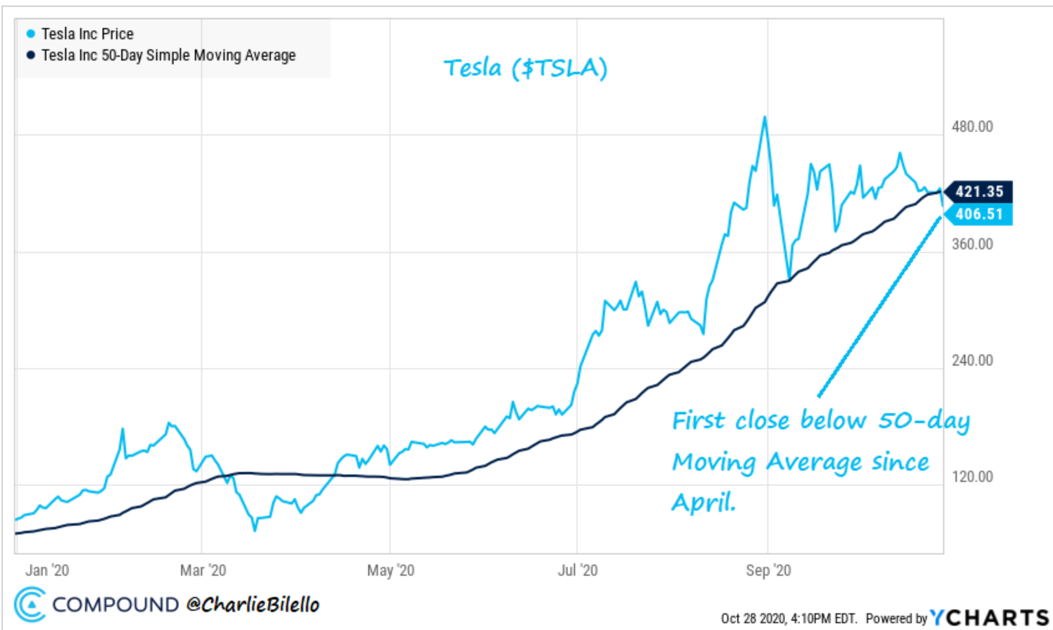
ODE Assignment 12

Problem 1 — Trailing Averages

When discussing the previous assignment, we came close to getting a nice expression for the Laplace transform of a trailing average, using the convolution property. First, just so you have an idea of how this kind of thing is used, consider this example from the stock market:



These kinds of things are produced by “chart watchers” who do “technical analyses” — rather than actually studying the fundamentals of a company — to try to guess whether it is time to buy or sell a stock. It is common in all sorts of fields, not just finance, to do a trailing moving average to smooth out noisy data. The period over which the average is computed is called the “window.” There are three times in a moving average:

(1) now, which we will call t
(2) the beginning of the window, which is a earlier than t, so it is $t - a$.
(3) and the end of the window, which is b earlier than t, so it is $t - b$.

We will assume $b < a$ (because otherwise what we have called the beginning of the window is $t - b$ and the end of the window is $t - a$). It is of course very common to choose $b = 0$, so that the end of the window is now, but to maintain a little bit of extra generality, let’s not do that yet. *Let us also assume that $a \leq t$ (because otherwise the beginning of the window could be at a negative time and we would have to deal with this as an annoying special case in our Laplace transforms).*

(a) Use two Heaviside functions to construct a function that is 1 only if $b \le t \le a$, and is zero everywhere else. You could do this with the product of two Heaviside functions, but don't. Use the difference of two Heaviside functions instead. I'd recommend graphing the two Heaviside functions and their difference (three graphs in total) to make sure you haven't got minus sign errors.

(b) Call the difference of two Heaviside functions you found in part (a), $g(t)$. What is the Laplace transform, $G(s)$ of the function $g(t)$? Use the definition of the Laplace transform to calculate this. You could of course use the table on p. 175, but it is nicer to do things from scratch. Double-check your work by computing the inverse Laplace transform of $G(s)$ (now go ahead and use the table) and make sure that you recover $g(t)$.

(c) Here is an expression of the average of a function over the time window $t - a$ to $t - b$:

$$\frac{1}{a-b} \int_{t-a}^{t-b} f(\tau)\, d\tau$$

The window goes from $t - a$ to $t - b$ and the function you constructed in (a) is nonzero from b to a. You have to be careful using that function, but once you do, you should be able to write that integral as:

$$\frac{1}{a-b} \int_0^t f(\tau) \text{ [times something involving the function } g\text{]}\, d\tau$$

(d) Using the convolution property of the Laplace transform and calling the Laplace transform of $f(t)$, $F(s)$, what is the Laplace transform of what you found in Part (c)? Now plug in $b = 0$ to simplify some. The bottom line is that the Laplace transform of a moving average of a function f is pretty simply related to the Laplace transform of f.

Problem 2 — Logan p. 173 #4

No comment.

Problem 3 — Logan p. 174 #6

Change the RHS from what Logan had, $\delta_2(t) - \delta_5(t)$, to the following (which will result in an answer that you can nicely simplify using properties of the sine function): $\delta(t - 2\pi) - \delta(t - 5\pi)$. This change also makes the problem more of a warm-up for the next one. Sketch your solution.

Problem 4 — Logan p. 174 #8

Simplify your solution using properties of the sine function. Sketch your solution.