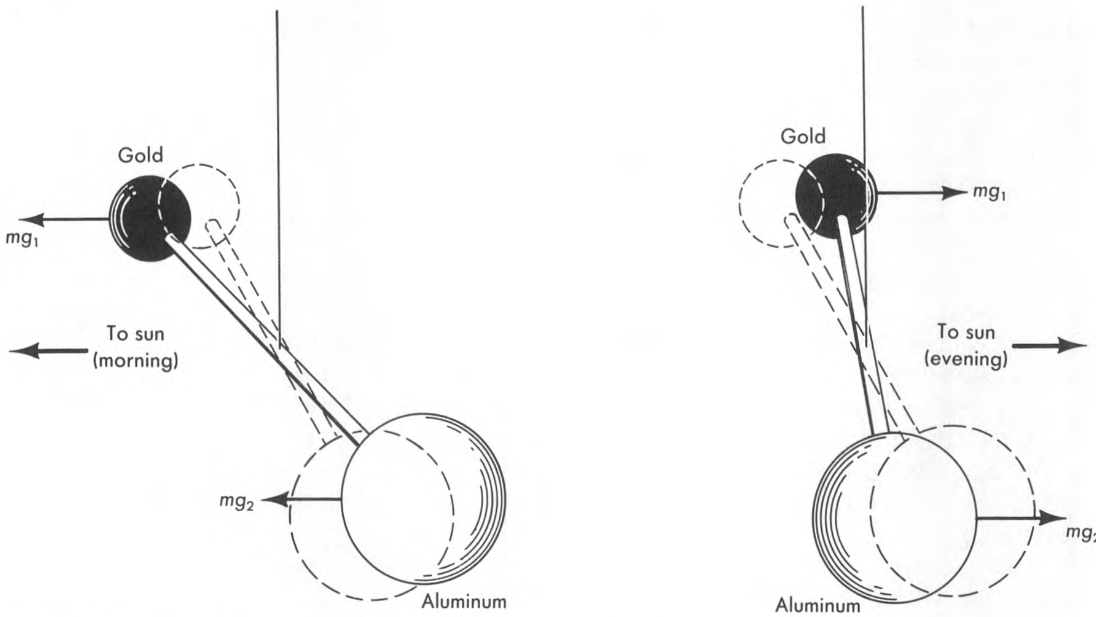
g Suppose that the fraction $(\Delta g/g_s)$ has the maximum value 3×10^{-11} consistent with the results of the final experiment, that L has the value 0.06 meters, and that each bob has a mass of 0.03 kilograms. What is the magnitude of the net torque? Compare this to the torque provided by the added weight of a bacterium of mass 10^{-15} kilogram placed on the end of a meter stick balanced at its center in the gravitational field of Earth.

h Sun moves around the heavens as seen from Earth. Twelve hours later Sun is located as shown at right in the second figure. Show that under these changed circumstances the net torque will have the same magnitude as that calculated in part **g** but now will be clockwise as viewed from above—in a sense opposite to that of part **g**. This change in the sense of the torque every twelve hours allows a small difference $\Delta g = g_1 - g_2$ in the acceleration of gold and aluminum to be detected using the torsion pendulum. As the torsion pendulum jiggles on its fiber because of random motion, passing trucks, Earth tremors and so forth, one needs to consider only those deflections that keep step with the changing position of Sun.

i A torque on the rod causes an angular rotation of the quartz fiber of θ radians given by the formula

$$\text{torque} = k\theta$$

where k is called the **torsion constant** of the fiber. Show that the maximum angular rotation of the torsion pendulum from one side to the other during one



EXERCISE 2-12, second figure. *Schematic diagram of the Dicke experiment.* ***Left:*** *Hypothetical effect: morning.* ***Right:*** *Hypothetical effect: evening. Any difference in the gravitational acceleration of Sun for gold and aluminum should result in opposite sense of net torque on torsion pendulum in the evening compared with the morning. The large aluminum ball has the same mass as the small high-density gold ball.*

rotation of Earth is given by the expression

$$\theta_{\text{tot}} = \frac{mg_s L}{k}\left(\frac{\Delta g}{g_s}\right)$$

i In practice Dicke's torsion balance can be thought of as consisting of 0.030-kilogram gold and aluminum bobs mounted on the ends of a beam 6×10^{-2} meter in length suspended in a vacuum on a quartz fiber of torsion constant 2×10^{-8} newton meter/radian. A statistical analysis of the angular displacements of this torsion pendulum over long periods of time leads to the conclusion that the fraction $\Delta g/g$ for gold and aluminum is less than 3×10^{-11}. To what mean maximum angle of rotation from side to side during one rotation of Earth does this correspond? Random motions of the torsion pendulum—noise!—are of much greater amplitude than this; hence the need for the statistical analysis of the results.

References: R. H. Dicke, "The Eötvös Experiment," *Scientific American,* Volume 205, pages 84–94 (December, 1961). See also P. G. Roll, R. Krotkov, and R. H. Dicke, *Annals of Physics,* Volume 26, pages 442–517 (1964). The first of these articles is a popular exposition written early in the course of the Dicke experiment. The second article reports the final results of the experiment and takes on added interest because of its account of the elaborate precautions required to insure that no influence that might affect the experiment was disregarded. Galileo quote from Galileo Galilei, *Dialogues Concerning Two New Sciences,* translated by Henry Crew and Alfonso de Salvio (Northwestern University Press, Evanston, Illinois, 1950).

2-13 deflection of starlight by Sun

Estimate the deflection of starlight by Sun using an elementary analysis. **Discussion:** Consider first a simpler example of a similar phenomenon. An elevator car of width L is released from rest near the surface of Earth. At the instant of release a flash of light is fired horizontally from one wall of the car toward the other wall. After release the elevator car is an inertial frame. Therefore the light flash crosses the car in a straight line with respect to the car. With respect to Earth, however, the flash of light is falling—because the elevator is falling. Therefore a light flash is deflected in a gravitation field, as Newton would phrase it. (How would Einstein phrase it? See Chapter 9.) As another example, a ray of starlight in its passage tangentially across Earth's surface receives a gravitational deflection (over and above any refraction by Earth's atmosphere). However, the time to cross Earth is so short, and in consequence the deflection so slight, that this effect has not yet been detected on Earth. At the surface of Sun, however, the acceleration of gravity has the much greater value of 275 meters/second/second. Moreover, the time of passage across the surface is much increased because Sun has a greater diameter, 1.4×10^9 meters. In the following, assume that the light just grazes the surface of Sun in passing.

a Determine an "effective time of fall" from the diameter of Sun and the speed of light. From this time of fall deduce the net velocity of fall toward Sun produced by the end of the whole period of gravitational interaction. (The maximum acceleration acting for this "effective time" produces the same net effect [calculus proof!] produced by the actual acceleration — changing in magnitude and direction along the path — in the entire passage of the ray through Sun's field of force.)

b Comparing the lateral velocity of the light with its forward velocity, deduce the angle of deflection. The accurate analysis of special relativity gives the same result. However, Einstein's 1915 general relativity predicted a previously neglected effect, associated with the change of lengths in a gravitational field, that produces something like a supplementary refraction of the ray of light and doubles the predicted deflection. [Deflection observed in 1947 eclipse of Sun: $(9.8 \pm 1.3) \times 10^{-6}$ radian; in the 1952 eclipse: $(8.2 \pm 0.5) \times 10^{-6}$ radian.]

CHAPTER 3

SAME LAWS FOR ALL

The name relativity theory was an unfortunate choice: The relativity of space and time is not the essential thing, which is the independence of laws of Nature from the viewpoint of the observer.

Arnold Sommerfeld

3.1 THE PRINCIPLE OF RELATIVITY

fundamental science needs only a closed room

How do you know you are moving? Or at rest? In a car, you pause at a stoplight. You see the car next to you easing forward. With a shock you suddenly realize that, instead, your own car is rolling backward. On an international flight you watch a movie with the cabin shades drawn. Can you tell if the plane is traveling at minimum speed or full speed? In an elaborate joke, could the plane actually be sitting still on the runway, engines running? How would you know?

Everyday observations such as these form the basis for a conjecture that Einstein raised to the status of a postulate and set at the center of the theory of special relativity. He called it the **Principle of Relativity.** Roughly speaking, the Principle of Relativity says that without looking out the window you cannot tell which reference frame you are in or how fast you are moving.

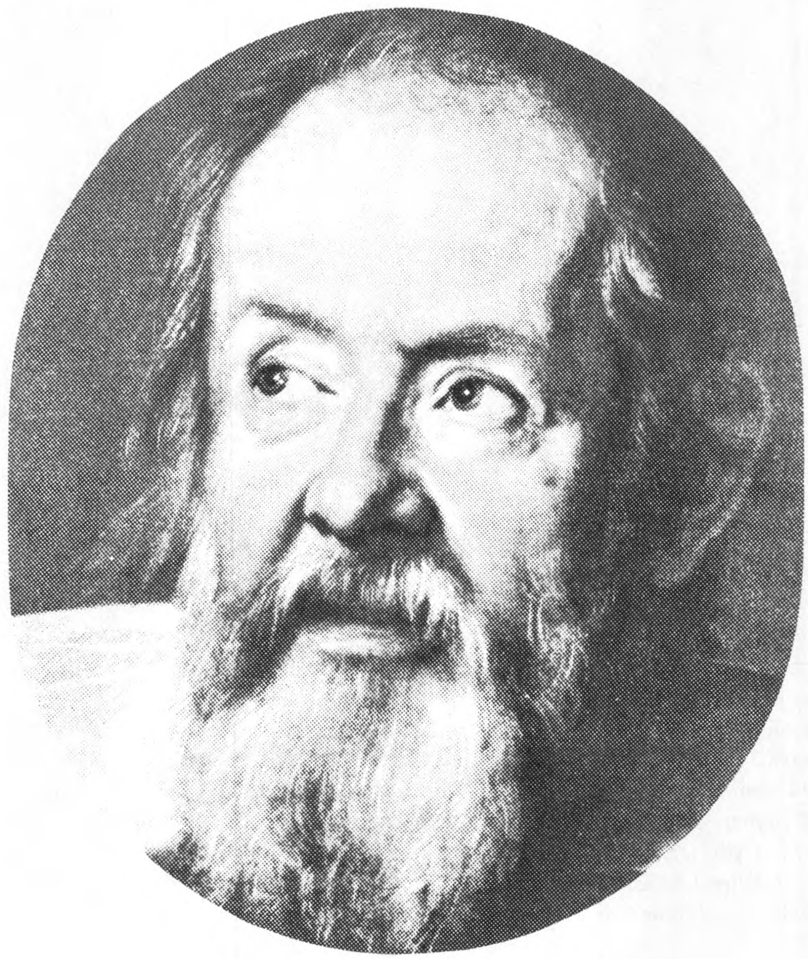
Principle of Relativity: With shades drawn you cannot tell your speed

Galileo Galilei made the first known formulation of the Principle of Relativity. Listen to the characters in his book:

Galileo: First known formulation of Principle of Relativity

> SALVATIUS: Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though there is no doubt that when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any

of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air . . .



GALILEO GALILEI

Pisa, February 15, 1564—Arcetri, near Florence, January 8, 1642

"My portrait is now finished, a very good likeness, by an excellent hand."
—*September 22, 1635*

* * *

"If ever any persons might challenge to be signally distinguished for their intellect from other men, Ptolemy and Copernicus were they that had the honor to see farthest into and discourse most profoundly of the World's systems."

* * *

"My dear Kepler, what shall we make of all this? Shall we laugh, or shall we cry?"

* * *

"When shall I cease from wondering?"

SAGREDUS: Although it did not occur to me to put these observations to the test when I was voyaging, I am sure that they would take place in the way you describe. In confirmation of this I remember having often found myself in my cabin wondering whether the ship was moving or standing still; and sometimes at a whim I have supposed it to be going one way when its motion was the opposite . . .

The Galilean Principle of Relativity is simple in this early formulation, yet not as simple as it might be. In what way is it simple? Physics looks the same in a ship moving uniformly as in a ship at rest. Relative uniform motion of the two ships does not affect the laws of motion in either ship. A ball falling straight down onto one ship appears from the other ship to follow a parabolic course; a ball falling straight down onto that second ship also appears to follow a parabolic course when observed from the first ship. The simplicity of the Galilean Principle of Relativity lies in the equivalence of the two Earthbound frames and the symmetry between them.

Extension of Galileo's reasoning from ship to spaceship

In what way is this simplicity not as great as it might be? In Galileo's account the frames of reference are not yet free-float (inertial). To make them so requires only a small conceptual step: from two uniformly moving sea-going ships to two unpowered spaceships. Then up and down, north and south, east and west, all become alike. A ball untouched by force undergoes no acceleration. Its motion with respect to one spaceship is as uniform as it is with respect to the other. This identity of the law of free motion in all inertial reference frames is what one means today by the Galilean Principle of Relativity.

Galileo could not by any stretch of the imagination have asked his hearer to place himself in a spaceship in the year 1632. Yet he could have described the greater simplicity of physics when viewed from such a vantage point. Bottles, drops of water, and all the other test objects float at rest or move at uniform velocity. The zero acceleration of every nearby object relative to the spaceship would have been intelligible to Galileo of all people. Who had established more clearly than he that relative to Earth all nearby objects have a common acceleration?

Einstein's Principle of Relativity is a generalization of such experiments and many other kinds of experiments, involving not only mechanics but also electromagnetism, nuclear physics, and so on.

Principle of Relativity

All the laws of physics are the same in every free-float (inertial) reference frame.

Einstein's Principle of Relativity says that once the laws of physics have been established in one free-float frame, they can be applied without modification in any other free-float frame. Both the mathematical form of the laws of physics and the numerical values of basic physical constants that these laws contain are the same in every free-float frame. So far as concerns the laws of physics, all free-float frames are equivalent.

We can tell where we are on Earth by looking out of the window. Where we are in the Milky Way we can tell by the configuration of the Big Dipper and other constellations. How fast and in what direction we are going through the larger framework of the universe we measure with a set of microwave horns pointed to pick up the microwave radiation streaming through space from all sides. But now exclude all information from outside. Screen out all radiation from the heavens. Pull down the window shade. Then do whatever experiment we will on the movement and collision of particles and the action of electric and magnetic forces in whatever free-float frame we please. We find not the slightest difference in the fit to the laws of physics between measurements made in one free-float frame and those made in another. We arrive at the Principle of Relativity in its negative form:

Principle of Relativity, negative form

No test of the laws of physics provides any way whatsoever to distinguish one free-float frame from another.

BOX 3-1

THE PRINCIPLE OF RELATIVITY RESTS ON EMPTINESS!

In his paper on special relativity, Einstein says, "We will raise this conjecture (whose intent will from now on be referred to as the 'Principle of Relativity') to a postulate . . . " Is the Principle of Relativity just a postulate? All of special relativity rests on it. How do we know it is true? What lies behind the Principle of Relativity?

This is a philosophical question, not a scientific one. You will have your own opinion; here is ours. We think the Principle of Relativity as used in special relativity rests on one word: emptiness.

Space is empty; there are no kilometer posts or mileposts in space. Do you want to measure distance and time? Then set up a latticework of meter sticks and clocks. Pace off the meter sticks, synchronize the clocks. Use the latticework to carry out your measurements. Discover the laws of physics. This latticework is your construction, not Nature's. Do not ask Nature to choose your latticework in preference to the similar latticework that I have constructed. Why not? Because space is empty. Space accommodates both of us as we go about our constructions and our investigations. But it does not choose either one of us in preference to the other. How can it? Space is empty. Nothing whatever can distinguish your latticework from mine. If we decide in secret to exchange latticeworks, Nature will never be the wiser! It follows that whatever laws of physics you discover employing your latticework must be the same laws of physics I discover using my latticework. The same is true even when our lattices move relative to one another. Which one of us is at rest? There is no way to tell in empty space! This is the Principle of Relativity.

But is space *really* empty? "Definitely not!" says modern quantum physics. "Space is a boiling cauldron of virtual particles. To observe this cauldron,

3.2 WHAT IS *NOT* THE SAME IN DIFFERENT FRAMES

not the same: space separations, time separations, velocities, accelerations, forces, fields

Space and time separations not the same in different frames

Notice what the Principle of Relativity does *not* say. It does not say that the time between two events is the same when measured from two different free-float frames. Neither does it say that space separation between the two events is the same in the two frames. Ordinarily neither time nor space separations are the same in the two frames.

The catalog of differences between readings in the two frames does not end with laboratory and rocket records of pairs of events. Physics to the Greeks meant the science of change and so it does to us today. Motion gives us a stream of events, for example the blinks of a firefly or the pulses of a sparkplug flashing as it moves. These flashes trace out the sparkplug's trajectory. Record the positions of two sequential sample regions of space much smaller than the proton. Carry out this sampling during times much shorter than the time it takes light to cross the diameter of the proton." These words are familiar or utterly incomprehensible, depending on the amount of our experience with physics. In either case, we can avoid dealing with the "boiling cauldron of virtual particles" by observing events that are far apart compared with the dimensions of the proton, events separated from one another by times long compared with the time it takes light to cross the diameter of the proton.

In the realm of classical (nonquantum) physics is space really empty? "Of course not!" says modern cosmology. "Space is full of stars and dust and radiation and neutrinos and white dwarfs and neutron stars and (many believe) black holes. To observe these structures, sample regions of space comparable in size to that of our galaxy. These structures evolve and move with respect to one another in times comparable to millions of years."

So we choose regions far from massive structures, avoid dust, ignore neutrinos and radiation, and measure events that take place close together in time compared with a million years.

Notice that for the very small and also for the very large, the "regions" described span both space and time — they are regions of *spacetime*. "Emptiness" refers to spacetime. Therefore we should have said from the beginning, "*Spacetime* is empty" — except for us and our apparatus — with limitations described above.

In brief, we can find "effectively empty" regions of spacetime of spatial extent quite a few orders of magnitude larger and smaller than dimensions of our bodies and of time spread quite a few orders of magnitude longer and shorter than times that describe our reflexes. In spacetime regions of this general size, empty spacetime can be found. In empty spacetime the Principle of Relativity applies. Where the Principle of Relativity applies, special relativity correctly describes Nature.

spark emissions in the laboratory frame. Record also the laboratory time between these sparks. Divide the change in position by the increase in time, yielding the laboratory-measured velocity of the sparkplug.

Spark events have identities that rise above all differences between reference frames. These events are recorded not only in the laboratory but also by recording devices and clocks in the rocket latticework. From the printouts of the recorders in the rocket frame we read off rocket space and time separations between sequential sparks. We divide. The quotient gives the rocket-measured velocity of the sparkplug. But both the space separation and the time separation between events, respectively, are ordinarily different for the rocket frame than for the laboratory frame. Therefore the rocket-measured velocity of the sparkplug is different from the laboratory-measured velocity of that sparkplug. Same world. Same motion. Different records of that motion. Figures for velocity that differ between rocket and laboratory.

Velocity not the same

Apply force to a moving object: Its velocity changes; it accelerates. Acceleration is the signal that force is being applied. Two events are enough to reveal velocity; three reveal change in velocity, therefore acceleration, therefore force. The laboratory observer reckons velocity between the first and second events, then he reckons velocity

Acceleration not the same

BOX 3-2

THE SPEED OF LIGHT

A "fundamental constant of nature"?
Or a mere factor of conversion between two units of measurement?

METERS AND MILES IN THE PARABLE OF THE SURVEYORS

Meter?
Originally (adopted France, 1799) one ten-millionth of the distance along the surface of Earth from its equator to its pole (in a curved line of latitude passing through the center of Paris).

Mile?
Originally one thousand *paces*—double step: right to left to right—of the Roman soldier.

Modern conversion factor?
1609.344 meters per mile.

Authority for this number?
Measures of equator-to-pole distance eventually (1799 to today) lagged in accuracy compared to laboratory measurement of distance. So the platinum meter rod at Sèvres, Paris, approximating one ten-millionth of that distance, for awhile became—in and by itself—the standard of distance. During that time the British Parliament and the United States Congress redefined the inch to be *exactly* 2.54 centimeters. This decree made the conversion factor (5280 feet/mile) times (12 inches/foot) times (2.54 centimeters/inch) times (1/100 of a meter per centimeter) equal to 1609.344 meters per mile—exactly!

A fundamental constant of nature?
Hardly! Rather, the work of two centuries of committees.

SECONDS AND METERS IN SPACETIME

Second?
Originally 1/24 of 1/60 of 1/60 of the time from high noon one day to high noon the next day. Since 1967, "The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the fundamental state of the atom cesium 133."

Meter?
Definition evolved from geographic to platinum meter rod to today's "One meter is the distance traveled by light, in vacuum, in the fraction 1/299,792,458 of a second."

Modern conversion factor?
299,792,458 meters per second.

Authority for this number?
Meeting of General Conference on Weights and Measures, 1983. In the accepted definition of the meter important changes took place over the years, and likewise in the definition of the second. With the 1983 definition of the meter these two streams of development have merged. What used to be understood as a measurement of the speed of light is understood today as two ways to measure separation in spacetime.

A fundamental constant of nature?
Hardly! Rather, the work of two centuries of committees.

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between the second and third events. Subtracting, he obtains the change in velocity. From this change he figures the force applied to the object.

The rocket observer also measures the motion: velocity between the first and second events, velocity between second and third events; from these the change in velocity; from this the force acting on the object. But the rocket-observed velocities are not equal to the corresponding laboratory-observed velocities. The *change* in velocity also differs in the two frames; therefore the computed *force* on the object is different for

Force not the same

Commentary

Is the distance from Earth's equator to its pole a fundamental constant of nature? No. Earth is plastic and ever changing. Is the distance between the two scratches on the standard meter bar constant? No. Oxidation from decade to decade slowly changes it. Experts in the art and science of measurement move to ever-better techniques. They search out an ever-better object to serve as benchmark. Via experiment after experiment they move from old standards of measurement to new. The goals? Accuracy. Availability. Dependability. Reproducibility.

Make a better measurement of the speed of light. Gain in that way better knowledge about light? No. Win instead an improved value of the ratio between one measure of spacetime interval, the meter, and another such measure, the second — both of accidental and historical origin? Before 1983, yes. Since 1983, no. Today the meter is *defined* as the distance light travels in a vacuum in the fraction 1/299,792,458 of a cesium-defined second. The two great streams of theory, definition, and experiment concerning the meter and the second have finally been unified.

What will be the consequence of a future, still better, measuring technique? Possibly it will shift us from the cesium-atom-based second to a pulsar-based second or to a still more useful standard for the second. But will that improvement in precision change the speed of light? No. Every past International Committee on Weights and Measures has operated on the principle of minimum dislocation of standards; we have to expect that the speed of light will remain at the decreed figure of 299,792,458 meters per second, just as the number of meters in the mile will remain at 1609.344. Through the fixity of this conversion factor c, any substantial improvement in the accuracy of defining the second will bring with it an identical improvement in the accuracy of defining the meter.

Is 299,792,458 a fundamental constant of nature? Might as well ask if 5280 is a fundamental constant of nature!

rocket observer and laboratory observer. The Principle of Relativity does not deny that the force acting on an object is different as reckoned in two frames in relative motion.

Electric and magnetic fields not the same

An electric field or a magnetic field or some combination of the two, acting on the electron, is the secret of action of many a device doing its quiet duty day after day in home, factory, or car. An electromagnetic force acting on an electron changes its velocity as it moves from event P to event Q and from Q to R. Laboratory and rocket observers do not agree on this change in velocity. Therefore they do not agree on the value of the force that changes that velocity. Nor, finally, do they agree on the magnitudes of the electric and magnetic fields from which the force derives.

In brief, figures for electric and magnetic field strengths, for forces, and for accelerations agree no better between rocket and laboratory observers than do figures for velocity. The Principle of Relativity does not deny these differences. It celebrates them. It explains them. It systematizes them.

3.3 WHAT *IS* THE SAME IN DIFFERENT FRAMES

the same: physical laws, physical constants in those laws

Laws of physics the same in different frames

Different values of some physical quantities between the two frames? Yes, but identical physical *laws!* For example, the relation between the force acting on a particle and the change in velocity per unit time of that particle follows the same law in the laboratory frame as in the rocket frame. The force is not the same in the two frames. Neither is the change in velocity per unit time the same. But the law that relates force and change of velocity per unit time is the same in each of the two frames. All the laws of motion are the same in the one free-float frame as in the other.

Not only the laws of motion but also the laws of electromagnetism and all other laws of physics hold as true in one free-float frame as in any other such frame. This is what it means to say, "No test of the laws of physics provides any way whatsoever to distinguish one free-float frame from another."

Fundamental constants the same

Deep in the laws of physics are numerical values of fundamental physical constants, such as the elementary charge on the electron and the speed of light. The values of these constants must be the same as measured in overlapping free-float frames in relative motion; otherwise these frames could be distinguished from one another and the Principle of Relativity violated.

Speed of light the same

One basic physical constant appears in the laws of electromagnetism: the speed of light in a vacuum, $c = 299{,}792{,}458$ meters per second. According to the Principle of Relativity, this value must be the same in all free-float frames in uniform relative motion. Has observation checked this conclusion? Yes, many experiments demonstrate it daily and hourly in every particle-accelerating facility on Earth. Nevertheless, it has taken a long time for people to become accustomed to the apparently absurd idea that there can be one special speed, the speed of light, that has the same value measured in each of two overlapping free-float frames in relative motion.

Values of the speed of light as measured by laboratory and by rocket observer turn out identical. This agreement has cast a new light on light. Its speed rates no longer as a constant of nature. Instead, today the speed of light ranks as mere conversion factor between the meter and the second, like the factor of conversion from the centimeter to the meter. The value of this conversion factor has now been set by decree and the meter defined in terms of it (Box 3.2). This decree *assumes* the invariance of the speed of light. No experimental result contradicts this assumption.

In 1905 the Principle of Relativity was a shocking heresy. It offended most people's intuition and common-sense way of looking at Nature. Consequences of the Principle of Relativity are tried out every day in many experiments where it is continually under severe test. Never has this Principle been verified to lead to a single incorrect experimental prediction.

SAMPLE PROBLEM 3-1

EXAMPLES OF THE PRINCIPLE OF RELATIVITY

Two overlapping free-float frames are in uniform relative motion. According to the Principle of Relativity, which of the quantities on the following list must *necessarily* be the same as measured in the two frames? Which quantities are *not* necessarily the same as measured in the two frames?

- **a.** numerical value of the speed of light in a vacuum
- **b.** speed of an electron
- **c.** value of the charge on the electron
- **d.** kinetic energy of a proton (the nucleus of a hydrogen atom)
- **e.** value of the electric field at a given point
- **f.** time between two events
- **g.** order of elements in the periodic table
- **h.** Newton's First Law of Motion ("A particle initially at rest remains at rest, and . . . ")

SOLUTION

- **a.** The speed of light IS necessarily the same in the two frames. This is one of the central tenets of the Principle of Relativity and a basis of the theory of relativity.
- **b.** The speed of an electron IS NOT necessarily the same in the two frames. Determining the speed of a particle depends on space and time measurements between events — such as flashes emitted by the particle. Space and time separations between events, respectively, can be measured to be different for observers in relative motion. So the speed — ratio of distance covered to time elapsed — can be different.
- **c.** The value of the charge on the electron IS necessarily the same in the two frames. Suppose that the charge had one value for the laboratory frame and progressively smaller values for rocket frames moving faster and faster relative to the laboratory frame. Then we could detect the "absolute velocity" of the frame we are in by measuring the charge on the electron. But this violates the Principle of Relativity. Therefore the charge on the electron must have the same value in all free-float frames.
- **d.** The kinetic energy of a proton IS NOT necessarily the same in the two frames. The value of its kinetic energy depends on the speed of the proton. But speed is not necessarily the same as measured in the two frames (**b**).
- **e.** The value of the electric field at a given point IS NOT necessarily the same in the two frames. The argument is indirect but inescapable: The electric field is measured by determining the force on a test charge. Force can be measured by change in velocity that the force imparts to a particle of known mass. But the velocity — and the change in velocity — of a particle can be *different* for observers in relative motion (**b**). Therefore the electric field may be different for observers in relative motion.
- **f.** The time between two events IS NOT necessarily the same in the two frames. This is a direct result of the invariance of the interval (Chapter 1 and Section 3.7).

SAMPLE PROBLEM 3-1

g. The order of elements in the periodic table by atomic number IS necessarily the same in the two frames. For suppose that the atomic number (the number of protons in the nucleus) were smaller for helium than for uranium in the laboratory frame but greater for helium than for uranium in the rocket frame. Then we could tell which frame we were in by comparing the atomic numbers of helium and uranium.

h. Newton's First Law of Motion IS necessarily the same in the two frames. Newton's First Law is really a definition of the inertial (free-float) frame. We assume that all laboratory and rocket frames are inertial.

3.4 RELATIVITY OF SIMULTANEITY

"same time"? ordinarily true for only one frame!

The Principle of Relativity directly predicts effects that initially seem strange—even weird. Strange or not, weird or not; logical argument demonstrates them and experiment verifies them. One effect has to do with simultaneity: Let two events occur separated in space along the direction of relative motion between laboratory and rocket frames. These two events, even if simultaneous as measured by one observer, cannot be simultaneous as measured by both observers.

Einstein demonstrated the relativity of simultaneity with his famous Train Paradox. (When Einstein developed the theory of special relativity, the train was the fastest common carrier.) Lightning strikes the front and back ends of a rapidly moving train, leaving char marks on the train and on the track and emitting flashes of light that travel forward and backward along the train (Figure 3-1). An observer standing on the ground halfway between the two char marks on the track receives the two light flashes at the same time. He therefore concludes that the two lightning bolts struck the track at the same time—with respect to him they fell simultaneously.

Train Paradox: Two lightning bolts strike simultaneously for ground observer

A second observer rides in the middle of the train. From the viewpoint of the observer on the ground, the train observer moves toward the flash coming from the front of the train and moves away from the flash coming from the rear. Therefore the train observer receives the flash from the front of the train first.

Two lightning bolts *do not* strike simultaneously for train observer

This is just what the train observer finds: The flash from the front of the train arrives at her position first, the flash from the rear of the train arrives later. But she can verify that she stands equidistant from the front and rear of the train, where she sees char marks left by the lightning. Moreover, using the Principle of Relativity, she knows that the speed of light has the same value in her train frame as for the ground observer (Section 3.3 and Box 3-2), and is the same for light traveling in both directions in her frame. Therefore the arrival of the flash first from the front of the train leads her to conclude that the lightning fell first on the front end of the train. For her the lightning bolts did not fall simultaneously. (To allow the train observer to make only measurements with respect to the train, forcing her to ignore Earth, let the train be a cylinder without windows—in other words a spaceship!)

Did the two lightning bolts strike the front and the back of the train simultaneously? Or did they strike at different times? Decide!

Strange as it seems, there is no unique answer to this question. For the situation described above, the two events are simultaneous as measured in the Earth frame; they

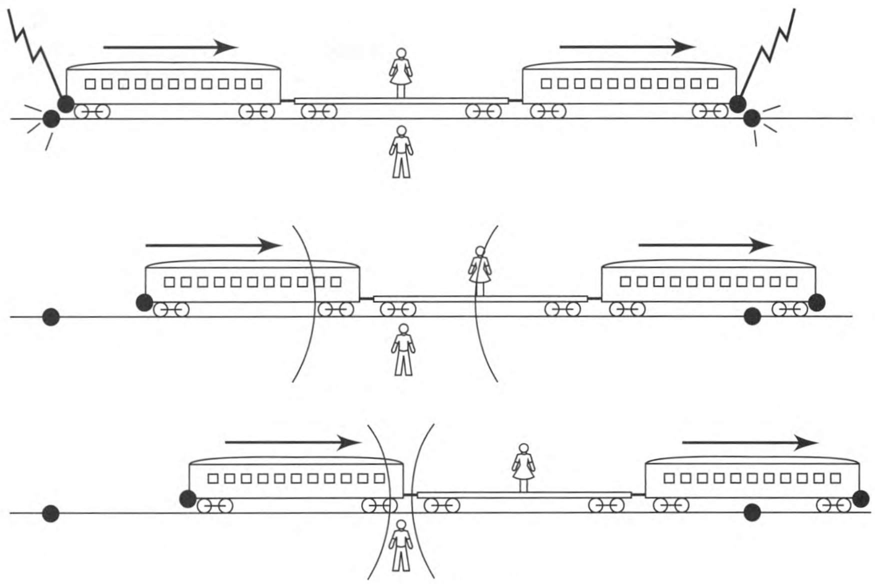


FIGURE 3-1. ***Einstein's Train Paradox illustrating the relativity of simultaneity. Top:*** *Lightning strikes the front and back ends of a moving train, leaving char marks on both track and train. Each emitted flash spreads out in all directions.* ***Center:*** *Observer riding in the middle of the train concludes that the two strokes are not simultaneous. Her argument: "(1) I am equidistant from the front and back char marks on the train. (2) Light has the standard speed in my frame, and equal speed in both directions. (3) The flash arrived from the front of the train first. Therefore, (4) the flash must have left the front of the train first; the front lightning bolt fell before the rear lightning bolt fell. I conclude that the lightning strokes* were not simultaneous." ***Bottom:*** *Observer standing by the tracks halfway between the char marks on the tracks concludes that the strokes* were simultaneous, *since the flashes from the strokes reach him at the same time.*

are not simultaneous as measured in the train frame. We say that the simultaneity of events is, in general, *relative,* different for different frames. Only in the special case of two or more events that occur at the same point (or in a plane perpendicular to the line of relative motion at that point—see Section 3.6) does simultaneity in the laboratory frame mean simultaneity in the rocket frame. When the events occur at different locations along the direction of relative motion, they cannot be simultaneous in both frames. This conclusion is called the **relativity of simultaneity.**

Simultaneity is relative

The relativity of simultaneity is a difficult concept to understand. Almost without exception, every puzzle and apparent paradox used to "disprove" the theory of relativity hinges on some misconception about the relativity of simultaneity.

3.5 LORENTZ CONTRACTION OF LENGTH

space separation between two length-measuring events? disagreement!

How do we measure the length of a moving rod—the distance between one end and the other end? One way is to use our latticework of clocks to mark the location of the two ends at the same time. But when the rod lies along the direction of relative motion, someone riding with the rod does not agree that our marking of the positions of the two ends occurs at the same time (Section 3.4). The relativity of simultaneity tells us

Length of a rod = separation between simultaneous sparks at its two ends that rocket and laboratory observers disagree about the simultaneity of two events (firecrackers exploding at the two ends of the rod) that occur at different locations along the direction of relative motion. Therefore the two observers disagree about whether or not a valid measurement of length has taken place.

Go back to the Train Paradox. For the observer standing on the ground, the two lightning bolts strike the front and back of the train at the same time. Therefore for him the distance between the char marks on the track constitutes a valid measure of the length of the train. In contrast, the observer riding on the train measures the front lightning bolt to strike first, the rear bolt later. The rider on the train exclaims to her Earth-based colleague, "See here! Your front mark was made before the back mark — since the flash from the front reached me (at the middle of the train) before the flash from the back reached me. Of course the train moved during the time lapse between these two lightning strikes. By the time the stroke fell at the back of the train, the front of the train had moved well past the front char mark on the track. Therefore your measurement of the length of the train is too small. The train is really longer than you measured."

Disagree about simultaneity? Then disagree about length.

There are other ways to measure the length of a moving rod. Many of these methods lead to the same result: the space separation between the ends of the rod is less as measured in a frame in which the rod is moving than as measured in a frame in which the rod is at rest. This effect is called **Lorentz contraction.** Section 5.8 examines the Lorentz contraction quantitatively.

Suppose we agree to measure the length of a rod by determining the position of its two ends at the same time. Then an observer for whom the rod is at rest measures the rod to be longer than does any other observer. This "rest length" of the rod is often called its **proper length.**

You keep using the word "measure." Occasionally you say "observe." You never talk about that most delicate, sensitive, and refined of our five senses: sight. Why not just look *and* see *these remarkable relativistic effects?*

We have been careful to say that the relativity of simultaneity and the Lorentz contraction are *measured,* not *seen* with the eye. *Measurement* employs the latticework of rods and clocks that constitutes a free-float frame. As mentioned in Chapter 2, seeing with the eye leads to confused images due to the finite speed of light. Stand in an open field in the southern hemisphere as Sun sets in the west and full Moon rises in the east: You see Moon as it was 1.3 seconds ago, Sun as it was eight minutes ago, the star Alpha Centauri (nearest star visible to the naked eye) as it was 4.34 years ago, the Andromeda nebula as it was *2 million years ago* — you see them all *now.* Similarly, light from the two separated ends of a speeding rod typically takes different times to reach your eye. This relative time delay results in visual distortion that is avoided when the location of each end is recorded locally, with zero or minimal delay, by the nearest lattice clock. Visual appearance of rapidly moving objects is itself an interesting study, but for most scientific work it is an unnecessary distraction. To avoid this kind of confusion we set up the free-float latticework of synchronized recording clocks and insist on its use — at least in principle!

Aha! Then I have caught you in a contradiction. Figure 3-1 shows lightning flashes and trains. Is this not a picture of what we would see with our eyes?

No. Strictly speaking, each of the three "pictures" in Figure 3-1 summarizes where parts of the train are as recorded by the Earth latticework of clocks at a given instant of Earth time. The position of each light flash at this instant is also recorded by the clocks in the lattice. The summary of data is then given to a draftsman, who draws the picture for that Earth time. To distinguish such a drafted picture from the visual view, we will often refer to it as a **plot.** For example, Figure 3-1 (top) is the Earth plot at the time when lightning bolts strike the two ends of the train.

Actually, all three plots in Figure 3-1 show approximately what you see through a telescope when you are very far from the scene in a direction perpendicular to the direction of motion of the train and at a position centered on the action. At such a remote location, light from all parts of the scene takes approximately equal times to reach your eye, so you would see events and objects at approximately the same time according to Earth clocks. Of course, you receive this information later than it actually occurs because of the time it takes light to reach you.

3.6 INVARIANCE OF TRANSVERSE DIMENSION

"faster" does not mean "thinner" or "fatter"

A rocket ship makes many trips past the laboratory observer, each at successively higher speed. For each new and greater speed of the rocket, the laboratory observer measures its length to be shorter than it was on the trip before. This observed contraction is **longitudinal**—along its direction of motion. Does the laboratory observer also measure contraction in the **transverse** dimension, perpendicular to the direction of relative motion? In brief, is the rocket measured to get thinner as well as shorter as it moves faster and faster?

Transverse dimension same for laboratory and rocket observers

The answer is No. This is confirmed experimentally by observing the width of electron and proton beams traveling in high-energy accelerators. It is also easily demonstrated by simple thought experiments.

Speeding-Train Thought Experiment: Return to Einstein's high-speed railroad train seen end-on (Figure 3-2). Suppose the Earthbound observer measures the train to get thinner as it moves faster. Then for the Earth observer the right and left wheels of the train would come closer and closer together as the train speeds up, finally slipping off *between the tracks* to cause a terrible wreck. In contrast, the train observer regards herself as at rest and the tracks as speeding by in the opposite direction. If she

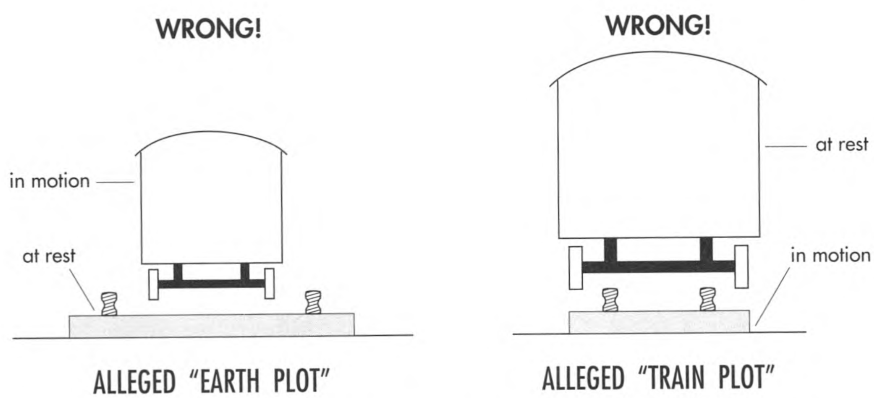


FIGURE 3-2. ***Two possible alternatives (both wrong!) if the moving train is measured to shrink transverse to its direction of motion.*** *The* ***"Earth plot"*** *assumes the speeding train to be measured as getting thinner with increasing speed. The train's wheels would slip off* between the tracks. *The* ***"train plot"*** *of the same circumstance assumes the speeding rails to be measured as getting closer together. In this case the wheels would slip off* outside the tracks. *But this is a contradiction. Therefore the wheel separation —and the transverse dimensions of train and track—must be* invariant, *the same for all free-float observers moving along the track. (If you think that the actual transverse contraction might be too small to cause a wreck for the train shown, assume that both the wheels and the track are knife edges; the same argument still applies.)*

measures the speeding tracks to get closer together as they move faster and faster, the train wheels will slip off *outside the tracks,* also resulting in a wreck. But this is absurd: the wheels cannot end up between the tracks and outside the tracks under the same circumstances. Conclusion: High speed leads to no measured change in transverse dimensions — no observed thinning or fattening of fast objects. We are left with the conclusion that high relative speed affects the measured values of longitudinal dimensions but not transverse dimension: a welcome simplification!

Thought experiments demonstrate invariance of transverse dimension

Speeding-Pipes Thought Experiment: Start with a long straight pipe. Paint one end with a checkerboard pattern and the other end with stripes. Cut out and discard the middle of the pipe, leaving only the painted ends. Now hurl the ends toward each other, with their cylindrical axes lying along a common line parallel to the direction of relative motion (Figure 3-3). Suppose that a moving object is measured to be thinner. Then someone riding on the checkerboard pipe will observe the striped pipe to pass inside her cylinder. *All* observers — everyone looking from the side — will see a checkerboard pattern. In contrast, someone riding on the striped pipe will observe the checkerboard pipe to pass inside his cylinder. In this case, all observers will see a striped pattern. Again, this is absurd: All observers must see stripes, or all must see checkerboard. The only tenable conclusion is that speed has no measurable effect on transverse dimensions and the pipe segments will collide squarely edge on.

A simple question leads to an even more fundamental argument against the difference of transverse dimensions of a speeding object as observed by different free-float observers in relative motion: *About what axis* does the contraction take place?

We try to define an "axis of shrinkage" parallel to the direction of relative motion. Can we claim that a speeding pipe gets thinner by shrinking uniformly toward an "axis of shrinkage" lying along its center? Then what happens when two pipe segments move along their lengths, side by side as a pair? Does each pipe shrink separately, causing the clear space between them to *increase?* Or does the combination of both pipes contract toward the line midway between them, causing the clear space between them to *decrease?* Is the answer different if one pipe is made of lead and the other one of paper? Or if one pipe is entirely in our imagination?

There is no logically consistent way to define an "axis of shrinkage." Given the direction of relative motion of two objects, we cannot select uniquely an "axis of shrinkage" from the infinite number of lines that lie parallel in this direction. For each different choice of axis a different pattern of distortions results. But this is logically intolerable. The only way out is to conclude that there is no transverse shrinkage at all (and, by a similar argument, no transverse expansion).

The above analysis leads to conclusions about events as well as about objects. A set of explosions occurs around the perimeter of the checkerboard pipe. More: These explosions occur simultaneously in this checkerboard frame. Then these events are simultaneous also in the striped frame. How do we know? By symmetry! For suppose the explosions were *not* simultaneous in the striped frame. Then which one of these

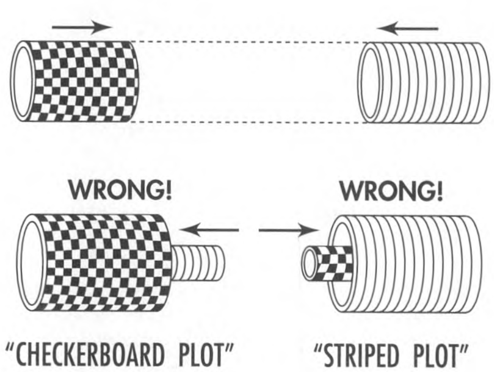


FIGURE 3-3. *Two identical-size pipe segments hurtle toward each other along a common centerline. What will happen when they meet? Here are two possible alternatives (both wrong!) if a moving object is observed to shrink transverse to direction of motion. Which pipe passes inside the other? The impossibility of a consistent answer to this question leads to the conclusion that neither pipe can be measured to change transverse dimension.*

events would occur first in the striped frame? The one on the right side of the pipe or the one on the left side of the pipe? But "left" and "right" cannot be distinguished by means of any physical effect: Each pipe is cylindrically symmetric. Moreover, space is the same in all directions — space is **isotropic,** the same to right as to left. So neither the event on the right side nor the event on the left side can be first. They must be simultaneous. The same argument can be made for events at the "top" and "bottom" of the pipe, and for every other pair of events on opposite sides of the pipe. Conclusion: If the explosions are simultaneous in the checkerboard frame, they must also be simultaneous in the striped frame.

"Same time" agreed on for events separated only transverse to relative motion

We make the following summary conclusions about dimensions transverse to the direction of relative motion:

> **Dimensions of moving objects transverse to the direction of relative motion are measured to be the same in laboratory and rocket frames (invariance of transverse distance).**
>
> **Two events with separation only transverse to the direction of relative motion and simultaneous in either laboratory or rocket frame are simultaneous in both.**

3.7 INVARIANCE OF THE INTERVAL PROVED

laboratory and rocket observers agree on something important

The Principle of Relativity has a major consequence. It demands that the spacetime interval have the same value as measured by observers in every overlapping free-float frame; in brief, it demands "invariance of the interval." Proof? Plan of attack: Determine the separation in space and the separation in time between two events, E and R, in the rocket frame. Then determine the quite different space and time separations between the same two events as measured in a free-float laboratory frame. Then look for — and find — what is invariant. It is the "interval." Now for the details (Figures 3-4 and 3-5).

Principle of Relativity leads to invariance of spacetime interval

Event E we take to be the reference event, the emission of a flash of light from the central laboratory and rocket reference clocks as they coincide at the zero of time (Section 2.6). The path of this flash is tracked by the recording clocks in the rocket lattice. Riding with the rocket, we examine that portion of the flash that flies straight "up" 3 meters to a mirror. There it reflects straight back down to the photodetector located at our rocket reference clock, where it is received and recorded. The act of reception constitutes the second event we consider. This event, R, is located at the rocket space origin, at the same location as the emission event E. Therefore, for the rocket observer, the space separation between event E and event R equals zero.

What is the time separation between events E and R in the rocket frame? The light travels 3 meters up to the mirror and 3 meters back down again, a total of 6 meters of distance. At the "standard" light speed of 1 meter of distance per meter of light-travel time, the flash takes a total of 6 meters of time to complete the round trip. In summary, for the rocket observer the event of reception, R, is separated from the event of emission, E, by zero meters in space and 6 meters in time.

What are the space and time separations of events E and R measured in the free-float laboratory frame? As measured in the laboratory, the rocket moves at high speed to the right (Figures 3-4 and 3-5). The rocket goes so fast that the simple

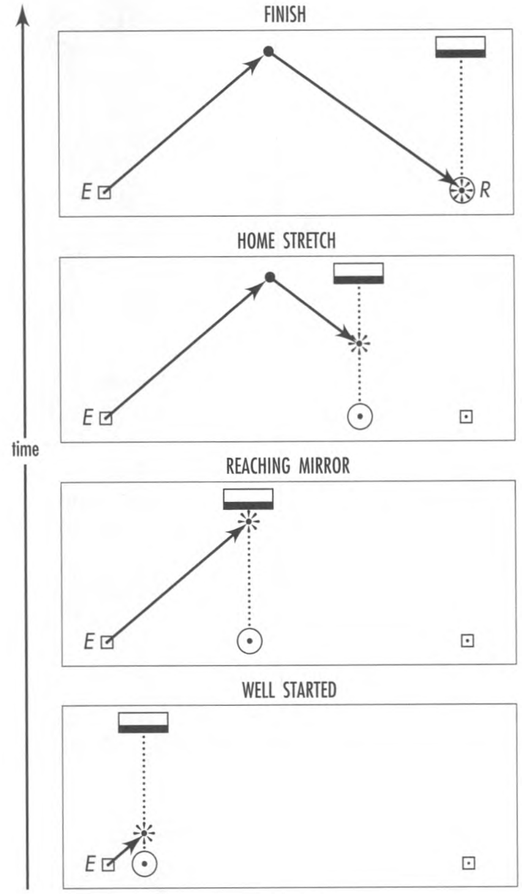


FIGURE 3-4. ***Plot of the flash path as recorded in the laboratory frame.*** *Time progresses from bottom to top:* ***Well started:*** *The flash (represented as an asterisk) has been emitted (event* E*) from a moving rocket clock (shown as a circle) that coincided with a laboratory clock (shown as a square).* ***Reaching mirror*** *and* ***Home stretch:*** *The flash reaches a mirror and reflects from it. The mirror moves along in step with the rocket clock.* ***Finish:*** *The flash is received (event* R*) back at the same rocket clock, which has moved in the laboratory frame to coincide with a second laboratory clock. Figure 3-5 shows the trajectory of the same flash in three different free-float frames.*

up-down track of the light in the rocket frame appears in the laboratory to have the profile of a tent, with its right-hand corner — the place of reception of the light — 8 meters to the right of the starting point.

When does the event of reception, R, take place as registered in the laboratory frame? Note that it occurs at the time 6 meters in the rocket frame. All we know about everyday events urges us to say, "Why, obviously it occurs at 6 meters of time in the laboratory frame too." But no. More binding than preconceived expectations are the demands of the Principle of Relativity. Among those demands none ranks higher than this: The speed of light has the standard value 1 meter of distance in 1 meter of light-travel time in every free-float frame.

Greater distance of travel for light flash: longer time!

Figure 3-6 punches us in the eye with this point: The light flash travels *farther* as recorded in the laboratory frame than as recorded in the rocket frame. The perpendicular "altitude" of the mirror from the line along which the rocket reference clock moves has the same value in laboratory frame as in rocket frame no matter how fast the rocket — as shown in Section 3.6. Therefore on its slanted path toward and away from the mirror the flash must cover more distance in the laboratory frame than it does in the rocket frame. More distance covered means more time required at the "standard" light speed. We conclude that the time between events E and R is greater in the laboratory frame than in the rocket frame — a staggering result that stood physics on its ear when first proposed. There is no way out.

In the laboratory frame the flash has to go "up" 3 meters, as before, and "down" again 3 meters. But in addition it has to go 8 meters to the right: 4 meters to the right while rising to hit the mirror, and 4 meters more to the right while falling again to the receptor. The Pythagorean Theorem, applied to the right triangles of Figure 3-6, tells

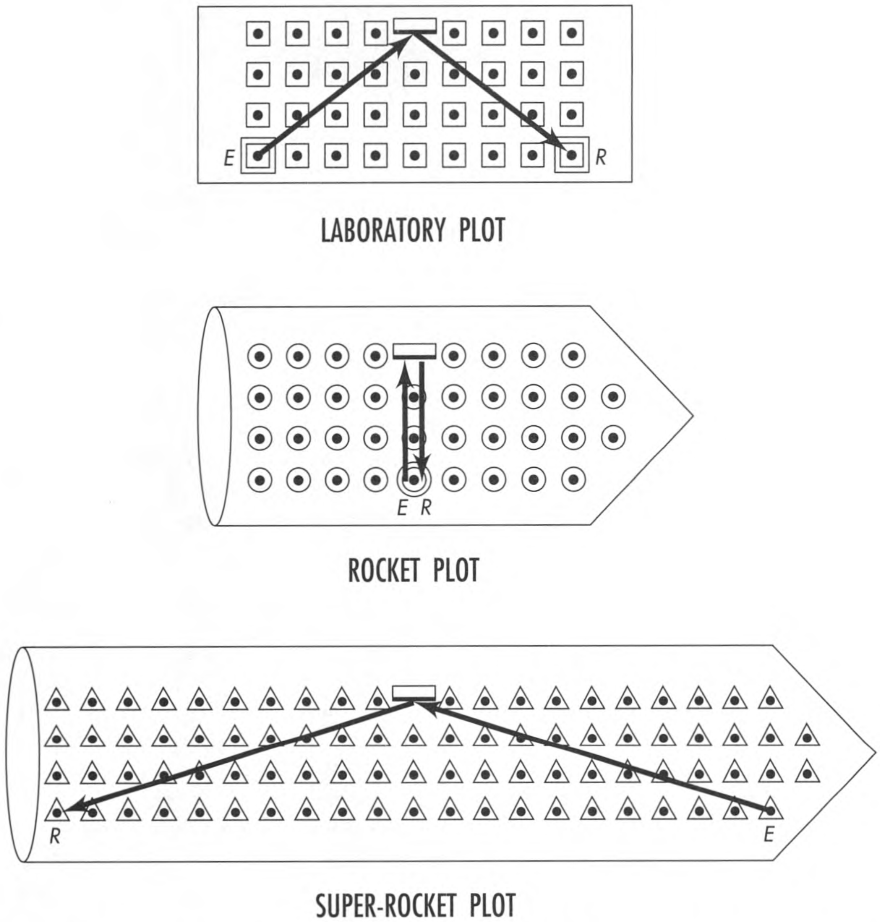


FIGURE 3-5. ***Plots of the path in space of a reflected flash of light as measured in three different frames, showing event* E*, emission of the flash, and event* R*, its reception after reflection.*** *Squares, circles, and triangles represent latticeworks of recording clocks in laboratory, rocket, and super-rocket frames, respectively. The super-rocket frame moves to the right with respect to the rocket, and with such relative speed that the event of reception,* R*, occurs to the left of the event of emission,* E*, as measured in the super-rocket frame. The reflecting mirror is fixed in the rocket, hence appears to move from left to right in the laboratory and from right to left in the super-rocket.*

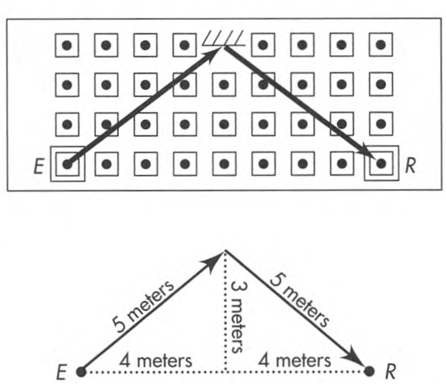


FIGURE 3-6. ***Laboratory plot of the path of the light flash.*** *The flash rises 3 meters while it moves to the right 4 meters. Then it falls 3 meters as it moves an additional 4 meters to the right. From the Pythagorean Theorem, the total length of the flash path equals 5 meters plus 5 meters or 10 meters. Therefore 10 meters of light-travel time is the separation in time between emission event* E *and reception event* R *as measured in the laboratory frame.*

us that each slanted leg of the trip has length 5 meters:

$$(3 \text{ meters})^2 + (4 \text{ meters})^2 = (5 \text{ meters})^2$$

Thus the total length of the trip equals 10 meters, definitely longer than the length of the round trip, 6 meters, as observed in the rocket frame. Moreover, the light can cover that slanted and greater distance only at the standard rate of 1 meter of distance in 1 meter of light-travel time. Therefore there is no escape from saying that the time of reception as recorded in the laboratory frame equals 10 meters. Thus there is a great variance between what is recorded in the two frames (Figure 3-5, Laboratory plot and Rocket plot): separation in time and in space between the emission E of a pulse of light and its reception R after reflection.

In spite of the difference in space separation between events E and R and the difference in time lapse between these events as measured in laboratory and rocket frames, there exists a measure of their separation that has the same value for both observers. This is the interval calculated from the difference of squares of time and space separations (Table 3-1). For both observers the interval has the value 6 meters. The interval is an **invariant** between free-float frames.

Two central results are to be seen here, one of variance, the other of invariance. We discover first that typically there is not and cannot be an absolute time difference between two events. The difference in time depends on our choice of the free-float frame, which inertial frame we use to record events. There is no such thing as a simple concept of universal and absolute separation in time.

Between events: No absolute time, but invariant interval

Second, despite variance between the laboratory frame and the rocket frame in the values recorded for time and space separations individually, the difference between the squares of those separations is identical, that is, invariant with respect to choice of reference frame. The difference of squares obtained in this way defines the square of the interval. The invariant interval itself has the value 6 meters in this example.

TABLE 3-1

RECKONING THE SPACETIME INTERVAL FROM ROCKET AND LABORATORY MEASUREMENTS

	Rocket measurements		Laboratory measurements
Time from emission of the flash to its reception	6 meters	← DIFFERENT! →	10 meters
Distance from the point of emission of the flash to its point of reception	0 meters	← DIFFERENT! →	8 meters
Square of time	36 (meters)^2		100 (meters)^2
Square distance and subtract	−0 (meters)^2		−64 (meters)^2
Result of subtraction	36 (meters)^2		36 (meters)^2
This is the square of what measurement?	6 meters		6 meters

SAME SPACETIME
INTERVAL

3.8 INVARIANCE OF THE INTERVAL FOR *ALL* FREE-FLOAT FRAMES

super-rocket observer joins the agreement

The interval between two events has the same value for *all possible* relative speeds of overlapping free-float frames. As an example of this claim, consider a third free-float frame moving at a different speed with respect to the laboratory frame—a speed different from that of the rocket frame.

Super-rocket: Same interval between events

We now measure the same events of emission and reception from a "super-rocket frame" moving faster than the rocket (but not faster than light!) along the line between events E and R (Figure 3-5, Super-rocket plot). For convenience we arrange that the reference clock of this frame also coincides with reference clocks of the other two frames at event E.

Events E and R occur at the same place in the rocket frame. Between these two events the super-rocket moves to the *right* with respect to the rocket. As a result, the super-rocket observer records event R as occurring to the *left* of the emission event. How far to the left? That depends on the relative speed of the super-rocket frame.

The super-rocket is not super-size; rather it has super-speed. We adjust this super-speed so that the reception occurs 20 meters to the left of the emission for the super-rocket observer. Then the flash of light that rises vertically in the rocket must travel the same 3 meters upward in the super-rocket but also 10 meters to the left as it slants toward the mirror. Hence the distance it travels to the mirror in the super-rocket frame is the length of a hypotenuse, 10.44 meters:

$$\begin{aligned}(3 \text{ meters})^2 + (10 \text{ meters})^2 &= 9 \text{ meters}^2 + 100 \text{ meters}^2 = 109 \text{ meters}^2 \\ &= (10.44 \text{ meters})^2\end{aligned}$$

It must travel another 10.44 meters as it slants downward and leftward to the event of reception. The total distance traveled equals 20.88 meters. It follows that the total time lapse between E and R equals 20.88 meters of light-travel time for the super-rocket observer.

The speed of the super-rocket is very high. As a result the space separation between emission and reception is very great. But then the time separation is also very great. Moreover, the magnitude of the time separation is perfectly tailored to the size of the space separation. In consequence, the particular quantity equal to the difference of their squares has the value (6 meters)^2, no matter how great the space separation and time separation individually may be. For the super-rocket frame:

$$\begin{aligned}(20.88 \text{ meters})^2 - (20 \text{ meters})^2 &= 436 \text{ meters}^2 - 400 \text{ meters}^2 = 36 \text{ meters}^2 \\ &= (6 \text{ meters})^2\end{aligned}$$

In spite of the difference in space separation observed in the three frames (0 meters for the rocket, 8 meters for the laboratory, 20 meters for the super-rocket) and the difference in time separation (6 meters for the rocket, 10 meters for the laboratory, 20.88 meters for the super-rocket), the interval between the two events has the same value for all three observers:

$$\begin{aligned}&\text{In general:} && (\text{time separation})^2 - (\text{space separation})^2 = (\text{interval})^2 \\ &\text{Rocket frame:} && (6 \text{ meters})^2 - (0 \text{ meters})^2 = (6 \text{ meters})^2 \\ &\text{Laboratory frame:} && (10 \text{ meters})^2 - (8 \text{ meters})^2 = (6 \text{ meters})^2 \\ &\text{Super-rocket frame:} && (20.88 \text{ meters})^2 - (20 \text{ meters})^2 = (6 \text{ meters})^2\end{aligned}$$

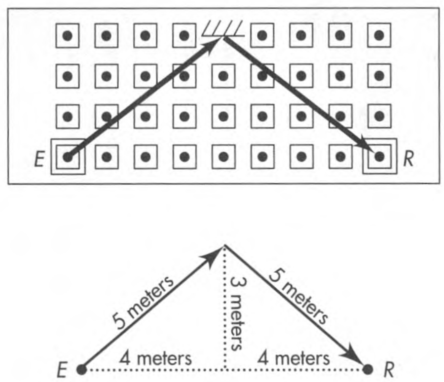


FIGURE 3-6 (repeated). *Laboratory plot of the path of the light flash.*

The laboratory observer clocks the time between the flash and its reception as 10 meters, in total disagreement with the 6 meters of timelike interval he figures between those two events. The observer in the super-rocket frame marks an even greater discrepancy, 20.88 meters of her time versus the 6 meters of timelike interval. Only for the rocket observer does clock time agree with interval. Why? Because only she sees reception at the same place as emission.

Invariance of interval from invariance of transverse dimension

The invariance of the interval can be seen at a glance in Figure 3-6. The hypotenuse of the first right triangle has a length equal to half the time separation between E and R. Its base has a length equal to half the space separation. To say that (time separation)^2 — (space separation)^2 has a standard value, and consequently to state that (half the time separation)^2 — (half the space separation)^2 has a standard value, is simply to say that the altitude of this right triangle has a fixed magnitude (3 meters in the diagram) for rocket and all super-rocket frames, no matter how fast they move. And this altitude has a length equal to half the interval between these two events.

SAMPLE PROBLEM 3-2

THE K^+ MESON

A beam of (unstable) K^+ mesons, traveling at a speed of $v = 0.868$, passes through two counters 9 meters apart. The particles suffer negligible loss of speed and energy in passing through the counters but give electrical pulses that can be counted. The first counter records 1000 pulses (1000 passing particles); the second records 250 counts (250 passing particles). This decrease arises almost entirely from decay of particles in flight. Determine the half-life of the K^+ meson in its own rest frame.

SOLUTION

Unstable particles of different kinds decay at different rates. By definition, the half-life of unstable particles of a particular species measures the particle wristwatch time during which — on the average — half of the particles decay. Half of the remaining particles decay in an additional time lapse equal to the same half-life, and so forth. In this case, one quarter of the K^+ particles remain after passage from counter to counter. Therefore the particles that survive experience the passage of two half-lives between counter and counter. We make the interval between those two passages, those two events, the center of our attention, because it has the same value in the laboratory frame where we do our measuring as it does in the free-float frame of the representative particle.

The keystone of the argument establishing the invariance of the interval between two events for all free-float frames? The Principle of Relativity, according to which there is no difference in the laws of physics between one free-float frame and another. This principle showed here in two very different ways. First, it said that distances at right angles to the direction of relative motion are recorded as of equal magnitude in the laboratory frame and the rocket frame (Section 3.6). Otherwise one frame could be distinguished from the other as the one with the shorter perpendicular distances.

Basis of invariance of interval: Principle of Relativity

Second, the Principle of Relativity demanded that the speed of light be the same in the laboratory frame as in the rocket frame. The speed being the same, the fact that the light-travel path in the laboratory frame (the hypotenuse of two triangles) is longer than the simple round-trip path in the rocket frame (the altitudes of these two triangles: up 3 meters and down again) directly implies a longer time in the laboratory frame than in the rocket frame.

In brief, one elementary triangle in Figure 3-6 displays four great ideas that underlie all of special relativity: invariance of perpendicular distance, invariance of the speed of light, dependence of space and time separations upon the frame of reference, and invariance of the interval.

3.9 SUMMARY

same laws for all; invariant interval for all

The **Principle of Relativity** says that the laws of physics are the same in every inertial (free-float) reference frame (Section 3.1). This simple principle has important consequences. Specifically:

$$(\text{interval})^2 = \begin{pmatrix}\text{separation}\\ \text{in lab}\\ \text{time}\end{pmatrix}^2 - \begin{pmatrix}\text{separation}\\ \text{in lab}\\ \text{position}\end{pmatrix}^2 = \begin{pmatrix}\text{separation}\\ \text{in moving-}\\ \text{particle time}\end{pmatrix}^2 - \begin{pmatrix}\text{separation}\\ \text{in moving-}\\ \text{particle position}\end{pmatrix}^2$$

$$= \left(\frac{\text{9 meters of distance}}{\begin{matrix}\text{0.868 meters of distance}\\ \text{per meter of time}\end{matrix}}\right)^2 - \begin{pmatrix}\text{9 meters}\\ \text{of distance}\end{pmatrix}^2 = (\text{2 half-lives})^2 - \begin{pmatrix}\text{zero separation}\\ \text{in space (in}\\ \text{particle frame)}\\ \text{between those}\\ \text{two events}\end{pmatrix}^2$$

$$= \begin{pmatrix}\text{10.368 meters}\\ \text{of light-travel time}\end{pmatrix}^2 - \begin{pmatrix}\text{9 meters}\\ \text{of distance}\end{pmatrix}^2 = (\text{2 half-lives})^2$$

A little arithmetic tells us that two half-lives total 5.15 meters of light-travel time. Consequently the K^+ half-life itself is 2.57 meters of time or (2.57 meters)/(3.00×10^8 meters/second) = 8.5×10^{-9} second or 8.5 nanoseconds.

1. Two events that lie along the direction of relative motion between two frames cannot be simultaneous as measured in both frames **(relativity of simultaneity).** (Section 3.4)
2. An object in high-speed motion is measured to be shorter along its direction of motion than its **proper length,** measured in its rest frame **(Lorentz contraction).** (Section 3.5)
3. The dimensions of moving objects transverse to their direction of relative motion are measured to be the same, whatever the relative speed **(invariance of transverse distances).** (Section 3.6)
4. Two events with separation only transverse to the direction of relative motion and simultaneous in either frame are simultaneous in both. (Section 3.6)

BOX 3-3

FASTER THAN LIGHT?

We always want to go faster. Faster than what? Faster than anything has gone before. What is our greatest possible speed, according to the theory of relativity? The speed of light in a vacuum! How do we know that this is the greatest possible speed that we can travel? Many lines of evidence reach this conclusion. Rocket speed greater than the speed of light would lead to the destruction of the essential relation between cause and effect, a result explored in Special Topic: Lorentz Transformation (especially Box L-1) and in Chapter 6. In particular, we could find a frame in which a faster-than-light object arrives before it starts! Moreover, in particle accelerators built over several decades we have spent hundreds of millions of dollars effectively trying to accelerate electrons and protons to the greatest possible speed—which by experiment never exceeds light speed.

The conclusion that no thing can move faster than light arises also from the invariance of the interval. To see this, let a rocket emit two flashes of light a time t' apart as measured in the rocket frame. (Use a prime to distinguish rocket measurements from laboratory measurements.) In the rocket frame the two emissions occur at the same place: the separation x' between them equals zero. Let t and x be the corresponding separations in time and space as measured in the laboratory frame. Then the invariance of the interval tells us that the three quantities t', t, and x are related by the equation

$$(t')^2 - (x')^2 = (t')^2 - (0)^2 = t^2 - x^2$$

whence

$$(t')^2 = t^2 - x^2 \tag{3-1}$$

In the laboratory frame the rocket is moving with some speed; give this speed the symbol v. The distance x between emissions is just the distance that the rocket moves in time t in the laboratory frame. The relation between

5. The spacetime interval between two events is invariant — it has the same value in laboratory and rocket frames (Sections 3.7 and 3.8):

$$(\text{interval})^2 = \overset{\textbf{Laboratory}}{\begin{pmatrix}\text{time}\\ \text{separation}\end{pmatrix}^2} - \overset{\textbf{Laboratory}}{\begin{pmatrix}\text{space}\\ \text{separation}\end{pmatrix}^2}$$

$$= \overset{\textbf{Rocket}}{\begin{pmatrix}\text{time}\\ \text{separation}\end{pmatrix}^2} - \overset{\textbf{Rocket}}{\begin{pmatrix}\text{space}\\ \text{separation}\end{pmatrix}^2}$$

6. In any free-float frame, no object moves with a speed greater than the speed of light (Box 3-3).

distance, time, and speed is

$$x = vt \tag{3-2}$$

Substitute this into equation (3-1) to obtain $(t')^2 = t^2 - (vt)^2 = t^2\,[1 - v^2]$, or

$$t' = t\,(1 - v^2)^{1/2} \tag{3-3}$$

Now, v is the speed of the rocket. How large can that speed be? Equation (3-3) makes sense for any rocket speed less than the speed of light, or when v has a value less than one.

Suppose we try to force the rocket to move faster than the speed of light. If we should succeed, v would have a value greater than one. Then v^2 also would have a value greater than one. But in this case the expression $1 - v^2$ would have a negative value and its square root would have no physical meaning. In a formal mathematical sense, the rocket time t' would be an imaginary number for the case of rocket speed greater than the speed of light. But clocks do not read imaginary time; they read real time — three hours, for example. Therefore a rocket speed greater than the speed of light leads to an impossible consequence.

Equation (3-3) does not forbid a rocket to go as close to the speed of light as we wish, as long as this speed remains less than the speed of light. For v very close to the speed of light, equation (3-3) tells us that the rocket time can be very much smaller than the laboratory time. Now suppose that emission of the first flash occurs when the rocket passes Earth on its outward trip to a distant star. Let emission of the second flash occur as the rocket *arrives* at that distant star. No matter how long the laboratory time t between these two events, we can find a rocket speed, v, such that the rocket time t' is as small as we wish. This means that in principle we can go to any remote star in as short a rocket time as we want. In brief, although our speed is limited to less than the speed of light, the distance we can travel in a lifetime has no limitation. We can go anywhere! This result is explored further in Chapter 4.

BOX 3-4

DOES A MOVING CLOCK *REALLY* "RUN SLOW"?

You keep saying, "The time between clock-ticks is shorter as MEASURED in the rest frame of the clock than as MEASURED in a frame in which the clock is moving." I am interested in reality, not someone's measurements. Tell me what really happens!

What is reality? You will have your own opinion and speculations. Here we pose two related scientific questions whose answers may help you in forming your opinion.

Are differences in clock rates *really* verified by experiment?
Different values of the time between two events as observed in different frames? Absolutely! Energetic particles slam into solid targets in accelerators all over the world, spraying forward newly created particles, some of which decay in very short times as measured in their rest frames. But these "short-lived" particles survive much longer in the laboratory frame as they streak from target to detector. In consequence, the detector receives a much larger fraction of the undecayed fast-moving particles than would be predicted from their decay times measured at rest. This result has been tested thousands of times with many different kinds of particles. Such experiments carried out over decades lead to dependable, consistent, repeatable results. As far as we can tell, they are correct, true, and reliable and cannot effectively be denied. If that is what you personally mean by "real," then these results are "what really happens."

Does something about a clock *really* change when it moves, resulting in the observed change in tick rate?
Absolutely not! Here is why: Whether a free-float clock is at rest or in motion in the frame of the observer is controlled by the observer. You want the clock

REFERENCES

Introductory quote: A. Sommerfeld, *Naturwissenschaftliche Rundschau,* Volume 1, pages 97–100, reprinted in *Gesammelte Schriften* (Vieweg, Braunschweig, 1968), Volume IV, pages 640–643.

Galileo quote, Section 3.1: Galileo Galilei, *Dialogue Concerning the Two Chief World Systems — Ptolemaic and Copernican,* first published February 1632; the translation quoted here is by Stillman Drake (University of California Press, Berkeley, 1962), pages 186ff. Galileo's writings, along with those of Dante, by reason of their strength and aptness, are treasures of human thought, studied today in Italy by secondary school students as part of a great literary heritage.

Einstein quote, Box 3-1: Albert Einstein, "On the Electrodynamics of Moving Bodies," *Annalen der Physik,* Volume 17, pages 891–921 (1905), translated by Arthur I. Miller in *Albert Einstein's Special Theory of Relativity* (Addison-Wesley, Reading, Mass., 1981), page 392.

to be at rest? Move along with it! Now do you want the clock to move? Simply change your own velocity! This is true even when you and the clock are separated by the diameter of the solar system. The magnitude of the clock's steady velocity is entirely under your control. Therefore the time between its ticks as measured in your frame is determined by your actions. How can your change of motion affect the inner mechanism of a distant clock? It cannot and does not.

Every time you change your motion on Earth — and even when you sit down, letting the direction of your velocity change as Earth rotates — you change the rate at which the planets revolve around Sun, as measured in your frame. (You also change the shape of planetary orbits, contracting them along the direction of your motion relative to Sun.) Do you think this change on your velocity really affects the workings of the "clock" we call the solar system? If so, what about a person who sits down on the other side of Earth? That person moves in the opposite direction around the center of Earth, so the results are different from yours. Are each of you having a different effect on the solar system? And are there still different effects — different solar-system clocks — for observers who could in principle be scattered on other planets?

We conclude that free-float motion does not affect the structure or operation of clocks (or rods). If this is what you mean by reality, then there are *really* no such changes due to uniform motion.

Is there some unity behind these conflicting measurements of time and space? Yes! The interval: the proper time (wristwatch time) between ticks of a clock as measured in a frame in which ticks occur at the same place, in which the clock is at rest. Proper time can also be calculated by all free-float observers, whatever their state of motion, and all agree on its value. Behind the confusing clutter of conflicting measurements stands the simple, consistent, powerful view provided by spacetime.

ACKNOWLEDGMENTS

The idea for Box 3-1 was suggested by Kenneth L. Laws. Box 3-4 and the argument for Section 3.6, Invariance of Transverse Dimension, is adapted from material by William A. Shurcliff, private communications. Sample Problem 3-2 is adapted from A. P. French, *Special Relativity* (W. W. Norton, New York, 1968), page 121.

CHAPTER 3 EXERCISES

PRACTICE

3-1 relativity and swimming

The idea here is to illustrate how remarkable is the invariance of the speed of light (light speed same in all free-float frames) by contrasting it with the case of a swimmer making her way through water.

Light goes through space at 3×10^8 meters/second, and the swimmer goes through the water at 1 meter/second. "But how can there otherwise be any difference?" one at first asks oneself.

For a light flash to go down the length of a 30-meter spaceship and back again takes

$$\begin{aligned} \text{time} &= (\text{distance})/(\text{speed}) \\ &= 2 \times (30 \text{ meters})/(3 \times 10^8 \text{ meters/second}) \\ &= 2 \times 10^{-7} \text{ second} \end{aligned}$$

as measured in the spaceship, regardless of whether the ship is stationary at the spaceport or is zooming past it at high speed.

Check how very different the story is for the swimmer plowing along at 1 meter/second with respect to the water.

a How long does it take her to swim down the length of a 30-meter pool and back again?

b How long does it take her to swim from float A to float B and back again when the two floats, A and B, are still 30 meters apart, but now are being towed through a lake at 1/3 meter/second? **Discussion:** When the swimmer is swimming in the same direction in which the floats are being towed, what is her speed relative to the floats? And how great is the distance she has to travel expressed in the "frame of reference" of the floats? So how long does it take to travel that leg of her trip? Then consider the same three questions for the return trip.

c Is it true that the total time from A to B and back again is independent of the reference system ("stationary" pool ends vs. moving floats)?

d Express in the cleanest, clearest, sharpest one-sentence formulation you can the difference between what happens for the swimmer and what happens for a light flash.

3-2 Einstein puzzler

When Albert Einstein was a boy of 16, he mulled over the following puzzler: A runner looks at herself in a mirror that she holds at arm's length in front of her. If she runs with nearly the speed of light, will she be able to see herself in the mirror? Analyze this question using the Principle of Relativity.

3-3 construction of clocks

For the measurement of time, we have made no distinction among spring clocks, quartz crystal clocks, biological clocks (aging), atomic clocks, radioactive clocks, and a clock in which the ticking element is a pulse of light bouncing back and forth between two mirrors (Figure 1-3). Let all these clocks be adjusted by the laboratory observer to run at the same rate when at rest in the laboratory. Now let the clocks all be accelerated gently to a high speed in a rocket, which then turns off its engines. Make a simple but powerful argument that the free-float rocket observer will also measure these different clocks all to run at the same rate as one another. Does it follow that the (common) clock rate of these clocks measured by the rocket observer is the same as their (common) rate measured by the laboratory observer as they pass by in the rocket?

3-4 the Principle of Relativity

Two overlapping free-float frames are in uniform relative motion. On the following list, mark with a "yes" the quantities that must *necessarily* be the same as measured in the two frames. Mark with a "no" the quantities that are *not* necessarily the same as measured in the two frames.

a time it takes for light to go one meter of distance in a vacuum
b spacetime interval between two events
c kinetic energy of an electron
d value of the mass of the electron
e value of the magnetic field at a given point
f distance between two events
g structure of the DNA molecule
h time rate of change of momentum of a neutron

3-5 many unpowered rockets

In the laboratory frame, event 1 occurs at $x = 0$ light-years, $t = 0$ years. Event 2 occurs at $x = 6$ light-years, $t = 10$ years. In all rocket frames, event 1 also occurs at the position 0 light-years and the time 0 years. The y- and z-coordinates of both events are zero in both frames.

a In rocket frame A, event 2 occurs at time $t' = 14$ years. At what position x' will event 2 occur in this frame?

b In rocket frame B, event 2 occurs at position $x'' = 5$ light-years. At what time t'' will event 2 occur in this frame?

c How fast must rocket frame C move if events 1 and 2 occur at the same place in this rocket frame?

d What is the time between events 1 and 2 in rocket frame C of part *c*?

3-6 down with relativity!

Mr. Van Dam is an intelligent and reasonable man with a knowledge of high school physics. He has the following objections to the theory of relativity. Answer each of Mr. Van Dam's objections decisively—without criticizing him. If you wish, you may present a single connected account of how and why one is driven to relativity, in which these objections are all answered.

a "Observer A says that B's clock goes slow, and observer B says that A's clock goes slow. This is a logical contradiction. Therefore relativity should be abandoned."

b "Observer A says that B's meter sticks are contracted along their direction of relative motion, and observer B says that A's meter sticks are contracted. This is a logical contradiction. Therefore relativity should be abandoned."

c "Relativity does not even have a unique way to *define* space and time coordinates for the instantaneous position of an object. Laboratory and rocket observers typically record different coordinates for this position and time. Therefore anything relativity says about the velocity of the object (and hence about its motion) is without meaning."

d "Relativity postulates that light travels with a standard speed regardless of the free-float frame from which its progress is measured. This postulate is certainly wrong. Anybody with common sense knows that travel at high speed in the direction of a receding light pulse will decrease the speed with which the pulse recedes. Hence a flash of light *cannot* have the same speed for observers in relative motion. With this disproof of the basic postulate, all of relativity collapses."

e "There isn't a single experimental test of the *results* of special relativity."

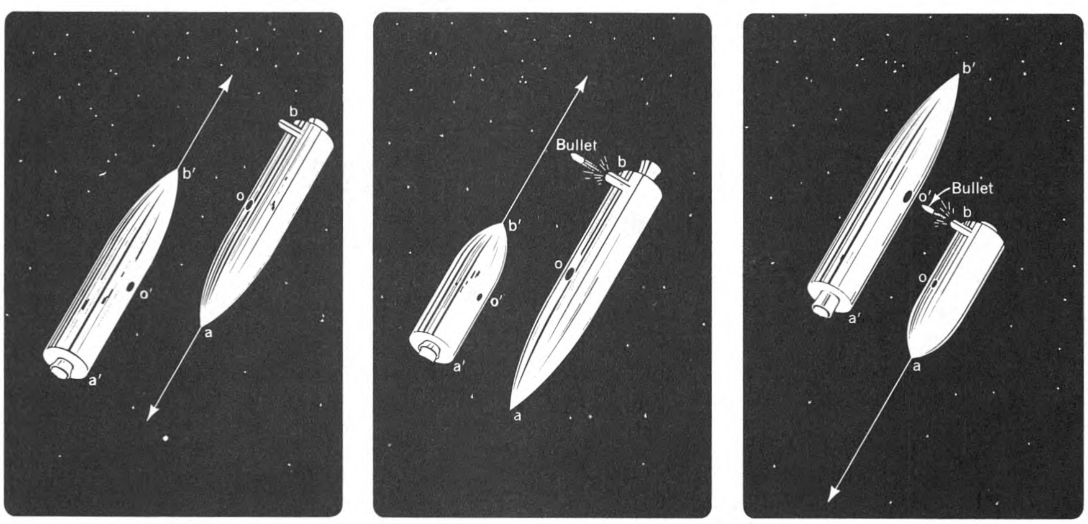
f "Relativity offers no way to describe an event without coordinates—and no way to speak about coordinates without referring to one or another particular reference frame. However, physical events have an existence independent of all choice of coordinates and all choice of reference frame. Hence relativity—with its coordinates and reference frames—cannot provide a valid description of these events."

g "Relativity is preoccupied with how we *observe* things, not what is *really* happening. Hence it is not a scientific theory, since science deals with reality."

PROBLEMS

3-7 space war

Two rockets of equal rest length are passing "head on" at relativistic speeds, as shown in the figure (left). Observer *o* has a gun in the tail of her rocket pointing perpendicular to the direction of relative motion



EXERCISE 3-7. ***Left:*** *Two rocket ships passing at high speed.* ***Center:*** *In the frame of* o *one expects a bullet fired when* a *coincides with* a′ *to miss the other ship.* ***Right:*** *In the frame of* o′ *one expects a bullet fired when* a *coincides with* a′ *to hit the other ship.*

(center). She fires the gun when points a and a' coincide. In her frame the other rocket ship is Lorentz contracted. Therefore o expects her bullet to miss the other rocket. But in the frame of the other observer o' it is the rocket ship of o that is measured to be Lorentz contracted (right). Therefore when points a and a' coincide, observer o' should observe a hit.

Does the bullet actually hit or miss? Pinpoint the looseness of the language used to state the problem and the error in one figure. Show that your argument is consistent with the results of the Train Paradox (Section 3.4).

3-8 Čerenkov radiation

No particle has been observed to travel faster than the speed of light in a *vacuum*. However particles have been observed that travel in a material medium faster than the speed of light *in that medium*. When a charged particle moves through a medium faster than light moves in that medium, it radiates coherent light in a cone whose axis lies along the path of the particle. (Note the rough similarity to waves created by a motorboat speeding across calm water and the more exact similarity to the "cone of sonic boom" created by a supersonic aircraft.) This is called Čerenkov radiation (Russian Č is pronounced as "ch"). Let v be the speed of the particle in the medium and v_{light} be the speed of light in the medium.

a From this information use the first figure to show that the half-angle ϕ, of the light cone is given by the expression

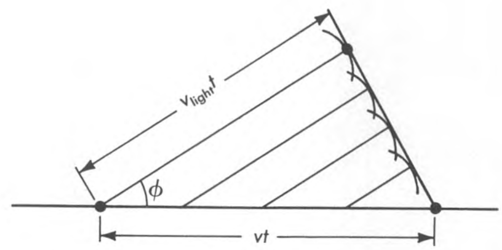
$$\cos \phi = v_{light}/v$$

b Consider the plastic with the trade name Lucite, for which $v_{light} = 2/3$. What is the minimum velocity that a charged particle can have if it is to produce Čerenkov radiation in Lucite? What is the *maximum* angle ϕ at which Čerenkov radiation can be produced in Lucite? Measurement of the angle provides a good way to measure the velocity of the particle.

c In water the speed of light is approximately $v_{light} = 0.75$. Answer the questions of part **b** for the case of water. See the second figure for an application of Čerenkov radiation in water.

3-9 aberration of starlight

A star lies in a direction generally perpendicular to Earth's direction of motion around Sun. Because of Earth's motion, the star appears to an Earth observer to lie in a slightly different direction than it would



EXERCISE 3-8, first figure. *Calculation of Čerenkov angle ϕ.*

EXERCISE 3-8, second figure. *Use of Čerenkov radiation for indirect detection of neutrinos in the Deep Underwater Muon and Neutrino Detector (DUMAND) 30 kilometers off Keahole Point on the island of Hawaii. Neutrinos have no electric charge and their mass, if any, has so far escaped detection (Box 8-1). Neutrinos interact extremely weakly with matter, passing through Earth with almost no collisions. Indeed, the DUMAND detector array selects for analysis only neutrinos that come upward through Earth. In this way Earth itself acts as a shield to eliminate all other cosmic-ray particles.*

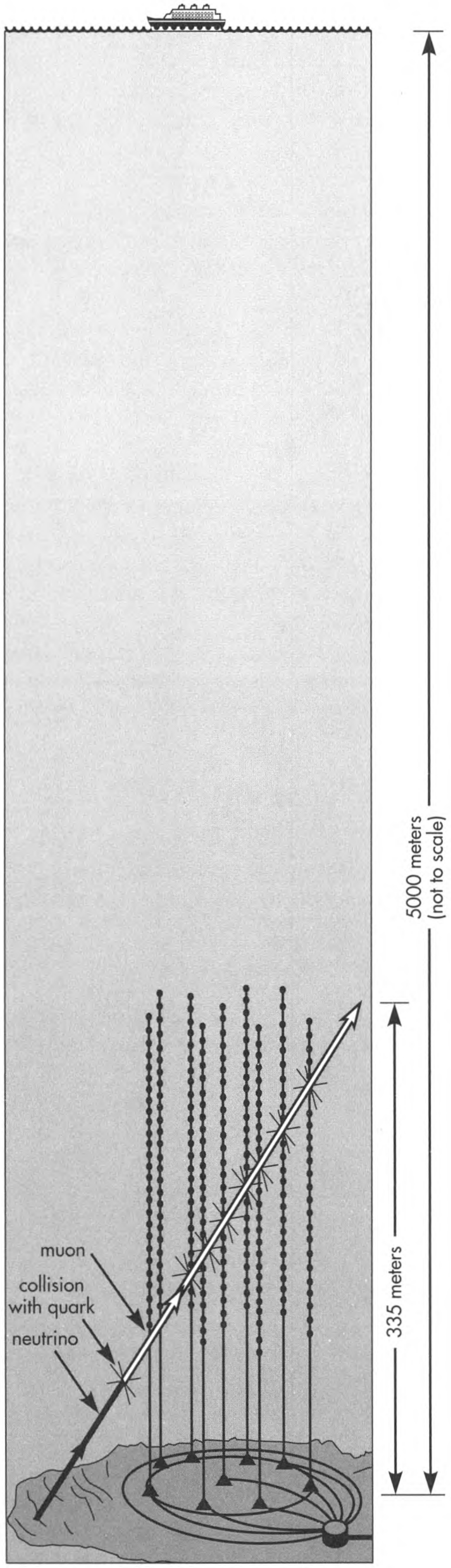
What are possible sources for these neutrinos? Theory predicts the emission of very high-energy (greater than 10^{12} electron-volt) neutrinos from matter plunging toward a black hole. Black holes may be the energy sources for extra-bright galactic nuclei and for quasars —small, distant, enigmatic objects shining with the light of hundreds of galaxies (Section 9.8). Information about conditions deep within these astronomical structures may be carried by neutrinos as they pierce Earth and travel upward through the DUMAND detector array.

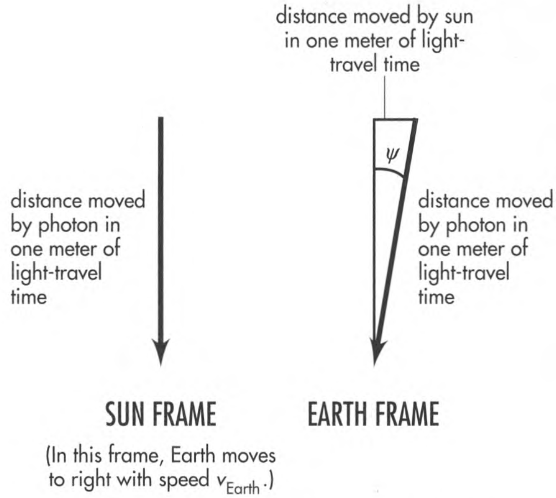
In a rare event, a neutrino moving through the ocean slams into one of the quarks that make up a proton or a neutron in, say, an oxygen nucleus in the water, creating a burst of particles. All of these particles are quickly absorbed by the surrounding water except a stable negatively charged muon, 207 times the mass of the electron (thus sometimes called a "fat electron"). This muon streaks through the water in the same direction as the neutrino that created it and at a speed greater than that of light in water, thus emitting Čerenkov radiation. The Čerenkov radiation is detected by photomultiplier tubes in an array anchored to the ocean floor.

Photomultipliers are strung along 9 vertical cables, 8 cables spaced around a circle 100 meters in diameter on the ocean floor, the ninth cable rising from the center of the circle. Each cable is 335 meters long and holds 24 glass spheres positioned 10 meters apart on the top 230 meters of its length. There are no detectors on the bottom 110 meters, in order to avoid any cloud of sediments from the bottom. Above the bottom, the water is so clear and modern photodetectors so sensitive that Čerenkov radiation can be detected from a muon that passes within 40 meters of a detector.

Photomultipliers in the glass spheres detect Čerenkov radiation from the passing muons, transmitting this signal through underwater optical fibers to computers on the nearby island of Hawaii. The computers select for examination only those events in which (1) several optical sensors detect bursts that are (2) within 40 meters or so of a straight line, (3) spaced in time to show that the particle is moving at essentially the speed of light in a vacuum, and (4) from a particle moving upward through the water. A system of sonar beacons and hydrophones tracks the locations of the photomultipliers as the strings sway with the slow ocean currents. As a result, the direction of motion of the original neutrino can be recorded to an accuracy of one degree.

The DUMAND facility is designed to create a new sky map of neutrino sources to supplement our knowledge of the heavens, so far obtained primarily from the electromagnetic spectrum (radio, infrared, optical, ultraviolet, X-ray, gamma ray).





EXERCISE 3-9. *Aberration of starlight. Not to scale.*

appear to an observer at rest relative to Sun. This effect is called **aberration.** Using the diagram, find this apparent difference of direction.

a Find a trigonometric expression for the aberration angle ψ shown in the figure.

b Evaluate your expression using the speed of Earth around Sun, $v_{\text{Earth conv}} = 30$ kilometers/second. Find the answer in radians and in seconds of arc. (One degree equals 60 minutes of arc; one minute equals 60 seconds of arc.) This change in apparent position can be detected with sensitive equipment.

c The nonrelativistic answer to this problem—the answer using nonrelativistic physics—is $\tan \psi = v_{Earth}$ (in meters/meter). Do you think that the experimental difference between relativistic and nonrelativistic answers for stellar aberration observed from Earth can be the basis of a crucial experiment to decide between the correctness of the two theories?

Discussion: Of course we cannot climb off Earth and view the star from the Sun frame. But Earth reverses direction every six months (with respect to what?), so light from a "transverse star" viewed in, say, July will appear to be shifted through twice the aberration angle calculated in part **b** compared with the light from the same star in January. New question: Since the background of stars behind the one under observation also shifts due to aberration, how can the effect be measured at all?

d A rocket in orbit around Earth suddenly changes its velocity from a very small fraction of the speed of light to $v = 0.5$ with respect to Sun, moving in the same direction as Earth is moving around Sun. In what direction will the rocket astronaut now see the star of parts **a** and **b**?

3-10 the expanding universe

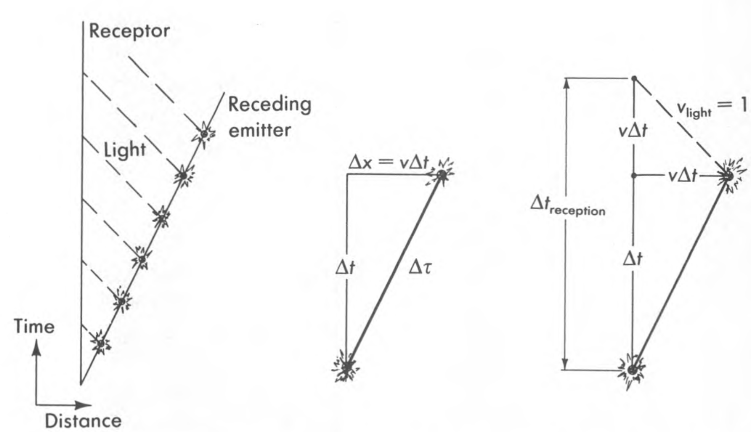
a A giant bomb explodes in otherwise empty space. What is the nature of the motion of one fragment relative to another? And how can this relative motion be detected? **Discussion:** Imagine each fragment equipped with a beacon that gives off flashes of light at regular, known intervals $\Delta\tau$ of time as measured in its own frame of reference (proper time!). Knowing this interval between flashes, what method of detection can an observer on one fragment employ to determine the velocity v — relative to her — of any other fragment? Assume that she uses, in making this determination, (1) the known proper time $\Delta\tau$ between flashes and (2) the time $\Delta t_{reception}$ between the arrival of consecutive flashes at her position. (This is *not* equal to the time Δt in her frame between the emission of the two flashes from the receding emitter; see the figure.) Derive a formula for v in terms of proper time lapse $\Delta\tau$ and $\Delta t_{reception}$. How will the measured recession velocity depend on the distance from one's own fragment to the fragment at which one is looking? Hint: In any given time in any given frame, fragments evidently travel distances in that frame from the point of explosion that are in direct proportion to their velocities in that frame.

b How can observation of the light from stars be used to verify that the universe is expanding? **Discussion:** Atoms in hot stars give off light of different frequencies characteristic of these atoms ("spectral lines"). The observed period of the light in each spectral line from starlight can be measured on Earth. From the pattern of spectral lines the kind of atom emitting the light can be identified. The same kind of atom can then be excited in the laboratory to emit light while at rest and the proper period of the light in any spectral line can be measured. Use the results of part **a** to describe how the observed period of light in one spectral line from starlight can be compared to the proper period of light in the same spectral line from atoms at rest in the laboratory to give the velocity of recession of the star that emits the light. This observed change in period due to the velocity of the source is called the Doppler shift. (For a more detailed treatment of Doppler shift, see the exercises for Chapters 5 and 8.) If the universe began in a gigantic explosion, how must the observed velocities of recession of different stars at different distances compare with one another? Slowing down during expansion — by gravitational attraction or otherwise — is to be neglected here but is considered in more complete treatments.

c The brightest steadily shining objects in the heavens are called quasars, which stands for "quasi-stellar objects." A single quasar emits more than 100 times the light of our entire galaxy. One possible source of quasar energy is the gravitational energy released as material falls into a black hole (Section 9.8). Because they are so bright, quasars can be observed at great distances. As of 1991, the greatest observed quasar red shift $\Delta t_{reception}/\Delta\tau$ has the value 5.9. According to the theory of this exercise, what is the velocity of recession of this quasar, as a fraction of the speed of light?

3-11 law of addition of velocities

In a spacebus a bullet shoots forward with speed 3/4 that of light as measured by travelers in the bus. The spacebus moves forward with speed 3/4 light speed as measured by Earth observers. How fast does the bullet move as measured by Earth observers: $3/4 + 3/4 = 6/4 = 1.5$ times the speed of light? No! Why not? Because (1) special relativity predicts that noth-



EXERCISE 3-10. *Calculation of the time $t_{reception}$ between arrival at observer of consecutive flashes from receding emitter. Light moves one meter of distance in one meter of time, so lines showing motion of light are tilted at $\pm 45°$ from the vertical.*

ing can travel faster than light, and (2) hundreds of millions of dollars have been spent accelerating particles ("bullets") to the fastest possible speed without anyone detecting a single particle that moves faster than light in a vacuum. Then where is the flaw in our addition of velocities? And what is the correct law of addition of velocities? These questions are answered in this exercise.

a First use Earth observers to record the motions of the spacebus (length L measured in the Earth frame, speed v_{rel}) and the streaking bullet (speed v_{bullet}). The bullet starts at the back of the bus. To give it some competition, let a light flash (speed = 1) race the bullet from the back of the bus toward the front. The light flash wins, of course, reaching the front of the bus in time t_{forward}. And t_{forward} is also equal to the distance that the light travels in this time. Show that this distance (measured in the Earth frame) equals the length of the bus plus the distance the bus travels in the same time:

$$t_{\text{forward}} = L + v_{\text{rel}}\, t_{\text{forward}} \text{ or } t_{\text{forward}} = \frac{L}{1 - v_{\text{rel}}} \qquad (1)$$

b In order to rub in its advantage over the bullet, the light flash reflects from the front of the bus and moves backward until, after an additional time t_{backward}, it rejoins the forward-plodding bullet. This meeting takes place next to the seat occupied by Fred, who sits a distance fL behind the front of the bus, where f is a fraction of the bus length L. Show that for this leg of the trip the Earth-measured distance t_{backward} traveled by the light flash can also be expressed as

$$t_{\text{backward}} = fL - v_{\text{rel}}\, t_{\text{backward}} \quad \text{or} \quad t_{\text{backward}} = \frac{fL}{1 + v_{\text{rel}}} \qquad (2)$$

c The light flash has moved forward and then backward with respect to Earth. What is the *net* forward distance covered by the light flash at the instant it rejoins the bullet? Equate this with the forward distance moved by the bullet (at speed v_{bullet}) to obtain the equation

$$v_{\text{bullet}}(t_{\text{forward}} + t_{\text{backward}}) = t_{\text{forward}} - t_{\text{backward}}$$

or

$$(1 + v_{\text{bullet}})\, t_{\text{backward}} = (1 - v_{\text{bullet}})\, t_{\text{forward}} \qquad (3)$$

d What are we after? We want a relation between the bullet speed v_{bullet} as measured in the Earth frame and the bullet speed, call it v'_{bullet} (with a prime), as measured in the spacebus frame. The times given in parts **a**, **b**, and **c** are of no use to this end. Worse, we already know that times between events are typically different as measured in the spacebus frame than times between the same events measured in the Earth frame. So get rid of these times! Moreover, the Lorentz-contracted length L of the spacebus itself as measured in the Earth frame will be different from its rest length measured in the bus frame (Section 3.5). So get rid of L as well. Equations (1), (2), and (3) can be treated as three equations in the three unknowns t_{forward}, t_{backward}, and L. Substitute equations for the times (1) and (2) into equation (3). Lucky us: The symbol L cancels out of the result. Show that this result can be written

$$f = \frac{(1 - v_{\text{bullet}})\,(1 + v_{\text{rel}})}{(1 + v_{\text{bullet}})\,(1 - v_{\text{rel}})} \qquad (4)$$

e Now repeat the development of parts **a** through **d** for the spacebus frame, with respect to which the spacebus has its rest length L' and the bullet has speed v'_{bullet} (both with primes). Show that the result is:

$$f = \frac{(1 - v'_{\text{bullet}})}{(1 + v'_{\text{bullet}})} \qquad (5)$$

Discussion: Instead of working hard, work smart! Why not use the old equations (1) through (4) for the spacebus frame? Because there is no relative velocity v_{rel} in the spacebus frame; the spacebus is at rest in its own frame! No problem: Set $v_{\text{rel}} = 0$ in equation (4), replace v_{bullet} by v'_{bullet} and obtain equation (5) directly from equation (4). If this is too big a step, carry out the derivation from the beginning in the spacebus frame.

f Do the two fractions f in equations (4) and (5) have the same value? In equation (4) the number f locates Fred's seat in the bus as a fraction of the total length of the bus in the Earth frame. In equation (5) the number f locates Fred's seat in the bus as a fraction of the total length of the bus in the bus frame. But this fraction must be the same: Fred cannot be halfway back in the Earth frame and, say, three quarters of the way back in the spacebus frame. Equate the two expressions for f given in equations (4) and (5) and solve for v_{bullet} to obtain the Law of Addition of Velocities:

$$v_{\text{bullet}} = \frac{v'_{\text{bullet}} + v_{\text{rel}}}{1 + v'_{\text{bullet}}\, v_{\text{rel}}} \qquad (6)$$

g Explore some consequences of the Law of Addition of Velocities.

(1) An express bus on Earth moves at 108 kilometers/hour (approximately 67 miles/hour or 30 meters per second). A bullet moves forward with speed 600 meters/second with respect to the bus. What are the values of v_{rel} and v'_{bullet} in meters/meter? What is the value of their product in the denominator of equation (6)? Does this product of speeds increase the value of the denominator significantly over the value unity? Therefore what approximate form does equation (6) take for everyday speeds? Is this the form you would expect from your experience?

(2) Analyze the example that began this exercise: Speed of bullet with respect to spacebus $v'_{bullet} = 3/4$; speed of spacebus with respect to Earth $v_{rel} = 3/4$. What is the speed of the bullet measured by Earth observers?

(3) Why stop with bullets that saunter along at less than the speed of light? Let the bullet itself be a flash of light. Then the bullet speed as measured in the bus is $v'_{bullet} = 1$. For $v_{rel} = 3/4$, with what speed does this light flash move as measured in the Earth frame? Is this what you expect from the Principle of Relativity?

(4) Suppose a light flash is launched from the front of the bus directed toward the back ($v'_{bullet} = -1$). What is the velocity of this light flash measured in the Earth frame? Is this what you expect from the Principle of Relativity?

Reference: N. David Mermin, *American Journal of Physics*, Volume 51, pages 1130–1131 (1983).

3-12 Michelson–Morley experiment

a An airplane moves with air speed c (not the speed of light) from point A to point B on Earth. A stiff wind of speed v is blowing from B toward A. (In this exercise only, the symbol v stands for velocity in conventional units, for example meters/second.) Show that the time for a round trip from A to B and back to A under these circumstances is greater by a factor $1/(1 - v^2/c^2)$ than the corresponding round trip time in still air. Paradox: The wind helps on one leg of the flight as well as hinders on the other. Why, therefore, is the round-trip time not the same in the presence of wind as in still air? Give a simple physical reason for this difference. What happens when the wind speed is nearly equal to the speed of the airplane?

b The same airplane now makes a round trip between A and C. The distance between A and C is the same as the distance from A to B, but the line from A to C is perpendicular to the line from A to B, so that in moving between A and C the plane flies across the wind. Show that the round-trip time between A and C under these circumstances is greater by a factor $1/(1 - v^2/c^2)^{1/2}$ than the corresponding round-trip time in still air.

c Two airplanes with the same air speed c start from A at the same time. One travels from A to B and back to A, flying first against and then with the wind (wind speed v). The other travels from A to C and back to A, flying across the wind. Which one will arrive home first, and what will be the difference in their arrival times? Using the first two terms of the binomial theorem,

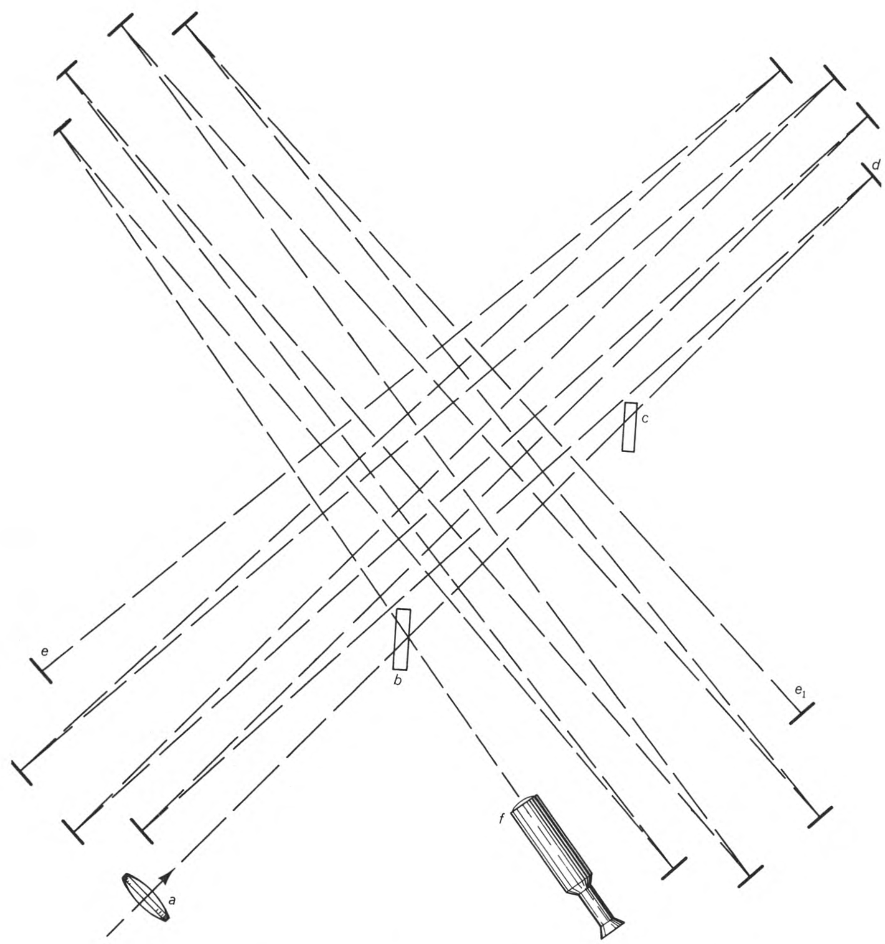
$$(1 + z)^n \approx 1 + nz \qquad \text{for } |z| << 1$$

show that if $v << c$, then an approximate expression for this time difference is $\Delta t \approx (L/2c)(v/c)^2$, where L is the round-trip distance between A and B (and between A and C).

d The South Pole Air Station is the supply depot for research huts on a circle of 300-kilometer radius centered on the air station. Every Monday many supply planes start simultaneously from the station and fly radially in all directions at the same altitude. Each plane drops supplies and mail to one of the research huts and flies directly home. A Fussbudget with a stopwatch stands on the hill overlooking the air station. She notices that the planes do not all return at the same time. This discrepancy perplexes her because she knows from careful measurement that (1) the distance from the air station to every research hut is the same, (2) every plane flies with the same air speed as every other plane—300 kilometers/hour—and (3) every plane travels in a straight line over the ground from station to hut and back. The Fussbudget finally decides that the discrepancy is due to the wind at the high altitude at which the planes fly. With her stopwatch she measures the time from the return of the first plane to the return of the last plane to be 4 seconds. What is the wind speed at the altitude where the planes fly? What can the Fussbudget say about the direction of this wind?

e In their famous experiment Michelson and Morley attempted to detect the so-called **ether drift**—the motion of Earth through the "ether," with respect to which light was supposed to have the velocity c. They compared the round-trip times for light to travel equal distances parallel and perpendicular to the direction of motion of Earth around Sun. They reflected the light back and forth between nearly parallel mirrors. (This would correspond to part **c** if each airplane made repeated round trips.) By this means they were able to use a total round-trip length of 22 meters for each path. If the "ether" is at rest with respect to Sun, and if Earth moves at 30×10^3 meters/second in its path around Sun, what is the approximate difference in time of return between light flashes that are emitted simultaneously and travel along the two perpendicular paths? Even with the instruments of today, the difference predicted by the ether-drift hypothesis would be too small to measure directly, and the following method was used instead.

f The original Michelson – Morley interferometer is diagrammed in the figure. Nearly monochromatic light (light of a single frequency) enters through the lens at *a*. Some of the light is reflected by the half-silvered mirror at *b* and the rest of the light continues toward *d*. Both beams are reflected back and forth until they reach mirrors *e* and e_1 respectively, where each beam is reflected back on itself and retraces its path to mirror *b*. At mirror *b* parts of each beam combine to enter telescope *f* together. The transparent piece of glass at *c*, of the same dimensions as the half-silvered mirror *b*, is inserted so that both beams pass the same number of times (three times) through this thickness of glass on their way to telescope *f*. Suppose that the perpendicular path lengths are exactly equal and the instrument is at rest with respect to the ether. Then monochromatic light from the two paths that leave mirror *b* in some relative phase will return to mirror *b* in the same phase. Under these circumstances the waves entering telescope *f* will add crest to crest and the image in this telescope will be bright. On the other hand, if one of the beams has been delayed a time corresponding to one half period of the light, then it will arrive at mirror *b* one half period later and the waves entering the telescope will cancel (crest to trough), so the image in the telescope will be dark. If one beam is retarded a time corresponding to one whole period, the telescope image will be bright, and so forth. What time corresponds to



EXERCISE 3-12. *Michelson – Morley interferometer mounted on a rotating marble slab.*

one period of the light? Michelson and Morley used sodium light of wavelength 589 nanometers (one nanometer is equal to 10^{-9} meter). Use the equations $f\lambda = c$ and $f = 1/T$ that relate frequency f, period T, wavelength λ, and speed c of an electromagnetic wave. Show that one period of sodium light corresponds to about 2×10^{-15} seconds.

Now there is no way to "turn off" the alleged ether drift, adjust the apparatus, and then turn the alleged ether drift on again. Instead of this, Michelson and Morley floated their interferometer in a pool of mercury and rotated it slowly about its center like a phonograph record while observing the image in the telescope (see the figure). In this way if light is delayed on either path when the instrument is oriented in a certain direction, light on the other path will be delayed by the same amount of time when the instrument has rotated 90 degrees. Hence the total change in delay time between the two paths observed as the interferometer rotates should be twice the difference calculated using the expression derived in part **c**. By refinements of this method Michelson and Morley were able to show that the time change between the two paths as the instrument rotated corresponded to less than one one-hundredth of the shift from one dark image in the telescope to the next dark image. Show that this result implies that the motion of the ether at the surface of Earth — if it exists at all — is less than one sixth of the speed of Earth in its orbit. In order to eliminate the possibility that the ether was flowing past Sun at the same rate as Earth was moving its orbit, they repeated the experiment at intervals of three months, always with negative results.

g **Discussion question:** Does the Michelson–Morley experiment, by itself, disprove the theory that light is propagated through an ether? Can the ether theory be modified to agree with the results of this experiment? How? What further experiment can be used to test the modified theory?

Reference: A. A. Michelson and E. W. Morley, *American Journal of Science,* Volume 134, pages 333–345 (1887).

3-13 the Kennedy–Thorndike experiment

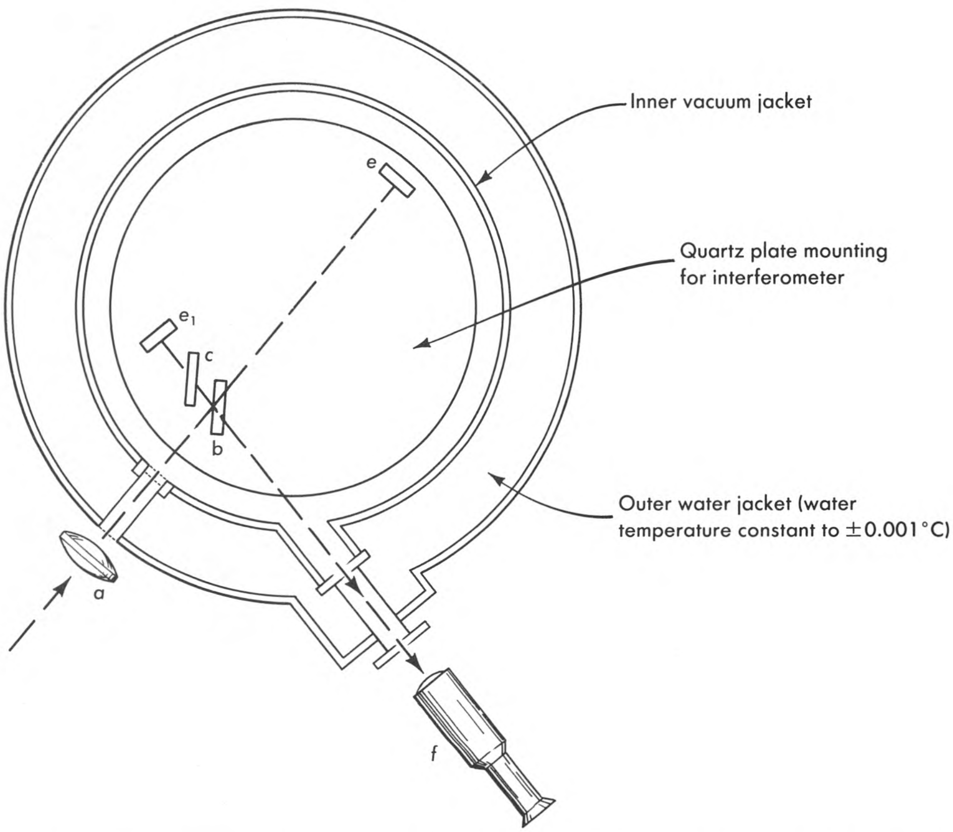
Note: Part **d** of this exercise uses elementary calculus.

The Michelson–Morley experiment was designed to detect any motion of Earth relative to a hypothetical fluid — the ether — a medium in which light was supposed to move with characteristic speed c. No such relative motion of earth and ether was detected. Partly as a result of this experiment the concept of ether has since been discarded. In the modern view, light requires no medium for its transmission. What significance does the negative result of the Michelson–Morley experiment have for us who do not believe in the ether theory of light propagation? Simply this: (1) The round-trip speed of light measured on earth is the same in every direction — the speed of light is isotropic. (2) The speed of light is isotropic not only when Earth moves in one direction around Sun in, say, January (call Earth with this motion the "laboratory frame"), but also when Earth moves in the opposite direction around Sun six months later, in July (call Earth with this motion the "rocket frame"). (3) The generalization of this result to any pair of inertial frames in relative motion is contained in the statement, The round-trip speed of light is isotropic both in the laboratory frame and in the rocket frame. This result leaves an important question unanswered: Does the round-trip speed of light — which is isotropic in both laboratory and rocket frames — also have the same numerical value in laboratory and rocket frames? The assumption that this speed has the same numerical value in both frames played a central role in demonstrating the invariance of the interval (Section 3.7). But is this assumption valid?

a An experiment to test the assumption of the equality of the round-trip speed of light in two inertial frames in relative motion was conducted in 1932 by Roy J. Kennedy and Edward M. Thorndike. The experiment uses an interferometer with arms of unequal length (see the figure). Assume that one arm of the interferometer is Δl longer than the other arm. Show that a flash of light entering the apparatus will take a time $2\Delta l/c$ longer to complete the round trip along the longer arm than along the shorter arm. The difference in length Δl used by Kennedy and Thorndike was approximately 16 centimeters. What is the approximate difference in time for the round trip of a light flash along the alternative paths?

b Instead of a pulse of light, Kennedy and Thorndike used continuous monochromatic light of period $T = 1.820 \times 10^{-15}$ seconds ($\lambda = 546.1$ nanometers $= 546.1 \times 10^{-9}$ meters) from a mercury source. Light that traverses the longer arm of the interferometer will return approximately how many periods n later than light that traverses the shorter arm? If in the actual experiment the number of periods is an integer, the reunited light from the two arms will add (crest-to-crest) and the field of view seen through the telescope will be bright. In contrast, if in the actual experiment the number of periods is a half-integer, the reunited light from the two arms will cancel (crest-to-trough) and the field of view of the telescope will be dark.

c Earth continues on its path around Sun. Six months later Earth has reversed the direction of its velocity relative to the fixed stars. In this new frame of



EXERCISE 3-13. *Schematic diagram of apparatus used for the Kennedy–Thorndike experiment. Parts of the interferometer have been labeled with letters corresponding to those used in describing the Michelson–Morley interferometer (Exercise 3-12). The experimenters went to great lengths to insure the optical and mechanical stability of their apparatus. The interferometer is mounted on a plate of quartz, which changes dimension very little when temperature changes. The interferometer is enclosed in a vacuum jacket so that changes in atmospheric pressure will not alter the effective optical path length of the interferometer arms (slightly different speed of light at different atmospheric pressure). The inner vacuum jacket is surrounded by an outer water jacket in which the water is kept at a temperature that varies less than ±0.001 degrees Celsius. The entire apparatus shown in the figure is enclosed in a small darkroom (not shown) maintained at a temperature constant within a few hundredths of a degree. The small darkroom is in turn enclosed in a larger darkroom whose temperature is constant within a few tenths of a degree. The overall size of the apparatus can be judged from the fact that the difference in length of the two arms of the interferometer (length* eb *compared with length* e_1b) *is 16 centimeters.*

reference will the round-trip speed of light have the same numerical value c as in the original frame of reference? One can rewrite the answer to part **b** for the original frame of reference in the form

$$c = (2/n)(\Delta l/T)$$

where Δl is the difference in length between the two interferometer arms, T is the time for one period of the atomic light source, and n is the number of periods that elapse between the return of the light on the shorter path and the return of the light on the longer path. Suppose that as Earth orbits Sun no shift is observed in the telescope field of view from, say, light toward dark. This means that n is observed to be constant. What would this hypothetical result tell about the numerical value c of the speed of light? Point out the standards of distance and time used in determining this result, as they appear in the equation. Quartz has the greatest stability of dimension of any known material. Atomic time standards have proved to be the most dependable earth-bound time-keeping mechanisms.

d In order to carry out the experiment outlined in the preceding paragraphs, Kennedy and Thorndike would have had to keep their interferometer operating perfectly for half a year while continuously observing the field of view through the telescope. Uninterrupted operation for so long a time was not feasible. The actual durations of their observations varied from eight days to a month. There were several such periods of observation at three-month time separations. From the data obtained in these periods, Kennedy and Thorndike were able to estimate that over a single six-month observation the number of periods n of relative delay would vary by less than the fraction 3/1000 of one period. Take the differential of the equation in part **c** to find the largest fractional change dc/c of the round-trip speed of light between the two frames consistent with this estimated change in n (frame 1 — the "laboratory" frame — and frame 2 — the "rocket" frame — being in the present analysis Earth itself at two different times of year, with a relative velocity twice the speed of Earth in its orbit: 2×30 kilometers/second).

Historical note: At the time of the Michelson–Morley experiment in 1887, no one was ready for the idea that physics — including the speed of light — is the same in every inertial frame of reference. According to today's standard Einstein interpretation it seems obvious that both the Michelson–Morley and the Kennedy–Thorndike experiments should give null results. However, when Kennedy and Thorndike made their measurements in 1932, two alternatives to the Einstein theory were open to consideration (designated here as theory A and theory B). Both A and B assumed the old idea of an absolute space, or "ether," in which light has the speed c. Both A and B explained the zero fringe shift in the Michelson–Morley experiment by saying that all matter that moves at a velocity v (expressed as a fraction of light-speed) relative to "absolute space" undergoes a shrinkage of its space dimensions in the direction of motion to a new length equal to $(1 - v^2)^{1/2}$ times the old length ("Lorentz-FitzGerald contraction hypothesis"). The two theories differed as to the effect of "motion through absolute space" on the running rate of a clock. Theory A said, No effect. Theory B said that a standard seconds clock moving through absolute space at velocity v has a time between ticks of $(1 - v^2)^{1/2}$ seconds. In theory B the ratio $\Delta l/T$ in the equation in part **b** will not be affected by the velocity of the clock, and the Kennedy–Thorndike experiment will give a null result, as observed ("complicated explanation for simple effect"). In theory A the ratio $\Delta l/T$ in the equation will be multiplied by the factor $(1 - v_1{}^2)^{1/2}$ at a time of year when the "velocity of Earth relative to absolute space" is v_1 and multiplied by $(1 - v_2{}^2)^{1/2}$ at a time of year when this velocity is v_2. Thus the fringes should shift from one time of year ($v_1 = v_{\text{orbital}} + v_{\text{Sun}}$) to another time of year ($v_2 = v_{\text{orbital}} - v_{\text{Sun}}$) unless by accident Sun happened to have "zero velocity relative to absolute space" — an accident judged so unlikely as not to provide an acceptable explanation of the observed null effect. Thus the Kennedy–Thorndike experiment ruled out theory A (length contraction alone) but allowed theory B (length contraction plus time contraction) — and also allowed the much simpler Einstein theory of equivalence of all inertial reference frames.

The "sensitivity" of the Kennedy–Thorndike experiment depends on the theory under consideration. In the context of theory A the observations set an upper limit of about 15 kilometers/second to the "speed of Sun through absolute space" (sensitivity reported in the Kennedy–Thorndike paper). In the context of Einstein's theory the observations say that the round-trip speed of light has the same numerical magnitude — within an error of about 3 meters/second — in inertial frames of reference having a relative velocity of 60 kilometers/second.

Reference: R. J. Kennedy and E. M. Thorndike, *Physical Review*, Volume 42, pages 400–418 (1932).

3-14 things that move faster than light

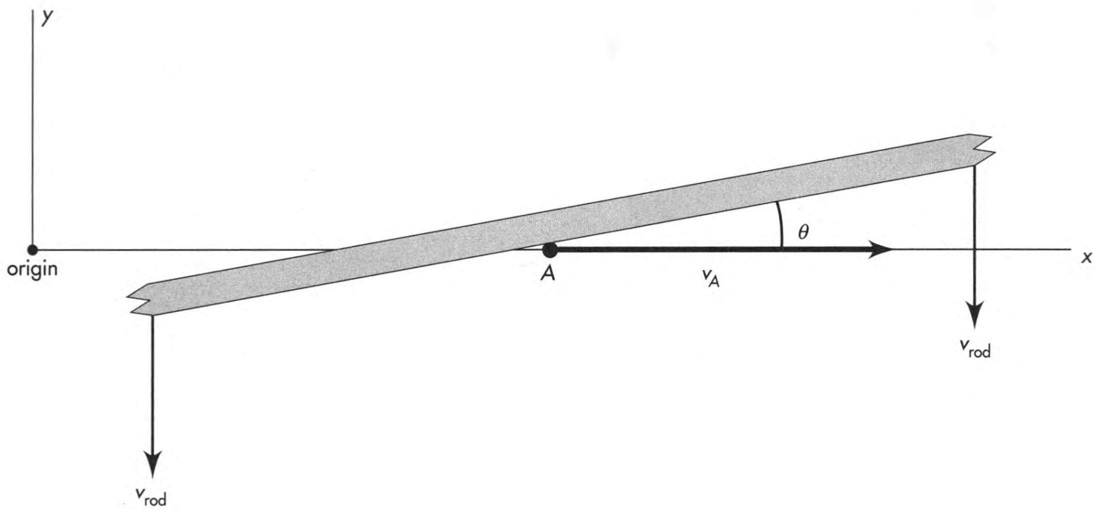
Can "things" or "messages" move faster than light? Does relativity really say "No" to this possibility? Explore these questions further using the following examples.

a The Scissors Paradox. A very long straight rod, inclined at an angle θ to the x-axis, moves downward with uniform speed v_{rod} as shown in the figure. Find the speed v_A of the point of intersection A of the lower edge of the stick with the x-axis. Can this speed be greater than the speed of light? If so, for what values of the angle θ and v_{rod} does this occur? Can the motion of intersection point A be used to transmit a message faster than light from someone at the origin to someone far out on the x-axis?

b Transmission of a Hammer Pulse. Suppose the same rod is initially at rest in the laboratory with the point of intersection initially at the origin. The region of the rod centered at the origin is struck sharply with the downward blow of a hammer. The point of intersection moves to the right. Can this motion of the point of intersection be used to transmit a message faster than the speed of light?

c Searchlight Messenger? A very powerful searchlight is rotated rapidly in such a way that its beam sweeps out a flat plane. Observers A and B are at rest on the plane and each the same distance from the searchlight but not near each other. How far from the searchlight must A and B be in order that the searchlight beam will sweep from A to B faster than a light signal could travel from A to B? Before they took their positions, the two observers were given the following instruction:

> To A: "When you see the searchlight beam, fire a bullet at B."
>
> To B: "When you see the searchlight beam, duck because A has fired a bullet at you."



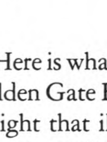
EXERCISE 3-14. *Can the point of intersection* A *move with a speed* v_A *greater than the speed of light?*

Under these circumstances, has a warning message traveled from A to B with a speed faster than that of light?

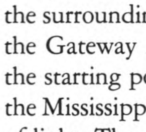
d Oscilloscope Writing Speed. The manufacturer of an oscilloscope claims a writing speed (the speed with which the bright spot moves across the screen) in excess of the speed of light. Is this possible?

3-15 four times the speed of light?

We look westward across the United States and see the rocket approaching us at four times the speed of light.



How can this be, since nothing moves faster than light?



We did not say the rocket *moves* faster than light; we said only that we *see* it moving faster than light.

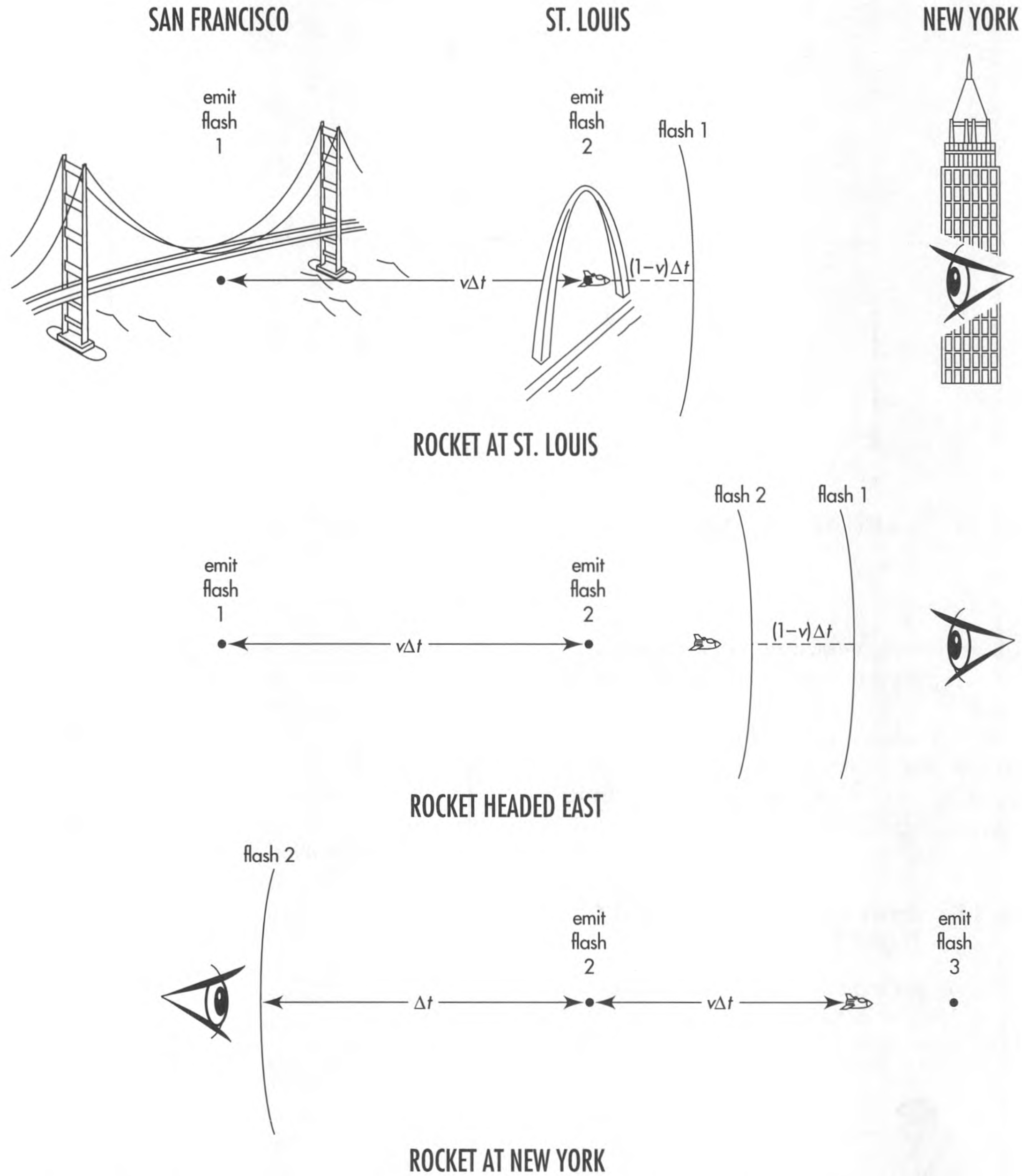
Here is what happens: The rocket streaks under the Golden Gate Bridge in San Francisco, emitting a flash of light that illuminates the rocket, the bridge, and the surroundings. At time Δt later the rocket threads the Gateway Arch in St. Louis that commemorates the starting point for covered wagons. The arch and the Mississippi riverfront are flooded by a second flash of light. The top figure is a visual summary of measurements from our continent-spanning latticework of clocks taken at this moment.

Now the rocket continues toward us as we stand in New York City. The center figure summarizes data taken as the first flash is about to enter our eye. Flash 1 shows us the rocket passing under the Golden Gate Bridge. An instant later flash 2 shows us the rocket passing through the Gateway Arch.

a Answer the following questions using symbols from the first two figures. The images carried by the two flashes show the rocket how far apart in space? What is the time lapse between our reception of these two images? Therefore, what is the apparent speed of the approaching rocket we see? For what speed v of the rocket does the apparent speed of approach equal four times the speed of light? For what rocket speed do we see the approaching rocket to be moving at 99 times the speed of light?

b Our friend in San Francisco is deeply disappointed. Looking eastward, she sees the retreating rocket traveling at less than half the speed of light (bottom figure). She wails, "Which one of us is wrong?" "Neither one." we reply. "No matter how high the speed v of the rocket, you will never see it moving directly away from you at a speed greater than half the speed of light."

Use the bottom figure to derive an expression for the apparent speed of recession of the rocket. When we in New York see the rocket approaching at four times the speed of light, with what speed does our San Francisco friend see it moving away from her? When we see a faster rocket approaching at 99 times the speed of light, what speed of recession does she behold?

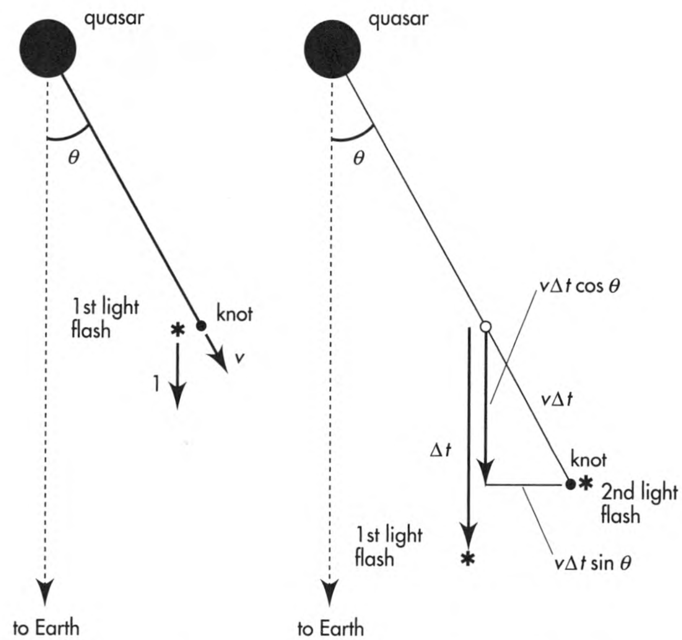


EXERCISE 3-15. ***Top:*** *Rocket headed east, shown at the instant it passes under the Gateway Arch in St. Louis and emits flash 2. The rocket is chasing flash 1, emitted earlier as it passed under the Golden Gate Bridge in San Francisco.* ***Center:*** *The two image-carrying flashes are close together, so they enter the eye in rapid succession. This gives the viewer the visual impression that the rocket moved from San Francisco to St. Louis in a very short time.* ***Bottom:*** *Rocket headed east, shown at the instant it approaches the Empire State Building in New York City and emits flash 3. When the rocket moves away from the viewer, the distance of rocket travel is added to the separation between flashes. This increases the apparent time between flashes, giving the viewer the impression that the rocket moved from St. Louis to New York at less than one half light-speed.*

3-16 superluminal expansion of quasar 3C273?

The most powerful sources of energy we know or conceive or see in all the universe are so-called quasi-stellar objects, or **quasars,** starlike sources of light located billions of light-years away. Despite being far smaller than any galaxy, the typical quasar manages to put out more than 100 times as much energy as our own Milky Way, with its hundred billion stars. Quasars, unsurpassed in brilliance and remoteness, we count today as lighthouses of the heavens.

One of the major problems associated with quasars is that some are composed of two or more components



EXERCISE 3-16, first figure. *Left: Bright "knot" of plasma ejected from a quasar at high speed* v *emits a first flash of light toward Earth.* ***Right:*** *The knot emits a second light flash toward Earth a time* Δt *later. This time* Δt *is measured locally near the knot using the Earth-linked latticework of rods and clocks (har! har!).*

that appear to be separating from each other with relative velocity greater than the speed of light ("superluminal" velocity). One theory that helps explain this effect pictures the quasar as a core that ejects a jet of plasma at relativistic speed. Disturbances or instabilities in such a jet appear as discrete "knots" of plasma. The motion and light emission from a knot may account for its apparent greater-than-light speed, as shown using the first figure.

a The first figure shows two Earth-directed light flashes emitted from the streaking knot. The time between emissions is Δt as measured locally near the knot using the Earth-linked latticework of rods and clocks. Of course the clock readings on this portion of the Earth-linked latticework are not available to us on Earth; therefore we cannot measure Δt directly. Rather, we see the time separation between the arrivals of the two flashes at Earth. From the figure, show that this Earth-seen time separation Δt_{seen} is given by the expression

$$\Delta t_{seen} = \Delta t(1 - v \cos \theta)$$

b We have another disability in viewing the knot from Earth. We do not see the motion of the knot toward us, only the apparent motion of the knot across our field of view. Find an expression for this transverse motion (call it Δx_{seen}) between emissions of the two light flashes in terms of Δt.

c Now calculate the speed v^x_{seen} of the rightward motion of the knot as seen on Earth. Show that the result is

$$v^x_{seen} = \frac{\Delta x_{seen}}{\Delta t_{seen}} = \frac{v \sin \theta}{1 - v \cos \theta}$$

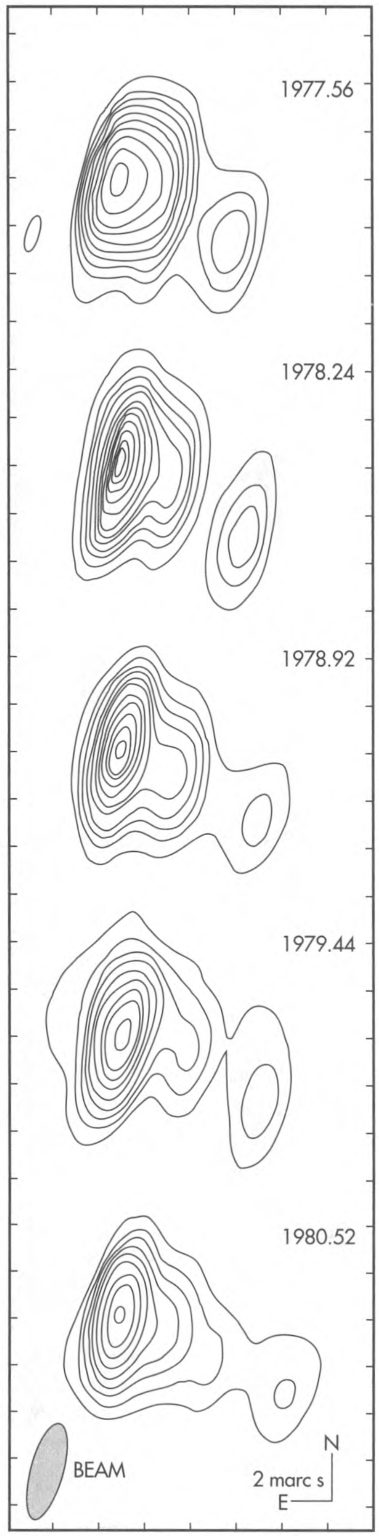
d What is the value of v^x_{seen} when the knot is emitted in the direction exactly toward Earth? when it is emitted perpendicular to this direction? Find an expression that gives the range of angles θ for which v^x_{seen} is greater than the speed of light. For $\theta = 45$ degrees, what is the range of knot speeds v such that v^x_{seen} is greater than the speed of light?

e If you know calculus, find an expression for the angle θ_{max} at which v^x_{seen} has its maximum value for a given knot speed v. Show that this angle satisfies the equation $\cos \theta_{max} = v$. Whether or not you derive this result, use it to show that the maximum apparent transverse speed is seen as

$$v^x_{seen,\, max} = \frac{v}{(1 - v^2)^{1/2}}$$

f What is this maximum transverse speed seen on Earth when $v = 0.99$?

g The second figure shows the pattern of radio emission from the quasar 3C273. The decreased pe-



EXERCISE 3-16, second figure. *Contour lines of radio emission from the quasar 3C273 showing a bright "knot" of plasma apparently moving away from it at a speed greater than the speed of light. The time of each image is given as calendar year and decimal fraction. Horizontal scale divisions are in units of 2 milli arc-seconds. (1 milli arc-second = $10^{-3}/3600$ degree = 4.85×10^{-9} radian)*

riod of radiation from this source (Exercise 3-10) shows that it is approximately 2.6×10^9 light-years from Earth. A secondary source is apparently moving away from the central quasar. Take your own measurements on the figure. Combine this with data from the figure caption to show that the apparent speed of separation is greater than 9 times the speed of light.

Note: As of 1990, apparent greater-than-light-speed ("superluminal") motion has been observed in approximately 25 different sources.

References: Analysis and first figure adapted from Denise C. Gabuzda, *American Journal of Physics,* Volume 55, pages 214–215 (1987). Second figure and data taken from T. J. Pearson, S. C. Unwin, M. H. Cohen, R. P. Linfield, A. C. S. Readhead, G. A. Seielstad, R. S. Simon, and R. C. Walker, *Nature,* Volume 290, pages 365–368 (2 April 1981).

3-17 contraction or rotation?

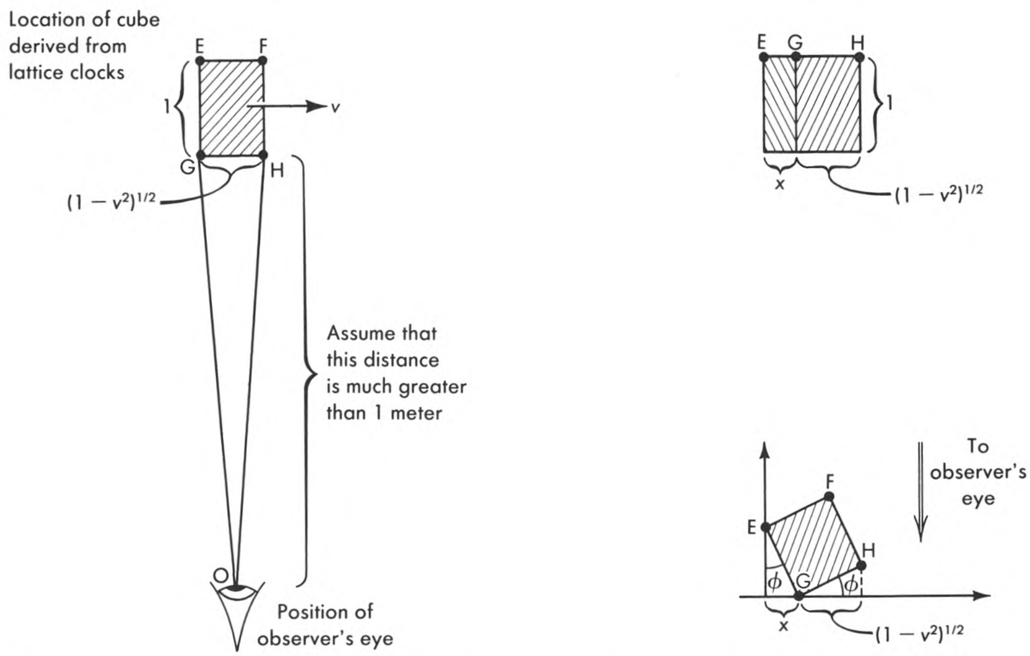
A cube at rest in the rocket frame has an edge of length 1 meter in that frame. In the laboratory frame the cube is Lorentz contracted in the direction of motion, as shown in the figure. Determine this Lorentz contraction, for example, from locations of four clocks at rest and synchronized in the laboratory lattice with which the four corners of the cube, *E, F, G, H,* coincide when all four clocks read the same time. This latticework measurement eliminates time lags in the travel of light from different corners of the cube.

Now for a different observing procedure! Stand in the laboratory frame and look at the cube with one eye as the cube passes overhead. What one sees at any time is light that enters the eye at that time, even if it left the different corners of the cube at different times. Hence, what one sees visually may not be the same as what one observes using a latticework of clocks. If the cube is viewed from the bottom then the distance *GO* is equal to the distance *HO*, so light that leaves *G* and *H* simultaneously will arrive at *O* simultaneously. Hence, when one sees the cube to be overhead one will see the Lorentz contraction of the bottom edge.

a Light from *E* that arrives at *O* simultaneously with light from *G* will have to leave *E* earlier than light from *G* left *G*. How much earlier? How far has the cube moved in this time? What is the value of the distance x in the right top figure?

b Suppose the eye interprets the projection in the figures as a rotation of a cube that is not Lorentz contracted. Find an expression for the angle of apparent rotation ϕ of this uncontracted cube. Interpret this expression for the two limiting cases of cube speed in the laboratory frame: $v \longrightarrow 0$ and $v \longrightarrow 1$.

c **Discussion question:** Is the word "really" an appropriate word in the following quotations?



EXERCISE 3-17. ***Left:*** *Position of eye of visual observer watching cube pass overhead.* ***Right top:*** *What the visual observer sees as she looks up from below.* ***Right bottom:*** *How the visual observer can interpret the projection of the second figure.*

(1) An observer using the rocket latticework of clocks says, "The stationary cube is really neither rotated nor contracted."

(2) Someone riding in the rocket who looks at the stationary cube agrees, "The cube is really neither rotated nor contracted."

(3) An observer using the laboratory latticework of clocks says, "The passing cube is really Lorentz contracted but not rotated."

(4) Someone standing in the laboratory frame looking at the passing cube says, "The cube is really rotated but not Lorentz contracted."

What can one rightfully say—in a sentence or two—to make each observer think it reasonable that the other observers should come to different conclusions?

d The analysis of parts **b** and **c** assumes that the visual observer looks with one eye and has no depth perception. How will the cube passing overhead be perceived by the viewer with accurate depth perception?

Reference: For a more complete treatment of this topic, see Edwin F. Taylor, *Introductory Mechanics* (John Wiley and Sons, New York, 1963), pages 346–360.

SPECIAL TOPIC

LORENTZ TRANSFORMATION

L.1 LORENTZ TRANSFORMATION: USEFUL OR NOT?

related events or lonely events?

Events, and the intervals between events, define the layout of the physical world. No latticework of clocks there! Only events and the relation between event and event as expressed in the interval. That's spacetime physics, lean and spare, as it offers itself to us to meet the needs of industry, science, and understanding.

Events and intervals only: Spacetime lean and spare

There's another way to express the same information and use it for the same purposes: Set up a free-float latticework of recording clocks, or the essential rudiments of such a latticework. The space and time coordinates of that Lorentz frame map each event as a lonesome individual, with no mention of any connection, any spacetime interval, to any other event.

Or isolated events described using latticework

This lattice-based method for doing spacetime physics has the advantage that it can be mechanized and applied to event after event, wholesale. These regimented space and time coordinates then acquire full usefulness only when we can translate them from the clock-lattice frame used by one analyst to the clock-lattice frame used by another.

This scheme of translation has acquired the name "Lorentz transformation." Its usefulness depends on the user. Some never need it because they deal always with intervals. Others use it frequently because it regiments records and standardizes analysis. For their needs we insert this Special Topic on the Lorentz transformation. The reader may wish to read it now, or skip it altogether, or defer it until after Chapter 4, 5, or 6. The later the better, in our opinion.

Lorentz transformation: Translate event description from lattice to lattice

L.2 FASTER THAN LIGHT?

a reason to examine the Lorentz transformation

No object travels faster than light.

So YOU say, but watch ME: I travel in a rocket that you observe to move at 4/5 light speed. Out the front of my rocket I fire a bullet that I observe to fly forward at 4/5 light speed. Then you measure this bullet to streak forward at 4/5 + 4/5 = 8/5 = 1.6 light speed, which is greater than the speed of light. There!

No!

Why not? Is it not true that 4/5 + 4/5 = 1.6?

Velocities do not add

As a mathematical abstraction: always true. As a description of the world: only sometimes true! Example 1: Add 4/5 liter of alcohol to 4/5 liter of water. The result? Less than 8/5 = 1.6 liter of liquid! Why? Molecules of water interpenetrate molecules of alcohol to yield a combined volume less that the sum of the separate volumes. Example 2: Add the speed you measure for the bullet (4/5) to the speed I measure for your rocket (4/5). The result? The speed I measure for the bullet is 40/41 = 0.9756. This remains less than the speed of light.

Why? And where did you get that number 40/41 for the bullet speed you measure?

I got the number from the Lorentz transformation, the subject of this Special Topic. The Lorentz transformation embodies a central feature of relativity: Space and time separations typically do not have the same values as observed in different frames.

Space and time separations between what?

Between events.

What events are we talking about here?

Event 1: You fire the bullet out the front of your rocket. Event 2: The bullet strikes a target ahead of you.

What do these events have to do with speed? We are arguing about speed!

Events define velocities

Let the bullet hit the target four meters in front of you, as measured in your rocket. Then the space separation between event 1 and event 2 is 4 meters. Suppose the time of flight is 5 meters as measured by your clocks, the time separation between the two events. Then your bullet speed measurement is (4 meters of distance)/(5 meters of time) = 4/5, as you said.

And what do YOU measure for the space and time separations in your laboratory frame?

For that we need the **Lorentz coordinate transformation equations.**

Phooey! I know how to reckon spacetime separations in different frames. We have been doing it for several chapters! From measurements in one frame we figure the spacetime interval, which has the same value in all frames. End of story.

No, not the end of the story, but at least its beginning. True, the invariant interval has the same value as derived from measurements in every frame. That allows you to predict the time between firing and impact as measured by the passenger riding on the bullet—and measured directly by the bullet passenger alone.

Interval: Only a start in reckoning spacetime separations in different frames

Predict how?

You know your space separation $x' = 4$ meters (primes for rocket measurements), and your time separation, $t' = 5$ meters. You know the space separation for the bullet rider, $x'' = 0$ (double primes for bullet measurements), since she is present at both the firing and the impact. From this you can use invariance of the interval to determine the wristwatch time between these events for the bullet rider:

$$(t'')^2 - (x'')^2 = (t')^2 - (x')^2$$

or

$$(t'')^2 - (0)^2 = (5 \text{ meters})^2 - (4 \text{ meters})^2 = (3 \text{ meters})^2$$

so that $t'' = 3$ meters. This is the proper time, agreed on by all observers but measured directly only on the wristwatch of the bullet rider.

Fine. Can't we use the same procedure to determine the space and time separations between these events in your laboratory frame, and thus the bullet speed for you?

Unfortunately not. We do reckon the same value for the interval. Use unprimed symbols for laboratory measurements. Then $t^2 - x^2 = (3 \text{ meters})^2$. That, however, is not sufficient to determine x or t separately. Therefore we cannot yet find their ratio x/t, which determines the bullet's speed in our frame.

Need more to compare velocities in different frames

So how can we reckon these x and t separations in your laboratory frame, thereby allowing us to predict the bullet speed you measure?

Use the Lorentz transformation. This transformation reports that our laboratory space separation between firing and impact is $x = 40/3$ meters and the time separation is slightly greater: $t = 41/3$ meters. Then bullet speed in my laboratory frame is predicted to be $v = x/t = 40/41 = 0.9756$. The results of our analysis in three reference frames are laid out in Table L-1.

Compare velocities using Lorentz transformation

Is the Lorentz transformation generally useful, beyond the specific task of reckoning speeds as measured in different frames?

Oh yes! Generally, we insert into the Lorentz transformation the coordinates x', t' of an event determined in the rocket frame. The Lorentz transformation then grinds and whirs, finally spitting out the coordinates x, t of the same event measured in the laboratory frame. Following are the Lorentz transformation equations. Here v_{rel} is the relative velocity between rocket and laboratory frames. For our convenience we lay the positive x-axis along the direction of motion of the rocket as observed in the laboratory frame and choose a common reference event for the zero of time and space for both frames.

TABLE L-1

HOW FAST THE BULLET?

	Bullet fired (coordinates of this event)	Bullet hits (coordinates of this event)	Speed of bullet (computed from frame coordinates)
Rocket frame (moves at $v_{rel} = 4/5$ as measured in laboratory)	$x' = 0$ $t' = 0$	$x' = 4$ meters $t' = 5$ meters	as measured in rocket frame: $v' = 4/5 = 0.8$
Bullet frame (moves at $v' = 4/5$ as measured in rocket)	$x'' = 0$ $t'' = 0$	$x'' = 0$ $t'' = 3$ meters (from invariance of the interval)	as measured in bullet frame: $v'' = 0$
Laboratory frame	$x = 0$ $t = 0$	$x = 40/3$ meters $t = 41/3$ meters (from Lorentz transformation)	as measured in laboratory frame: $v = 40/41 = 0.9756$

Lorentz transformation previewed

$$x = \frac{x' + v_{rel}\, t'}{(1 - v_{rel}^2)^{1/2}}$$

$$t = \frac{v_{rel}\, x' + t'}{(1 - v_{rel}^2)^{1/2}}$$

$$y = y' \qquad \text{and} \qquad z = z'$$

Check for yourself that for the impact event of bullet with target (rocket coordinates: $x' = 4$ meters, $t' = 5$ meters; rocket speed in laboratory frame: $v_{rel} = 4/5$) one obtains laboratory coordinates $x = 40/3$ meters and $t = 41/3$ meters. Hence $v = x/t = 40/41 = 0.9756$.

You say the Lorentz transformation is general. If it is so important, then why is this a special topic rather than a regular chapter?

Lorentz transformation: Useful but not fundamental

The Lorentz transformation is powerful; it brings the technical ability to transform coordinates from frame to frame. It helps us predict how to add velocities, as outlined here. It describes the Doppler shift for light (see the exercises for this chapter). On the other hand, the Lorentz transformation is not fundamental; it does not expose deep new features of spacetime. But no matter! Physics has to get on with the world's work. One uses the method of describing separation best suited to the job at hand. On some occasions the useful fact to give about a luxury yacht is the 50-meter distance between bow and stern, a distance independent of the direction in which the yacht is headed. On another occasion it may be much more important to know that the bow is 30 meters east of the stern and 40 meters north of it as observed by its captain, who uses North-Star north.

What does the Lorentz transformation rest on? On what foundations is it based?

Two foundations of Lorentz transformation

On two foundations: (1) The equations must be linear. That is, space and time coordinates enter the equations to the first power, not squared or cubed. This results from the requirement that you may choose any event as the zero of space and time.

(2) The spacetime interval between two events must have the same value when computed from laboratory coordinate separations as when reckoned from rocket coordinate separations.

All right, I'll reserve judgment on the validity of what you claim, but show me the derivation itself.

Read on!

L.3 FIRST STEPS

invariance of the interval gets us started

Recall that the coordinates y and z transverse to the direction of relative motion between rocket and laboratory have the same values in both frames (Section 3.6):

$$y = y' \\ z = z' \qquad \text{(L-1)}$$

where primes denote rocket coordinates. A second step makes use of the difference in observed clock rates when the clock is at rest or in motion (Section 1.3 and Box 3-3). Think of a sparkplug at rest at the origin of a rocket frame that moves with speed v_{rel} relative to the laboratory. The sparkplug emits a spark at time t' as measured in the rocket frame. The sparkplug is at the rocket origin, so the spark occurs at $x' = 0$.

Where and when (x and t) does this spark occur in the laboratory? That depends on how fast, v_{rel}, the rocket moves with respect to the laboratory. The spark must occur at the location of the sparkplug, whose position in the laboratory frame is given by

Derive difference in clock rates

$$x = v_{rel}t$$

Now the invariance of the interval gives us a relation between t and t',

$$(t')^2 - (x')^2 = (t')^2 - (0)^2 = t^2 - x^2 = t^2 - (v_{rel}t)^2 = t^2(1 - v_{rel}^2)$$

from which

$$t' = t\,(1 - v_{rel}^2)^{1/2}$$

or

$$t = \frac{t'}{(1 - v_{rel}^2)^{1/2}} \qquad \text{[when } x' = 0\text{]} \quad \text{(L-2)}$$

The awkward expression $1/(1 - v_{rel}^2)^{1/2}$ occurs often in what follows. For simplicity, this expression is given the symbol Greek lower-case gamma: γ.

$$\gamma \equiv \frac{1}{(1 - v_{rel}^2)^{1/2}}$$

Because it gives the ratio of observed clock rates, γ is sometimes called the **time stretch factor** (Section 5.8). Strictly speaking, we should use the symbol γ_{rel}, since the value of γ is determined by v_{rel}. For simplicity, however, we omit the subscript in the hope that this will cause no confusion. With this substitution, equation (L-2) becomes

Time stretch factor defined

$$t = \gamma t' \qquad \text{[when } x' = 0\text{]} \quad \text{(L-3)}$$

Substitute this into the equation $x = v_{rel}\, t$ above to find laboratory position in terms of rocket measurements:

$$x = v_{rel}\gamma t' \qquad \text{[when } x' = 0\text{]} \quad \text{(L-4)}$$

Equations (L-1), (L-3), and (L-4) give the first answer to the question, "If we know the space and time coordinates of an event in one free-float frame, what are its space and time coordinates in some other overlapping free-float frame?" These equations are limited, however, since they apply only to a particular situation: one in which both events occur at the same place ($x' = 0$) in the rocket.

L.4 FORM OF THE LORENTZ TRANSFORMATION

any event can be reference event? then transformation is linear

What general form does the Lorentz transformation have? It has the form that mathematicians call a **linear transformation.** This means that laboratory coordinates x and t are related to linear (first) power of rocket coordinates x' and t' by equations of the form

Lorentz transformation: Linear equations

$$t = Bx' + Dt'$$
$$x = Gx' + Ht' \qquad \text{(L-5)}$$

where our task is to find expressions for the coefficients B, D, G, and H that do not depend on either the laboratory or the rocket coordinates of a particular event, though they do depend on the relative speed v_{rel}.

Why must these transformations be linear? Because we are free to choose any event as our reference event, the common origin $x = y = z = t = 0$ in all reference frames. Let our rocket sparkplug emit the flashes at $t' = 1$ and 2 and 3 meters. These are equally spaced in rocket time. According to equation (L-3) these three events occur at laboratory times $t = 1\gamma$ and 2γ and 3γ meters of time. These are equally spaced in laboratory time. Moving the reference event to the first of these events still leaves them equally spaced in time for both observers: $t' = 0$ and 1 and 2 meters in the rocket and $t = 0$ and 1γ and 2γ in the laboratory.

In contrast, suppose that equation (L-3) were not linear, reading instead $t = Kt'^2$, where K is some constant. Rocket times $t' = 1$ and 2 and 3 meters result in laboratory times $t = 1K$ and $4K$ and $9K$ meters. These are not equally spaced in time for the laboratory observer. Moving the reference event to the first event would result in rocket times $t' = 0$ and 1 and 2 meters as before, but in this case laboratory times $t = 0$ and $1K$ and $4K$ meters, with a completely different spacing. But the choice of reference event is arbitrary: Any event is as qualified to be reference event as any other. A clock that runs steadily as observed in one frame must run steadily in the other, independent of the choice of reference event. We conclude that the relation between t and t' must be a linear one. A similar argument requires that events equally separated in space in the rocket must also be equally separated in space as measured in the laboratory. Hence the Lorentz transformation must be linear in both space and time coordinates.

Arbitrary event as reference event? Then Lorentz transformation must be linear.

L.5 COMPLETING THE DERIVATION

invariance of the interval completes the story

Equations (L-3) and (L-4) provide coefficients D and H called for in equation (L-5):

$$t = Bx' + \gamma t'$$
$$x = Gx' + v_{rel}\gamma t' \qquad \text{(L-6)}$$

About the two constants B and G we know nothing, for an elementary reason. All events so far considered occured at point $x' = 0$ in the rocket. Therefore the two coefficients B and G could have any finite values whatever without affecting the numerical results of the calculation. To determine B and G we turn our attention from an $x' = 0$ event to a more general event, one that occurs at a point with arbitrary rocket coordinates x' and t'. Then we demand that the spacetime interval have the same numerical value in laboratory and rocket frames for any event whatever:

Demanding invariance of interval . . .

$$t^2 - x^2 = t'^2 - x'^2$$

Substitute expressions for t and x from equation (L-6):

$$(Bx' + \gamma t')^2 - (Gx' + v_{rel}\gamma t')^2 = t'^2 - x'^2$$

On the left side, multiply out the squares. This leads to the rather cumbersome result

$$B^2 x'^2 + 2B\gamma x't' + \gamma^2 t'^2 - G^2 x'^2 - 2Gv_{rel}\gamma x't' - v_{rel}^2\gamma^2 t'^2 = t'^2 - x'^2$$

Group together coefficients of t'^2, coefficients of x'^2, and coefficients of the cross-term $x't'$ to obtain

$$\gamma^2(1 - v_{rel}^2)t'^2 + 2\gamma(B - v_{rel}G)\, x't' - (G^2 - B^2)x'^2 = t'^2 - x'^2 \qquad \text{(L-7)}$$

Now, t' and x' can each take on any value whatsoever, since they represent the coordinates of an arbitrary event. Under these circumstances, it is impossible to satisfy equation (L-7) with a single choice of values of B and G unless they are chosen in a very special way. The quantities B and G must first be such as to make the coefficient of $x't'$ on the left side of equation (L-7) vanish as it does on the right:

. . . between any pair of events whatsoever . . .

$$2\gamma(B - v_{rel}G) = 0$$

But γ can never equal zero. The value of $\gamma = 1/(1 - v_{rel}^2)^{1/2}$ equals unity when $v_{rel} = 0$ and is greater than this for any other values of v_{rel}. Hence the left side of this equation can be zero only if

$$(B - v_{rel}G) = 0 \qquad \text{or} \qquad B = v_{rel}G \qquad \text{(L-8)}$$

Second, B and G must be such as to make the coefficient of x'^2 equal on the left and right of equation (L-7); hence

$$G^2 - B^2 = 1 \qquad \text{(L-9)}$$

Substitute B from equation (L-8) into equation (L-9):

. . . leads to completed form of Lorentz transformation.

$$G^2 - (v_{rel}G)^2 = 1 \qquad \text{or} \qquad G^2(1 - v_{rel}^2) = 1$$

Divide through by $(1 - v_{\text{rel}}^2)$ and take the square root of both sides:

$$G = \frac{1}{(1 - v_{\text{rel}}^2)^{1/2}}$$

But the right side is just the definition of the time stretch factor γ, so that

$$G = \gamma$$

Substitute this into equation (L-8) to find B:

$$B = v_{\text{rel}}\gamma$$

These results plus equations (L-1) and (L-6) yield the Lorentz transformation equations:

The Lorentz transformation

$$\begin{aligned} t &= v_{\text{rel}}\gamma x' + \gamma t' \\ x &= \gamma x' + v_{\text{rel}}\gamma t' \\ y &= y' \\ z &= z' \end{aligned} \qquad \text{(L-10a)}$$

or, substituting for the value of gamma, $\gamma = 1/(1 - v_{\text{rel}}^2)^{1/2}$:

$$\begin{aligned} t &= \frac{v_{\text{rel}}x' + t'}{(1 - v_{\text{rel}}^2)^{1/2}} \\ x &= \frac{x' + v_{\text{rel}}t'}{(1 - v_{\text{rel}}^2)^{1/2}} \\ y &= y' \qquad \text{and} \qquad z = z' \end{aligned} \qquad \text{(L-10b)}$$

In summary, the Lorentz transformation equations rest fundamentally on the required linearity of the transformation and on the invariance of the spacetime interval. Invariance of the interval was used twice in the derivation. First, we examined a pair of events both of which occur at the same fixed location in the rocket, so that rocket time between these events—proper time, wristwatch time—equals the space-time interval between them (Section L.3). Second, we demanded that the interval also be invariant between every possible event and the reference event (the present section).

L.6 INVERSE LORENTZ TRANSFORMATION

from laboratory event coordinates, reckon rocket coordinates

Equations (L-10) provide laboratory coordinates of an event when one knows the rocket coordinates of the same event. But suppose that one already knows the laboratory coordinates of the event and wishes to predict the coordinates of the event measured by the rocket observer. What equations should be used for this purpose?

An algebraic manipulation of equations (L-10) provides the answer. The first two of these equations can be thought of as two equations in the two unknowns x' and t'. Solve for these unknowns in terms of the now-knowns x and t. To do this, multiply both sides of the second equation by v_{rel} and subtract corresponding sides of the resulting second equation from the first. Terms in x' cancel to yield

Long derivation of inverse Lorentz transformation

$$t - v_{\text{rel}} x = \gamma t' - v_{\text{rel}}^2 \gamma t' = \gamma(1 - v_{\text{rel}}^2) t' = \frac{\gamma}{\gamma^2} t' = \frac{t'}{\gamma}$$

Here we have used the definition $\gamma^2 = 1/(1 - v_{\text{rel}}^2)$. The equation for t' can then be written

$$t' = -v_{\text{rel}} \gamma x + \gamma t$$

A similar procedure leads to the equation for x'. Multiply the first of equations (L-10) by v_{rel} and subtract corresponding sides of the first equation from the second — try it! The y and z components are respectively equal in both frames, as before. Then the **inverse Lorentz transformation equations** become

$$\begin{aligned} t' &= -v_{\text{rel}} \gamma x + \gamma t \\ x' &= \gamma x - v_{\text{rel}} \gamma t \\ y' &= y \\ z' &= z \end{aligned} \qquad \text{(L-11a)}$$

Inverse Lorentz transformation

Or, substituting again for gamma, $\gamma = 1/(1 - v_{\text{rel}}^2)^{1/2}$:

$$\begin{aligned} t' &= \frac{-v_{\text{rel}} x + t}{(1 - v_{\text{rel}}^2)^{1/2}} \\ x' &= \frac{x - v_{\text{rel}} t}{(1 - v_{\text{rel}}^2)^{1/2}} \\ y' &= y \qquad \text{and} \qquad z' = z \end{aligned} \qquad \text{(L-11b)}$$

Equations (L-11) transform coordinates of an event known in the laboratory frame to coordinates in the rocket frame.

A simple but powerful *argument from symmetry* leads to the same result. The symmetry argument is based on the relative velocity between laboratory and rocket frames. With respect to the laboratory, the rocket by convention moves with known speed in the *positive* x-direction. With respect to the rocket, the laboratory moves with the same speed but in the opposite direction, the *negative* x-direction. This convention about positive and negative directions — not a law of physics! — is the only difference between laboratory and rocket frames that can be observed from either frame. Lorentz transformation equations must reflect this single difference. In consequence, the "inverse" (laboratory-to-rocket) transformation can be obtained from the "direct" (rocket-to-laboratory) transformation by changing the sign of relative velocity, v_{rel}, in the equations and interchanging laboratory and rocket labels (primed and unprimed coordinates). Carrying out this operation on the Lorentz transformation equations (L-10) yields the inverse transformation equations (L-11).

Short derivation of inverse Lorentz transformation

L.7 ADDITION OF VELOCITIES

add light velocity to light velocity: get light velocity!

The Lorentz transformation permits us to answer decisively the apparent contradiction to special relativity outlined in Section L.2, namely the apparent addition of velocities to yield a resultant velocity greater than that of light.

Return to velocity addition paradox

I travel in a rocket that you observe to move at 4/5 light speed. Out the front of my rocket I fire a bullet that I observe to fly forward at 4/5 light speed. Then you measure this bullet to streak forward at 4/5 + 4/5 = 8/5 = 1.6 light speed, which is greater than the speed of light. There!

SAMPLE PROBLEM L-1

TRANSFORMING OVER AND BACK

A rocket moves with speed $v_{rel} = 0.866$ (so $\gamma = 2$) along the x-direction in the laboratory. In the rocket frame an event occurs at coordinates $x' = 10$ meters, $y' = 7$ meters, $z' = 3$ meters, and $t' = 20$ meters of light-travel time with respect to the reference event.

a. What are the coordinates of the event as observed in the laboratory?

b. Transform the laboratory coordinates back to the rocket frame to verify that the resulting coordinates are those given above.

SOLUTION

a. We already know from Section 3.6 — as well as from the Lorentz transformation, equation (L-10) — that coordinates transverse to direction of relative motion are equal in laboratory and in rocket. Therefore we know immediately that

$$
\begin{aligned}
y &= y' = 7 \text{ meters} \\
z &= z' = 3 \text{ meters}
\end{aligned}
$$

The x and t coordinates of the event as observed in the laboratory make use of the first two equations (L-10):

$$
\begin{aligned}
t &= v_{rel}\gamma x' + \gamma t' = (0.866)(2)(10 \text{ meters}) + (2)(20 \text{ meters}) \\
&= 17.32 + 40 = 57.32 \text{ meters}
\end{aligned}
$$

and

$$
\begin{aligned}
x &= \gamma x' + v_{rel}\gamma t' = 2(10 \text{ meters}) + (0.866)(2)(20 \text{ meters}) \\
&= 20 + 34.64 = 54.64 \text{ meters}
\end{aligned}
$$

So the coordinates of the event in the laboratory are $t = 57.32$ meters, $x = 54.64$ meters, $y = 7$ meters, and $z = 3$ meters.

b. Use equation (L-11) to transform back from laboratory to rocket coordinates.

$$
\begin{aligned}
t' &= -v_{rel}\gamma x + \gamma t = -(0.866)(2)(54.64 \text{ meters}) + (2)(57.32 \text{ meters}) \\
&= -94.64 + 114.64 = 20.00 \text{ meters}
\end{aligned}
$$

and

$$
\begin{aligned}
x' &= \gamma x - v_{rel}\gamma t = 2(54.64 \text{ meters}) - (0.866)(2)(57.32 \text{ meters}) \\
&= 109.28 - 99.28 = 10.00 \text{ meters}
\end{aligned}
$$

as given in the original statement of the problem.

To analyze this experiment, convert statements about the bullet to statements about events, since event coordinates are what the Lorentz transformation transforms. Event 1 is the firing of the gun, event 2 the arrival of the bullet at the target. The Lorentz transformation equations can give locations x_1, t_1 and x_2, t_2 of these events in the laboratory frame from their known locations x'_1, t'_1 and x'_2, t'_2 in the rocket frame. In particular:

$$x_2 = \gamma x_2' + v_{rel}\gamma t'_2$$
$$x_1 = \gamma x_1' + v_{rel}\gamma t_1'$$

Subtract corresponding sides of these two equations:

$$(x_2 - x_1) = \gamma\,(x'_2 - x'_1) + v_{rel}\gamma(t'_2 - t'_1)$$

We are interested in the *differences* between the coordinates of the two emissions. Indicate these differences with the Greek uppercase delta, Δ, for example Δx. Then this x-equation and the corresponding t-equation become

Incremental event separations define velocities

$$\Delta x = \gamma\Delta x' + v_{rel}\gamma\Delta t'$$
$$\Delta t = v_{rel}\gamma\Delta x' + \gamma\Delta t' \qquad \text{(L-12)}$$

The subscript "rel" distinguishes *relative* speed between laboratory and rocket frames from other speeds, such as particle speeds in one frame or the other.

Bullet speed in any frame is simply space separation between two events on its trajectory measured in that frame divided by time between them, observed in the same frame. In the special case chosen, only the x-coordinate needs to be considered, since the bullet moves along the direction of relative motion. Divide the two sides of the first equation (L-12) by the corresponding sides of the second equation to obtain laboratory speed:

$$\frac{\Delta x}{\Delta t} = \frac{\gamma\Delta x' + v_{rel}\gamma\Delta t'}{v_{rel}\gamma\Delta x' + \gamma\Delta t'}$$

Then the time stretch factor γ cancels from the numerator and denominator on the right. Divide every term in numerator and denominator on the right by $\Delta t'$.

$$\frac{\Delta x}{\Delta t} = \frac{(\Delta x'/\Delta t') + v_{rel}}{v_{rel}(\Delta x'/\Delta t') + 1}$$

Now, $\Delta x'/\Delta t'$ is just distance covered per unit time by the particle as observed in the rocket, its speed—call it v', with a prime. And $\Delta x/\Delta t$ is particle speed in the laboratory—call it simply v. Then (reversing order of terms in the denominator to give the result its usual form) the equation becomes

Law of Addition of Velocities

$$v = \frac{v' + v_{rel}}{1 + v'v_{rel}} \qquad \text{(L-13)}$$

This is called the **Law of Addition of Velocities** in one dimension. A better name is the **Law of Combination of Velocities,** since velocities do not "add" in the usual sense. Using the Law of Combination of Velocities, we can predict bullet speed in the laboratory. The bullet travels at $v' = 4/5$ with respect to the rocket and the rocket moves at $v_{rel} = 4/5$ with respect to the laboratory. Therefore, speed v of the bullet

(continued on page 110)

SAMPLE PROBLEM L-2

"ET TU, SPACETIME!"

Julius Caesar was murdered on March 15 in the year 44 B.C. at the age of 55 approximately 2000 years ago. Is there some way we can use the laws of relativity to save his life?

Let Caesar's death be the reference event, labeled *O*: $x_o = 0$, $t_o = 0$. Event *A* is you reading this exercise. In the Earth frame the coordinates of event *A* are $x_A = 0$ light-years, $t_A = 2000$ years. Simultaneous with event *A* in your frame, Starship Enterprise cruising the Andromeda galaxy sets off a firecracker: event *B*. The Enterprise moves along a straight line in space that connects it with Earth. Andromeda is 2 million light-years distant in our frame. Compared with this distance, you can neglect the orbit of Earth around Sun. Therefore, in our frame, event B has the coordinates $x_B = 2 \times 10^6$ light-years, $t_B = 2000$ years. Take Caesar's murder to be the reference event for the Enterprise too ($x_o' = 0$, $t_o' = 0$).

a. How fast must the Enterprise be going in the Earth frame in order that Caesar's murder is happening NOW (that is, $t_B' = 0$) in the Enterprise rest frame? Under these circumstances is the Enterprise moving toward or away from Earth?

b. If you are acquainted with the spacetime diagram (Chapter 5), draw a spacetime diagram for the Earth frame that displays event *O* (Caesar's death), event *A* (you reading this exercise), event *B* (firecracker exploding in Andromeda), your line of NOW simultaneity, the position of the Enterprise, the worldline of the Enterprise, and the Enterprise NOW line of simultaneity. The spacetime diagram need not be drawn to scale.

c. In the Enterprise frame, what are the *x* and *t* coordinates of the firecracker explosion?

d. Can the Enterprise firecracker explosion warn Caesar, thus changing the course of Earth history? Justify your answer.

SOLUTION

a. From the statement of the problem,

$$\begin{array}{lll} x_o = x_o' = 0 & x_A = 0 & x_B = 2 \times 10^6 \text{ light-years} \\ t_o = t_o' = 0 & t_A = 2000 \text{ years} & t_B = 2000 \text{ years} \end{array}$$

We want the speed v_{rel} of the Enterprise such that $t_B' = 0$. The first two Lorentz transformation equations (L-10) with $t_B' = 0$ become

$$t_B = v_{rel}\gamma x_B'$$
$$x_B = \gamma x_B'$$

We do not yet know the value of x_B'. Solve for v_{rel} by dividing the two sides of the first equation by the respective sides of the second equation. The unknown x_B' drops out (along with γ), and we are left with v_{rel} in terms of the known quantities t_B and x_B:

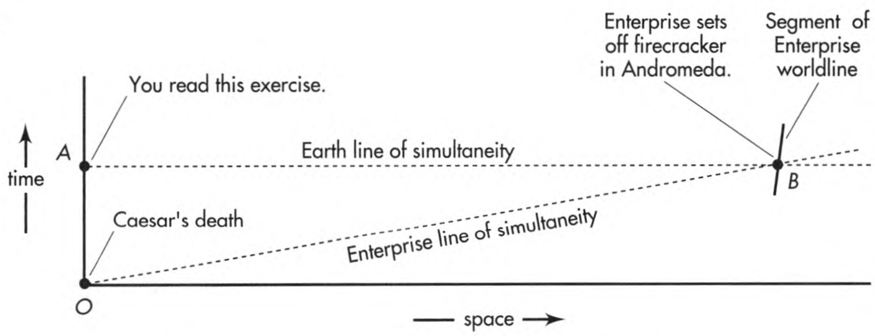
$$v_{rel} = \frac{t_B}{x_B} = \frac{2 \times 10^3 \text{ years}}{2 \times 10^6 \text{ years}} = 10^{-3} = 0.001$$

This is the desired speed v_{rel} between Earth and Enterprise frames. This velocity is a positive quantity, so the Enterprise moves in the positive *x*-direction, namely away from Earth.

Surprised to see a speed given as the ratio of a time separation to a space separation: t_B/x_B? Then realize that x_B and t_B are not displacements of any particle. Nothing can travel the distance x_B in the time t_B, as discussed in **d**. The goal here is to find a frame in which Caesar's death and the firecracker explosion are simultaneous. For this limited purpose the rocket speed $v_{rel} = t_B/x_B$ is correct.

Why is the relative velocity v_{rel} so small compared with the speed of light? Because of the large denominator x_B in the equation that leads to this value. Consider the string of Earth clocks stretching toward Andromeda when all Earth clocks read zero time (Caesar's death). Enterprise clocks read (from equations L-11 with $t = 0$) as follows: $t' = -v_{rel}\gamma x$. This is an example of the relativity of simultaneity (Section 3-4). The farther the x-distance from Earth, the earlier will Enterprise clock read. With $x = 2$ million light-years, the relative speed v_{rel} does not have to be large to carry Enterprise time back 2000 years for Earth.

b.



Earth spacetime diagram, showing events O, A, and *B. Not to scale.*

c. We need the value of gamma, γ, for the inverse Lorentz transformation equation (L-11). This value is very close to unity, and from it come $t_B{}'$ and $x_B{}'$.

$$\gamma = \frac{1}{[1 - v_{rel}^2]^{1/2}} = \frac{1}{[1 - (10^{-3})^2]^{1/2}} = \frac{1}{[1 - 10^{-6}]^{1/2}} \approx 1 + \frac{10^{-6}}{2}$$

$$t_B{}' = -v_{rel}\gamma x_B + \gamma t_B = \gamma(-10^{-3} \times 2 \times 10^6 + 2 \times 10^3)$$

$$= \gamma(-2 \times 10^3 + 2 \times 10^3) = 0 \text{ years}$$

$$x_B{}' = \gamma x_B - v_{rel}\gamma t_B = \gamma(2 \times 10^6 - 10^{-3} \times 2 \times 10^3) = 2\gamma(1 - 10^{-6})\,10^6$$

$$= 2\left(1 + \frac{10^{-6}}{2}\right)(1 - 10^{-6})10^6 = 2\left(1 - \frac{10^{-6}}{2} - \frac{10^{-12}}{2}\right)10^6$$

$$\approx 1.999999 \times 10^6 \text{ light-years.}$$

We *chose* the relative velocity so that the time of the firecracker explosion as observed in the rocket is the same as the time of Caesar's death, namely $t_B{}' = 0$. The x-coordinate of this explosion is not much different in the two frames because their relative velocity is so small.

d. There exists a frame — the rest frame of the Enterprise — in which Caesar's death and the firecracker explosion occur at the same time. In this frame a signal connecting the two events would have to travel at infinite speed. But this is impossible. Therefore the Enterprise cannot warn Caesar; his death is final. Sorry. (Note: In the language of Chapter 6, the relation between the two events is spacelike, and spacelike events cannot have a cause–effect relationship.)

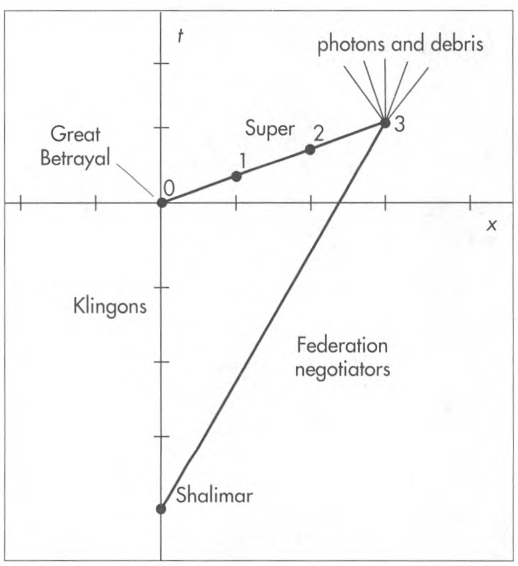
BOX L-1

WHY NO THING TRAVELS FASTER THAN LIGHT

A material object traveling faster than light? No! If one did, we could violate the normal order of cause and effect in a million testable ways, totally contrary to all experience. Here we investigate one example, making use of Lorentz transformation equations.

The Peace Treaty of Shalimar was signed four years before the Great Betrayal. So pivotal an event was the Great Betrayal that it was taken as zero of space and time.

By the Treaty of Shalimar, the murderous Klingons agreed to stop attacking Federation outposts in return for access to the Federation Technical Database. Federation negotiators left immediately after signing the Shalimar Treaty in a ship moving at 0.6 light speed.



Klingon ("laboratory") spacetime diagram. *The Klingon worldline is the vertical time axis. The Treaty of Shalimar is followed four years later by the Great Betrayal (event 0) at which Klingons launch the Super, which moves at three times light speed. Traveling from left to right, the Super passes one Federation colony (event 1) and then another (event 2). Finally the Super destroys the retreating ship of Federation negotiators (event 3).*

Within four years the Klingons used the Federation Technical Database to develop a faster-than-light projectile, the slaughtering Super. On that dark day of Great Betrayal (reference event 0), the Klingons launched the Super at three times light speed toward the retreating Federation ship.

Two Federation space colonies lay between the Klingons and the point of impact of the Super with the Federation ship. A lonely lookout at the first colony witnessed with awe the blinding passage of the Super (event 1). Later many citizens of the second colony gaped as the Super demolished one of their communication structures (event 2) and zoomed on. Both colonies desperately sent warnings toward the Federation ship, but to no avail since the Super outran the radio signals.

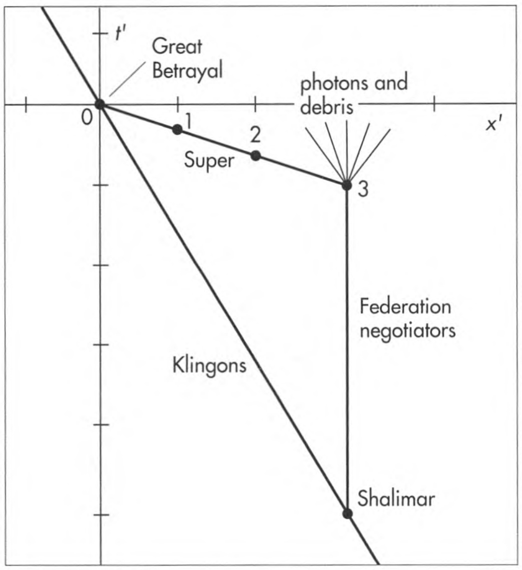
Finally, at event 3, the Super overtook and destroyed the Federation ship. All Federation negotiators were lost in a terrible flash of light and scattering of debris. A long dark period of renewed warfare began.

But wait! Look again at events of the Great Betrayal, this time from the point of view of the Federation rocket ship. Where and when does the Great Betrayal occur in this frame? The Great Betrayal is the "hinge of history," the reference event, the zero of space and time coordinates for all laboratory and rocket frames.

Where and when does the Super explode (event 3) in this rocket frame? In the Klingon "laboratory" frame, event 3 has coordinates $x_3 = 3$ light-years and $t_3 = 1$ year. Use the inverse Lorentz transformation equations to find the location of event 3 in the rocket frame of the Federation negotiators. Calculate the time stretch factor γ using speed of the Federation rocket, $v_{rel} = 0.6$, with respect to the Klingon frame:

$$\gamma = \frac{1}{[1 - v_{rel}^2]^{1/2}} = \frac{1}{[1 - (0.6)^2]^{1/2}} = \frac{1}{[1 - 0.36]^{1/2}}$$

$$= \frac{1}{[0.64]^{1/2}} = \frac{1}{0.8} = 1.25$$



"Rocket" spacetime diagram of departing Federation negotiators. In this frame their destruction comes first (event 3), followed by the passage of the Super from right to left past Federation colonies in reverse order (event 2 followed by event 1). Finally, the Super enters the Klingon launcher without doing further damage (event 0). The Great Betrayal has become the Great Confusion of Cause and Effect.

Substitute these values into equations (L-11) to reckon the rocket coordinates of event 3:

$$
\begin{aligned}
t'_3 &= -v_{rel}\gamma x_3 + \gamma t_3 \\
&= -(0.6)(1.25)(3 \text{ years}) + (1.25)(1 \text{ year}) \\
&= -2.25 \text{ years} + 1.25 \text{ years} = -1 \text{ year} \\
x'_3 &= \gamma x_3 - v_{rel}\gamma t_3 \\
&= (1.25)(3 \text{ years}) - (0.6)(1.25)(1 \text{ year}) \\
&= 3.75 \text{ years} - 0.75 \text{ year} = 3 \text{ years}
\end{aligned}
$$

Event 3 is plotted in the rocket diagram and the worldline of the Super drawn by connecting event 3 with the launching of the Super at event 0. Notice that this worldline slopes downward to the right. More about the significance of this in a minute.

In a similar manner find the rocket coordinates of the treaty signing at Shalimar (subscript Sh), which has laboratory coordinates $x_{Sh} = 0$ and $t_{Sh} = -4$ years:

$$
\begin{aligned}
t'_{Sh} &= -v_{rel}\gamma x_{Sh} + \gamma t_{Sh} \\
&= -(0.6)(1.25)(0 \text{ years}) + (1.25)(-4 \text{ years}) \\
&= -5 \text{ years} \\
x'_{Sh} &= \gamma x_{Sh} - v_{rel}\gamma t_{Sh} \\
&= (1.25)(0 \text{ years}) - (0.6)(1.25)(-4 \text{ years}) \\
&= +3 \text{ years}
\end{aligned}
$$

In the Federation (rocket) spacetime diagram, the worldline of Federation negotiators extends from treaty signing at Shalimar vertically to explosion of the Super (event 3). The worldline of the Klingons extends from Shalimar diagonally through the launch of the Super at event 0.

In the Federation spacetime diagram, the worldline for the Super tilts downward to the right. In this frame deaths of Federation negotiators (event 3) occur at a time $t'_3 =$ *minus 1 year,* that is, *before* the treacherous Klingons launch the Super at the event of Great Betrayal (reference event 0). From the diagram one would say that the Super moves with three times light speed *from* Federation ship *toward* the Klingons. This seems to be verified by the fact that in this frame the Super passes Federation colonies in reverse order, event 2 followed by event 1, going in the opposite direction. Yet Federation negotiators have created no such terrible weapon and in fact are destroyed by it at the moment they are supposed to launch it, as proved by the flying photons and debris. More: Klingons suffer no damage from the mighty impact of the slaughtering Super (event 0). Rather, in this frame it enters their launching cannon mild as a lamb.

What have we here? A confusion of cause and effect, a confusion that cannot be straightened out as long as we assume that the Super — or any other material object — travels faster than light in a vacuum.

Why does no signal and no object travel faster than light in a vacuum? Because if either signal or object did so, the entire network of cause and effect would be destroyed, and science as we know it would not be possible.

relative to the laboratory comes from the expression

Velocity addition paradox resolved

$$v = \frac{4/5 + 4/5}{1 + (4/5)(4/5)} = \frac{8/5}{1 + 16/25} = \frac{8/5}{41/25} = \frac{40}{41}$$

Thus the bullet moves in the laboratory at a speed less than light speed.

As a limiting case, suppose that the "bullet" shot out from the front of the rocket is, in fact, a pulse of light. Guess: What is the speed of this light pulse in the laboratory? Here is the calculated answer. Light moves with respect to the rocket at speed $v' = 1$ while the rocket continues along at a speed $v_{rel} = 4/5$ with respect to the laboratory. The light then moves with respect to the laboratory at speed v:

Light speed is invariant, as expected.

$$v = \frac{1 + 4/5}{1 + (1)(4/5)} = \frac{9/5}{9/5} = 1$$

So light moves with the same speed in both frames, as required by the Principle of Relativity. Question: Is this true also when a light pulse is shot out of the *rear* of the rocket?

SAMPLE PROBLEM L-3

THE FIRING MESON

A K^o (pronounced "K-naught") meson at rest in a rocket frame decays into π^+ ("pi plus") meson and a π^- ("pi minus") meson, each having a speed of $v' = 0.85$ with respect to the rocket. Now consider this decay as observed in a laboratory with respect to which the K^o meson travels at a speed of $v_{rel} = 0.9$. What is the greatest speed that one of the π mesons can have with respect to the laboratory? What is the least speed?

SOLUTION

Let the speeding K^o-meson move in the positive x-direction in the laboratory. In the rocket frame, daughter π-mesons come off in opposite directions. Their common line of motion can, however, be oriented arbitrarily in this frame. The maximum speed of a daughter π-meson in the laboratory results when it is emitted in the forward x-direction. For such a meson, the law of addition of velocities gives

$$v_{max} = \frac{v' + v_{rel}}{1 + v'v_{rel}} = \frac{0.85 + 0.9}{1 + (0.85)(0.9)} = \frac{1.75}{1.765} = 0.9915$$

Thus adding a speed of 0.85 to a speed of 0.9 does not yield a resulting speed greater than 1, light speed.

The slowest laboratory speed for a daughter meson occurs when it is emitted in the negative x-direction in the rocket frame. In this case the velocity of the daughter meson is negative and the law of addition of velocities becomes a law of subtraction of velocities:

$$v_{min} = \frac{-v' + v_{rel}}{1 - v'v_{rel}} = \frac{-0.85 + 0.9}{1 - (0.85)(0.9)} = \frac{0.05}{0.235} = 0.2128$$

Although the minimum-speed meson moves to the left in the rocket, it moves to the right in the laboratory because of the very great speed of the original K^o-meson in the laboratory.

L.8 SUMMARY

Lorentz transformation deals with coordinates, not invariant quantities

Given the space and time coordinates of an event with respect to the reference event in one free-float frame, the **Lorentz coordinate transformation equations** tell us the coordinates of the same event in an overlapping free-float frame in relative motion with respect to the first. The equations that transform rocket coordinates (primed coordinates) to laboratory coordinates (unprimed coordinates) have the form

$$
\begin{aligned}
t &= \frac{v_{rel}x' + t'}{(1 - v_{rel}^2)^{1/2}} \\
x &= \frac{x' + v_{rel}t'}{(1 - v_{rel}^2)^{1/2}} \qquad \text{(L-10b)} \\
y &= y' \qquad \text{and} \qquad z = z'
\end{aligned}
$$

where v_{rel} stands for relative speed of the two frames (rocket moving in the positive x-direction in the laboratory). The **inverse Lorentz transformation equations** transform laboratory coordinates to rocket coordinates:

$$
\begin{aligned}
t' &= \frac{-v_{rel}x + t}{(1 - v_{rel}^2)^{1/2}} \\
x' &= \frac{x - v_{rel}\,t}{(1 - v_{rel}^2)^{1/2}} \qquad \text{(L-11b)} \\
y' &= y \qquad \text{and} \qquad z' = z
\end{aligned}
$$

in which v_{rel} is treated as a positive quantity. In both these sets of equations, coordinates of events are measured with respect to a reference event. It is really only the *difference* in coordinates between events that matter, for example $x_2 - x_1 = \Delta x$ for any two events 1 and 2, not the coordinates themselves. This is important in deriving the Law of Addition of Velocities.

The **Law of Addition of Velocities** or **Law of Combination of Velocities** in one dimension follows from the Lorentz transformation equations. This law tells us the velocity v of a particle in the laboratory frame if we know its velocity v' with respect to the rocket and relative speed v_{rel} between rocket and laboratory,

$$
v = \frac{v' + v_{rel}}{1 + v'v_{rel}} \qquad \text{(L-13)}
$$

REFERENCE

Sample Problem L-3, The Firing Meson, was adapted from A. P. French, *Special Relativity* (W.W. Norton, New York, 1968), page 159.

SPECIAL TOPIC EXERCISES

PRACTICE

L-1 a super-speed super?

Take two more steps in the parable of the Great Betrayal (Box L-1).

a Find the speed of a new rocket frame moving relative to the Klingon frame such that the Super travels at 6 times the speed of light in this new frame. Hint: Examine the coordinates x' and t' of event 3 in the new frame. The ratio of these two, x'/t', is the speed of the Super in this frame. We know the coordinates of event 3 in the Klingon frame. Therefore . . .

b Find the speed of yet another rocket frame, relative to the Klingon frame, such that the Super travels with infinite speed in this frame. Hint: What does infinite speed imply about the time t' between events 0 and 3 in this new frame?

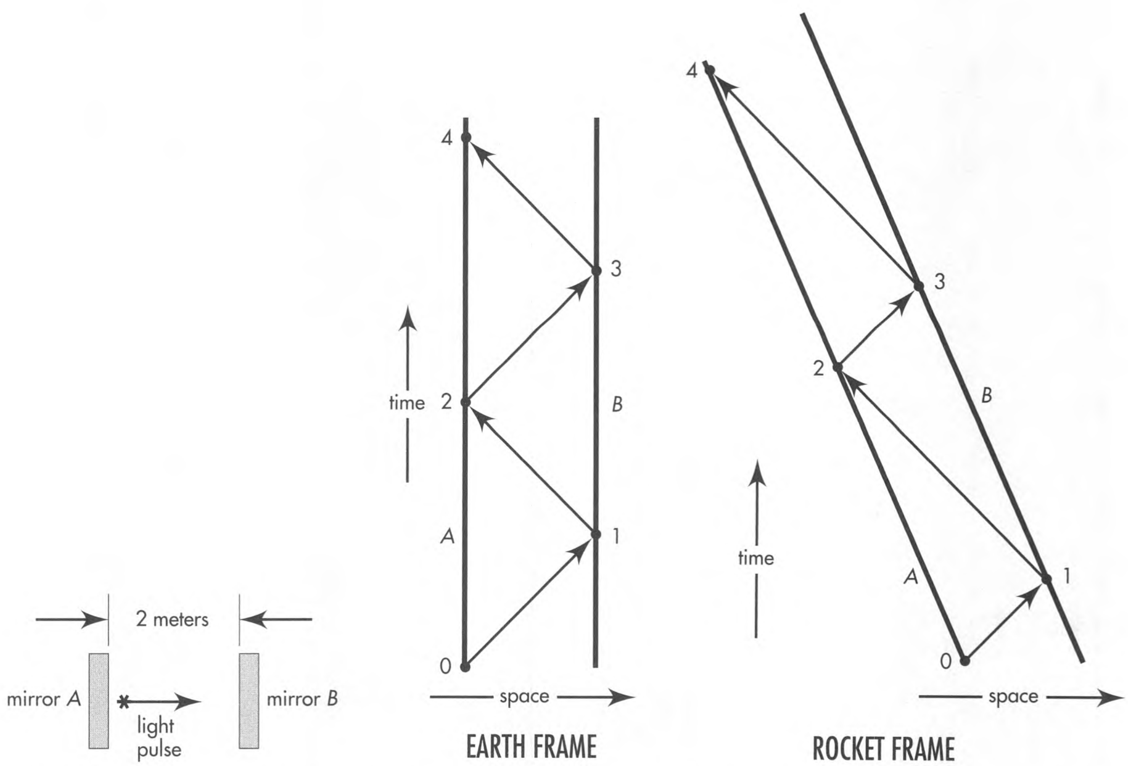
L-2 a bad clock

Note: This exercise uses spacetime diagrams, introduced in Chapter 5.

A pulse of light is reflected back and forth between mirrors A and B separated by 2 meters of distance in the x-direction in the Earth frame, as shown in the figure (left). A swindler tells us that this device constitutes a clock that "ticks" every time the pulse arrives at either mirror.

The swindler claims that events 1 through 6 are sequential "ticks" of this clock (center). However, we notice that the ticking of the clock is uneven in a rocket frame moving with speed v_{rel} in the Earth frame (right). For example, there is less time between events 0 and 1 than between events 1 and 2 as measured in the rocket frame.

a What is the physical basis for the "bad" behavior of this clock? Use the Lorentz transformation



EXERCISE L-2. ***Left:*** *Horizontal light-pulse clock as observed in the Earth frame.* ***Center:*** *Spacetime diagram showing worldlines of mirrors* A *and* B *and the "uniformly ticking" light pulse as observed in the Earth frame.* ***Right:*** *Time lapses between sequential ticks of the light-pulse clock are not uniform as observed in the rocket frame.*

equations to account for the uneven ticking of this clock in the rocket frame.

b Use some of the same events 0 through 4 to define a "good" clock that ticks evenly in both the laboratory frame and the rocket frame. From the spacetime diagrams, show qualitatively that your good clock "runs slow" as observed from the rocket frame—as it must, since the clock is in motion with respect to the rocket frame.

c Explain why the clock of Figure 1-3 in the text is a "good" clock.

L-3 the Galilean transformation

a Use everyday, nonrelativistic Newtonian arguments to derive transformation equations between reference frames moving at low relative velocities. Show that the result is

$$x' = x - v_{\text{conv}}\, t_{\text{sec}} \qquad \text{(Newtonian: } v_{\text{conv}} << c) \qquad (1)$$

$$t'_{\text{sec}} = t_{\text{sec}} \qquad \text{(Newtonian: } v_{\text{conv}} << c) \qquad (2)$$

where t_{sec} is time measured in seconds and v_{conv} is speed in conventional units (meters/second for example). List the assumptions you make in your derivation.

b Convert equations (1) and (2) to measure time t in meters and unitless measure of relative velocity, $v_{\text{rel}} = v_{\text{con}}/c$. Show the results are:

$$x' = x - v_{\text{rel}}\, t \qquad \text{(Newtonian: } v << 1) \qquad (3)$$

$$t' = t \qquad \text{(Newtonian: } v << 1) \qquad (4)$$

Do the new units make these equations correct at high relative velocity between frames?

c Use the first two terms in the binomial expansion to find a low-velocity approximation for γ in the Lorentz transformation.

$$\gamma = \frac{1}{(1 - v_{\text{rel}}^2)^{1/2}} = (1 - v_{\text{rel}}^2)^{-1/2} \approx 1 + \frac{v_{\text{rel}}^2}{2}$$

Show that this expression differs from unity by less than one percent provided v is less than $1/7$. A sports car can accelerate uniformly from rest to 60 miles/hour (about 27 meters/second) in 7 seconds. Roughly how many days would it take for the sports car to reach $v = 1/7$ at the same constant acceleration?

d Set $\gamma = 1$ in the Lorentz transformation equations. Show that the resulting "low-velocity Lorentz transformation" is

$$x' = x - v_{\text{rel}}\, t \qquad \text{(Lorentz: } v << 1) \qquad (5)$$

$$t' = -v_{\text{rel}}\, x + t \qquad \text{(Lorentz: } v << 1) \qquad (6)$$

What is the difference between the time transformations for the "Newtonian low-velocity limit" of equation (4) and the "Lorentz low-velocity limit" of equation (6)? How can they both be correct? The term $-v_{\text{rel}}x$ does not depend on any time lapse, but only on the separation x of the event from the laboratory origin. This term is due to the difference of synchronization of clocks in the two frames.

e In each of the following cases a laboratory clock (measuring t) at a distance x from the origin as measured in the laboratory frame is compared with a passing rocket clock (measuring t'). Say whether or not the time difference $t - t' = v_{\text{rel}}x$ can be detected using wristwatches (accuracy of 10^{-1} second $= 3 \times 10^7$ meters of light-travel time) and using modern electronic clocks (accuracy of 10^{-9} second $= 0.3$ meter of time).

(1) Sports car traveling at 100 kilometers/hour (roughly 30 meters/second) located 1000 kilometers down the road from the origin as measured in the Earth frame.

(2) Moon probe traveling at 30,000 kilometers/hour passing Moon, 3.8×10^5 kilometers from the origin on Earth as measured in the Earth frame.

(3) Distance from origin on Earth at which space probe traveling at 30,000 kilometers/hour leads to detectable time difference between rocket wristwatch and adjacent Earth-linked latticework clock. Compare with Earth–Sun distance of 1.5×10^{11} meters.

f Summarize in a sentence or two the conditions under which the regular Galilean transformation equations (3) and (4) will lead to correct predictions.

L-4 limits of Newtonian mechanics

Use the particle speed $v_{\text{crit}} = 1/7$ (Exercise L-3) as an approximate maximum limit for the validity of Newtonian mechanics. Determine whether or not Newtonian mechanics is adequate to analyze motion in each of the following cases, following the example.

Example: Satellite circling Earth at 30,000 kilometers/hour = 18,000 miles/hour. **Answer:** Light moves at a speed $v_{\text{conv}} = (3 \times 10^5$ kilometers/second$) \times (3600$ seconds/hour$) = 1.08 \times 10^9$ kilometers/hour. Therefore the speed of the satellite in meters/meter is $v = v_{\text{conv}}/c = 2.8 \times 10^{-5}$. This is much less than $v_{crit} = 1/7$, so the Newtonian description of satellite motion is adequate.

a Earth circling Sun at an orbital speed of 30 kilometers/second.

b Electron circling a proton in the orbit of smallest radius in a hydrogen atom. **Discussion:** The classical speed of the electron in the inner orbit of an atom of atomic number Z, where Z is the number of protons in the nucleus, is given, for low velocities, by the expression $v = Z/137$. For hydrogen, $Z = 1$.

c Electron in the inner orbit of the gold atom, for which $Z = 79$.

d Electron after acceleration from rest through a voltage of 5000 volts in a black-and-white television picture tube. **Discussion:** We say that this electron has a kinetic energy of 5000 electron-volts. One electron-volt is equal to 1.6×10^{-19} joule. Try using the Newtonian expression for kinetic energy.

e Electron after acceleration from rest through a voltage of 25,000 volts in a color television picture tube.

f A proton or neutron moving with a kinetic energy of 10 MeV (million electron-volts) in a nucleus.

PROBLEMS

L-5 Doppler shift

A sparkplug at rest in the rocket emits light with a frequency f' pulses or waves per second. What is the frequency f of this light as observed in the laboratory? Let this train of waves (or pulses) of light travel in the positive x-direction with speed c, so that in the course of one meter of light-travel time, f/c of these pulses pass the origin of the laboratory frame. It is understood that the *zero*th or "fiducial" crest or pulse passes the origin at the zero of time—and that the origin of the rocket frame passes the origin of the laboratory frame at this same time.

a Show that the x-coordinate of the nth pulse or wave crest is related to the time of observation t (in meters) by the equation

$$n = (f/c)(t - x)$$

b The same argument, applied in the rocket frame, leads to the relation

$$n = (f'/c)(t' - x')$$

Express this rocket formula in laboratory coordinates x and t using the Lorentz transformation. Equate the resulting expression for f' to the laboratory formula for f in terms of x and t to derive the simple formula for f in terms of f' and v_{rel}, the relative speed of laboratory and rocket frames.

$$f = \left(\frac{1 + v_{rel}}{1 - v_{rel}}\right)^{1/2} f' \qquad \text{[wave moves in positive } x\text{-direction]}$$

c Now observe a wave moving along the negative x-direction from the same source at rest in the rocket frame. Show that the frequency of the wave observed in the laboratory frame is

$$f = \left(\frac{1 - v_{rel}}{1 + v_{rel}}\right)^{1/2} f' \qquad \text{[wave moves in negative } x\text{-direction]}$$

d Astronomers define the **redshift** z of light from a receding astronomical object by the formula

$$z = \frac{f_{emit} - f_{obs}}{f_{obs}}$$

Here f_{emit} is the frequency of the light measured in the frame in which the emitter is at rest and f_{obs} the frequency observed in another frame in which the emitter moves directly away from the observer.

The most distant quasar reported as of 1991 has a redshift $z = 4.897$. With what fraction of the speed of light is this quasar receding from us?

Reference: D. P. Schneider, M. Schmidt, and J. E. Gunn, *Astronomical Journal,* Volume 102, pages 837–840 (1991).

L-6 transformation of angles

a A meter stick lies at rest in the rocket frame and makes an angle ϕ' with the x'-axis. Laboratory observers measure the x- and y-projections of the stick as it streaks past. What values do they measure for these projections, compared with the x'- and y'-projections measured by rocket observers? Therefore what angle ϕ does the same meter stick make with the x-axis of the laboratory frame? What is the length of the "meter stick" as observed in the laboratory frame?

b Make the courageous assumption that the directions of electric-field lines around a point charge transform in the same way as the directions of meter sticks that lie along these lines. (Electric field lines around a point charge are assumed to be infinite in length, so the length transformation of part **a** does not apply.) Draw qualitatively the electric-field lines due to an isolated positive point charge at rest in the rocket frame as observed in (1) the rocket frame and (2) the laboratory frame. What conclusions follow concerning the time variation of electric forces on nearby charges at rest in the laboratory frame?

L-7 transformation of y-velocity

A particle moves with uniform speed $v_y' = \Delta y'/\Delta t'$ along the y'-axis of the rocket frame. Transform $\Delta y'$ and $\Delta t'$ to laboratory displacements Δx, Δy, and Δt using the Lorentz transformation equations. Show that the x-component and the y-component of the velocity of this particle in the laboratory frame are given by the expressions

$$v_x = v_{rel}$$

$$v_y = v_y'(1 - v_{rel}^2)^{1/2}$$

L-8 transformation of velocity direction

A particle moves with velocity v' in the $x'y'$ plane of the rocket frame in a direction that makes an angle ϕ' with the x'-axis. Find the angle ϕ that the velocity vector of this particle makes with the x-axis of the laboratory frame. (Hint: Transform space and time displacements rather than velocities.) Why does this angle differ from that found in Exercise L-6 on transformation of angles? Contrast the two results when the relative velocity between the rocket and laboratory frames is very great.

L-9 the headlight effect

A flash of light is emitted at an angle ϕ' with respect to the x'-axis of the rocket frame.

a Show that the angle ϕ the direction of motion of this flash makes with respect to the x-axis of the laboratory frame is given by the equation

$$\cos\phi = \frac{\cos\phi' + v_{rel}}{1 + v_{rel}\cos\phi'}$$

b Show that your answer to Exercise L-8 gives the same result when the velocity v' is given the value unity.

c A particle at rest in the rocket frame emits light uniformly in all directions. Consider the 50 percent of this light that goes into the forward hemisphere in the rocket frame. Show that in the laboratory frame this light is concentrated in a narrow forward cone of half-angle ϕ_o whose axis lies along the direction of motion of the particle. The half-angle ϕ_o is the solution to the following equation:

$$\cos\phi_o = v_{rel}$$

This result is called the **headlight effect.**

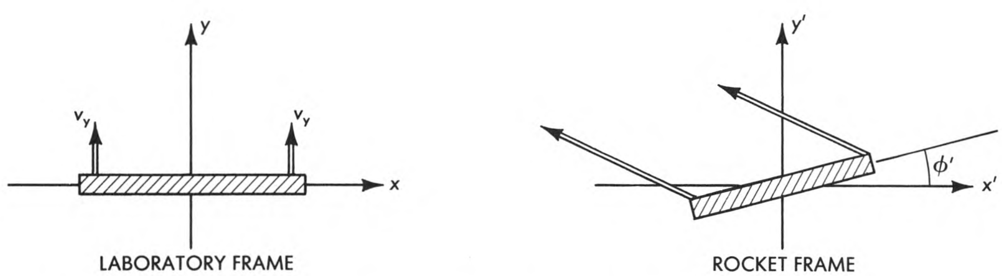
L-10 the tilted meter stick

Note: This exercise uses the results of Exercise L-7.

A meter stick lying parallel to the x-axis moves in the y-direction in the laboratory frame with speed v_y as shown in the figure (left).

a In the rocket frame the stick is tilted upward in the positive x'-direction as shown in the figure (right). Explain why this is, first without using equations.

b Let the center of the meter stick pass the point $x = y = x' = y' = 0$ at time $t = t' = 0$. Calculate the angle ϕ' at which the meter stick is inclined to the x'-axis as observed in the rocket frame. **Discussion:** Where and when does the right end of the meter stick cross the x-axis as observed in the laboratory frame? Where and when does this event of right-end crossing occur as measured in the rocket frame? What is the direction and magnitude of the velocity of the meter stick in the rocket frame (Exercise L-7)? Therefore where is the right end of the meter stick at $t' = 0$, when the center is at the origin? Therefore . . .



EXERCISE L-10. ***Left:*** *Meter stick moving transverse to its length as observed in the laboratory frame.* ***Right:*** *Meter stick as observed in rocket frame.*

L-11 the rising manhole

Note: This exercise uses the results of Exercise L-10.

A meter stick lies along the x-axis of the laboratory frame and approaches the origin with velocity v_{rel}. A very thin plate parallel to the xz laboratory plane moves upward in the y-direction with speed v_y as shown in the figure. The plate has a circular hole with a diameter of one meter centered on the y-axis. The center of the meter stick arrives at the laboratory origin at the same time in the laboratory frame as the rising plate arrives at the plane $y = 0$. Since the meter stick is Lorentz-contracted in the laboratory frame it will easily pass through the hole in the rising plate. Therefore there will be no collision between meter stick and plate as each continues its motion. However, someone who objects to this conclusion can make the following argument: "In the rocket frame in which the meter stick is at rest the meter stick is not contracted, while in this frame the hole in the plate is Lorentz-contracted. Hence the full-length meter stick cannot possibly pass through the contracted hole in the plate. Therefore there must be a collision between the meter stick and the plate." Resolve this paradox using your answer to Exercise L-10. Answer unequivocally the question, Will there be a collision between the meter stick and the plate?

Reference: R. Shaw, *American Journal of Physics,* Volume 30, page 72 (1962).

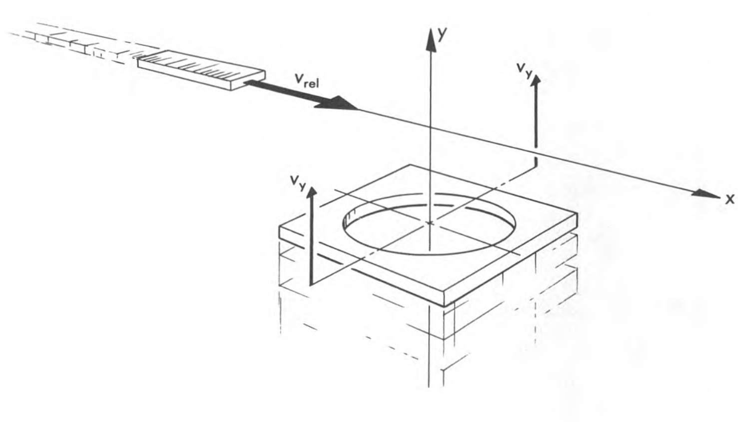
L-12 paradox of the skateboard and the grid

A girl on a skateboard moves very fast, so fast that the relativistic length contraction makes the skateboard very short. On the sidewalk she has to pass over a grid. A man standing at the grid fully expects the fast short skateboard to fall through the holes in the grid. Yet to the fast girl her skateboard has its usual length and it is the grid that has the relativistic contraction. To her the holes in the grid are much narrower than to the stationary man, and she certainly does not expect her skateboard to fall through them. Which person is correct? The answer hinges on the relativity of rigidity.

Idealize the problem as a one-meter rod sliding lengthwise over a flat table. In its path is a hole one meter wide. If the Lorentz contraction factor is ten, then in the table (laboratory) frame the rod is 10 centimeters long and will easily drop into the one-meter-wide hole. Assume that in the laboratory frame the meter stick moves fast enough so that it remains essentially horizontal as it descends into the hole (no "tipping" in the laboratory frame). Write an equation in the laboratory frame for the motion of the bottom edge of the meter stick assuming that $t = t' = 0$ at the instant that the back end of the meter stick leaves the edge of the hole. For small vertical velocities the rod will fall with the usual acceleration g. Note that in the laboratory frame we have assumed that every point along the length of the meter stick begins to fall simultaneously.

In the meter stick (rocket) frame the rod is one meter long whereas the hole is Lorentz-contracted to a 10-centimeter width so that the rod cannot possibly fit into the hole. Moreover, in the rocket frame different parts along the length of the meter stick begin to drop at different times, due to the relativity of simultaneity. Transform the laboratory equations into the rocket frame. Show that the front and back of the rod will begin to descend at different times in this frame. The rod will "droop" over the edge of the hole in the rocket frame—that is, it will not be rigid. Will the rod ultimately descend into the hole in both frames? Is the rod *really* rigid or nonrigid during the experiment? Is it possible to derive any physical characteristics of the rod (for example its flexibility or compressibility) from the description of its motion provided by relativity?

Reference: W. Rindler, *American Journal of Physics,* Volume 29, page 365–366 (1961).



EXERCISE L-11. *Will the "meter stick" pass through the "one-meter-diameter" hole without collision?*

L-13 paradox of the identically accelerated twins

Note: This exercise uses spacetime diagrams, introduced in Chapter 5.

Two fraternal twins, Dick and Jane, own identical spaceships each containing the same amount of fuel. Jane's ship is initially positioned a distance to the right of Dick's in the Earth frame. On their twentieth birthday they blast off at the same instant in the Earth frame and undergo identical accelerations to the right as measured by Mom and Dad, who remain at home on Earth. Mom and Dad further observe that the twins run out of fuel at the same time and move thereafter at the same speed v. Mom and Dad also measure the distance between Dick and Jane to be the same at the end of the trip as at the beginning.

Dick and Jane compare the ships' logs of their accelerations and find the entries to be identical. However when both have ceased accelerating, Dick and Jane, in their new rest frame, discover that Jane is older than Dick! How can this be, since they have an identical history of accelerations?

a Analyze a simpler trip, in which each spaceship increases speed not continuously but by impulses, as shown in the first spacetime diagram and the event table. How far apart are Dick and Jane at the beginning of their trip, as observed in the Earth frame? How far apart are they at the end of their accelerations? What is the final speed v (not the average speed) of the two spaceships? How much does each astronaut age along the worldline shown in the diagram? (The answer is not the Earth time of 12 years.)

b The second spacetime diagram shows the two worldlines as recorded in a rocket frame moving with the final velocity of the two astronauts. Copy the figure. On your copy extend the worldlines of Dick and Jane after each has ceased accelerating. Label your figure to show that Jane ceased accelerating before Dick as observed in this frame. Will Dick age the same between events 0 and 3 in this frame as he aged in the Earth frame? Will Jane age the same between events 4 and 7 in this frame as she aged in the Earth frame?

c Now use the Lorentz transformation to find the space and time coordinates of one or two critical events in this final rest frame of the twins in order to answer the following questions

(1) How many years earlier than Dick did Jane cease accelerating?

(2) What is Dick's age at event 3? (not the rocket time t' of this event!)

(3) What is Jane's age at event 7?

(4) What is Jane's age at the same time (in this frame) as event 3?

(5) What are the ages of Dick and Jane 20 years after event 3, assuming that neither moves again with respect to this frame?

(6) How far apart in space are Dick and Jane when both have ceased accelerating?

(7) Compare this separation with their initial (and final!) separation measured by Mom and Dad in the Earth frame.

d Extend your results to the general case in which Mom and Dad on Earth observe a period of identical *continuous* accelerations of the two twins.

(1) At the two start-acceleration events (the two events at which the twins start their rockets), the twins are the same age as observed in the Earth frame. Are they the same age at these events as observed in every rocket frame?

(2) At the two cease-acceleration events (the two events at which the rockets run out of fuel), are the twins the same age as observed in the Earth frame? Are they the same age at these events as observed in every rocket frame?

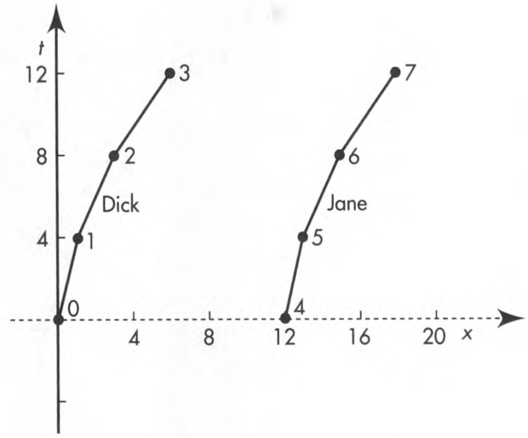
(3) The two cease-acceleration events are simultaneous in the Earth frame. Are they simultaneous as observed in every rocket frame? (No!) Whose cease-acceleration event occurs first as observed in the final frame in which both twins come to rest? (Recall the Train Paradox, Section 3.4.)

(4) "When Dick ceases accelerating, Jane is older than Dick." Is this statement true according to the astronauts in their final rest frame? Is the statement true according to Mom and Dad in the Earth frame?

(5) Criticize the lack of clarity (swindle?) of the word *when* in the statement of the problem: "However when both have ceased accelerating, Dick and Jane, in their new rest frame, discover that Jane is older than Dick!"

e Suppose that Dick and Jane both accelerate to the left, so that Dick is in front of Jane, but their history is otherwise the same. Describe the outcome of this trip and compare it with the outcome of the original trip.

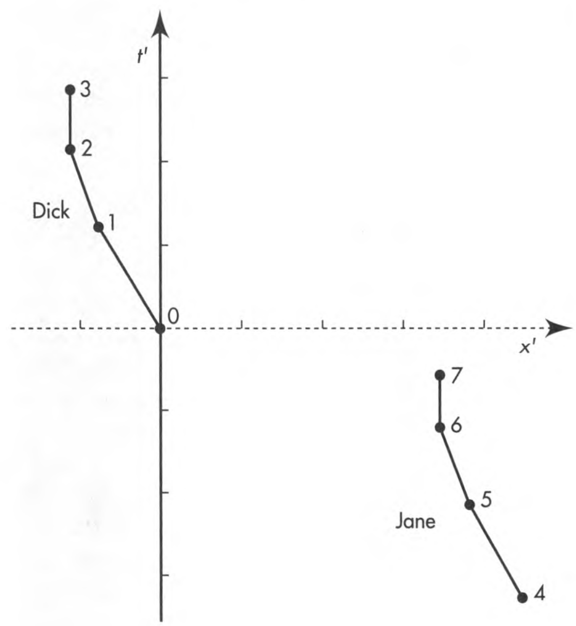
f Suppose that Dick and Jane both accelerate in a direction perpendicular to the direction of their separation. Describe the outcome of this trip and compare it with the outcome of the original trip.



Earth Frame Observations

Event number	x-position (light years)	Time (years)
0	0	0
1	1	4
2	3	8
3	6	12
4	12	0
5	13	4
6	15	8
7	18	12

EARTH FRAME



ROCKET FRAME

EXERCISE L-13. ***Top:*** *Worldlines of Dick and Jane as observed in the Earth frame of Mom and Dad.* ***Bottom:*** *Worldlines of Dick and Jane as observed in the "final" rocket frame in which both Dick and Jane come to rest after burnout.*

Discussion: Einstein postulated that physics in a uniform gravitational field is, locally and for small particle speeds, the same as physics in an accelerated frame of reference. In this exercise we have found that two accelerated clocks separated along the direction of acceleration do not remain in synchronism as observed simultaneously in their common frame. Rather, the forward clock reads a later time ("runs faster") than the rearward clock as so observed. Conclusion from Einstein's postulate: Two clocks one above the other in a uniform gravitational field do not remain in synchronism; rather the higher clock reads a later time ("runs faster") than the lower clock. General relativity also predicts this result, and experiment verifies it. (Read about the patrol plane experiment in Section 4.10.)

Reference: S. P. Boughn, *American Journal of Physics,* Volume 57, pages 791–793 (September 1989). Reference to general relativity result: Wolfgang Rindler, *Essential Relativity* (Springer, New York, 1977), pages 17 and 117.

L-14 how do rods Lorentz-contract?

Note: Calculus is used in the solution to this exercise; so is the formula for Lorentz contraction from Section 5.8.

Laboratory observers measure the length of a moving rod lying along its direction of motion in the laboratory frame. Then the rod speeds up a little. Again laboratory observers measure its length, which they find to be a little shorter than before. They call this shortening of length Lorentz contraction. How did this shortening of length come about? As happens so often in relativity, the answer lies in the relativity of simultaneity.

First, how much shortening takes place when the rod changes from speed v to speed $v + dv$? Let L_o be the proper length of the rod when measured at rest. At speed v its laboratory-measured length L will be shorter than this by the Lorentz contraction factor (Section 5.8):

$$L = (1 - v^2)^{1/2} L_o$$

a Using calculus, show that when the rod speeds up from v to a slightly greater speed $v + dv$, the change in length dL is given by the expression

$$dL = -\frac{L_o v dv}{(1 - v^2)^{1/2}}$$

The negative sign means that the change is a shortening of the rod. We want to explain this change in length.

How is the rod to be accelerated from v to $v + dv$? Fire a rocket attached to the rear of the rod? No. Why not? Because the rocket pushes only against the rear of the rod; this push is transmitted along the rod to the front at the speed of a compression wave — very slow! We want the front and back to change speed "at the same time" (exact meaning of this phrase to be determined later). How can this be done? Only by prearrangement! Saw the rod into a thousand equal pieces and tap each piece in the forward direction with a mallet "at exactly 12 noon" as read off a set of synchronized clocks. To simplify things for now, set aside all but the front and back pieces of the rod. Now tap the front and back pieces "at the same time." The change in length of the rod dL is then the change in distance between these two pieces as a result of the tapping. So much for how to accelerate the "rod."

Now the central question: What does it mean to tap the front and back pieces of the rod "at the same time"? To answer this question, ask another: What is our final goal? Answer: To account for the Lorentz contraction of a fast-moving rod of proper length L_o. More: We want a careful inspector riding on the fast-moving rod to certify that it has the same proper length L_o as it did when it was at rest in the laboratory frame. To achieve this goal, the inspector insists that the pair of accelerating taps be applied to the front and back rod pieces at the same time *in the current rest frame of the rod*. Otherwise the distance between these pieces would not remain the same in the frame of the rod; the rod would change proper length. [Notice that in Exercise L-13 the taps occur at the same time in the laboratory (Earth) frame. This leads to results different from those of the present exercise.]

b You are the inspector riding along with the front and back pieces of the rod. Consider the two events of tapping the front and back pieces. How far apart $\Delta x'$ are these events along the x-axis in your (rocket) frame? How far apart $\Delta t'$ in time are these events in your frame? Predict how far apart in time Δt these events are as measured in the laboratory frame. Use the Lorentz transformation equation (L-10):

$$\Delta t = v\,\gamma \Delta x' + \gamma \Delta t'$$

The relative velocity v_{rel} in equation (L-10) is just v, the current speed of the rod. In the laboratory frame is the tap on the rear piece earlier or later than the tap on the front piece?

Your answer to part **b** predicts how much earlier the laboratory observer measures the tap to occur on the back piece than on the front piece of the rod. Let the tap increase the speed of the back end by dv as measured in the laboratory frame. Then during laboratory time Δt the back end is moving at a speed dv faster than the front end. This relative motion will shorten the distance between the back and front ends. After time interval Δt the front end receives the identical tap, also speeds up by dv, and once again moves at the same speed as the back end.

c Show that the shortening dL predicted by this analysis is

$$dL = -dv\Delta t = -\gamma \Delta x' v dv = -v\gamma L_o dv$$
$$= -\frac{L_o v dv}{(1 - v^2)^{1/2}}$$

which is identical to the result of part **a**, which we wanted to explain. QED.

d Now start with the front and back pieces of the rod at rest in the laboratory frame and a distance L_o apart. Tap them repeatedly and identically. As they speed up, be sure these taps take place simultaneously in the rocket frame in which the two ends are currently at rest. (This requires you, the ride-along inspector, to resynchronize your rod-rest-frame clocks after each set of front-and-back taps.) Make a logically rigorous argument that after many taps, when the rod is moving at high speed relative to the laboratory, the length of the rod measured in the laboratory can be reckoned using the first equation given in this exercise.

e Now, by stages, put the rod back together. The full thousand pieces of the rod, lined up but not touching, are all tapped identically and at the same time in the current rest frame of the rod. One set of taps increases the rod's speed from v to $v + dv$ in the laboratory frame. Describe the time sequence of these thousand taps as observed in the laboratory frame. If you have studied Chapter 6 or the equivalent, answer the following questions: What kind of interval—timelike, lightlike, or spacelike—separates any pair of the thousand taps in this set? Can this pair of taps be connected by a light flash? by a compression wave moving along the rod when the pieces are glued back together? Regarding the "logic of acceleration," is there any reason why we should *not* glue these pieces back together? Done!

f During the acceleration process is the reglued rod *rigid*—unchanging in dimensions—as observed in the rod frame? As observed in the laboratory frame? Is the *rigidity* property of an object an invariant, the same for all observers in uniform relative motion? Show how an ideal rigid rod could be used to transmit signals instantaneously from one place to another. What do you conclude about the idea of a "rigid body" when applied to high-speed phenomena?

Reference: Edwin F. Taylor and A. P. French, *American Journal of Physics,* Volume 51, pages 889–893, especially the Appendix (1983).

L-15 the place where both agree

At any instant there is just one plane in which both the laboratory and the rocket clocks agree.

a By a symmetry argument, show that this plane lies perpendicular to the direction of relative motion. Using the Lorentz transformation equations, show that the velocity of this plane in the laboratory frame is equal to

$$v_{t=t'} = \frac{1}{v_{rel}}[1 - (1 - v_{rel}^2)^{1/2}]$$

b Does the expression for $v_{t=t'}$ seem strange? From our everyday experience we might expect that by symmetry the "plane of equal time" would move in the laboratory at half the speed of the rocket. Verify that indeed this is correct for the low relative velocities of our everyday experience. Use the first two terms of the binomial expansion

$$(1 + z)^n \approx 1 + nz \text{ for } |z| << 1$$

to show that for low relative velocity, $v_{t=t'} \rightarrow v_{rel}/2$.

c What is $v_{t=t'}$ for the extreme relativistic case in which $v_{rel} \rightarrow 1$? Show that in this case $v_{t=t'}$ is completely different from $v_{rel}/2$.

d Suppose we want to go from the laboratory frame to the rocket frame in two equal velocity jumps. Try a first jump to the plane of equal laboratory and rocket times. Now symmetry does work: Viewed from this plane the laboratory and rocket frames move apart with equal and opposite velocities, whose magnitude is given by the equation in part **a**. A second and equal velocity jump should then carry us to the rocket frame at speed v_{rel} with respect to the laboratory. Verify this directly by using the Law of Addition of Velocities (Section L.7) to show that

$$v_{rel} = \frac{v_{t=t'} + v_{t=t'}}{1 + v_{t=t'}v_{t=t'}}$$

L-16 Fizeau experiment

Light moves more slowly through a transparent material medium than through a vacuum. Let v_{medium} represent the reduced speed of light measured in the frame of the medium. Idealize to a case in which this reduced velocity is independent of the wavelength of the light. Place the medium at rest in a rocket moving at velocity v_{rel}, to the right relative to the laboratory frame, and let light travel through the medium, also to the right. Use the Law of Addition of Velocities (Section L.7) to find an expression for the velocity v of the light in the laboratory frame. Use the first two terms of the binomial expansion

$$(1 + z)^n \approx 1 + nz \text{ for } |z| << 1$$

to show that for small relative velocity v_{rel} between the rocket and laboratory frames, the velocity v of the light with respect to the laboratory frame is given approximately by the expression

$$v \approx v_{medium} + v_{rel}(1 - v_{medium}{}^2)$$

This expression has been tested by Fizeau using water flowing in opposite directions in the two arms of an interferometer similar (but not identical) to the interferometer used later by Michelson and Morley (Exercise 3-12).

Reference: H. Fizeau, *Comptes rendus,* Volume 33, pages 349–355 (1851). A fascinating discussion (in French) of some central themes in relativity theory—delivered more than fifty years before Einstein's first relativity paper.