Heavenly Mathematics Term 4 Exam

Thursday, Feb. 17, 2022

For Problems 2, 3, and 4, we will take the best two of those three. So if one of 2, 3, or 4, throws you for a loop, focus on the other two. Everyone should work Problems 1 and 5.

Problem 1 (4 pts)

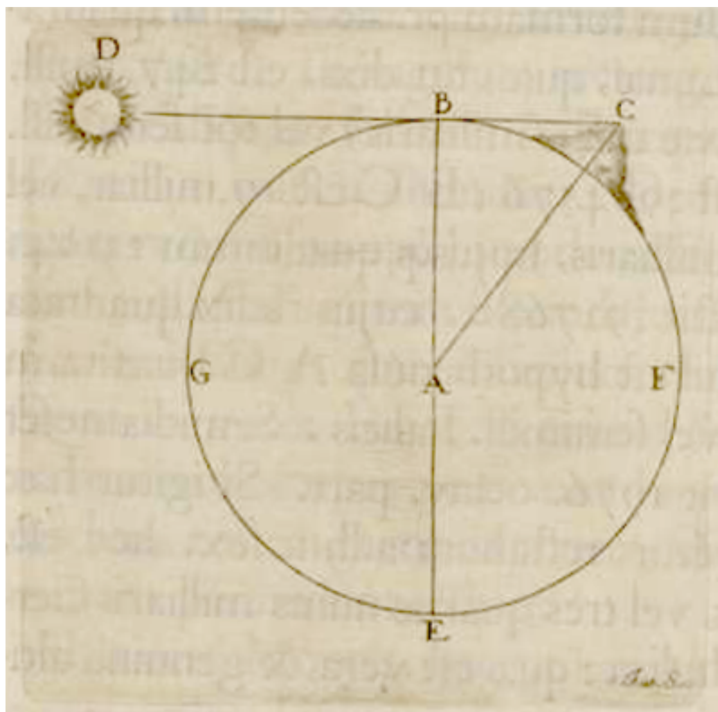
(a) Omega Centauri has declination about -47.5° degrees. From our latitude at Deep Springs, which is about 37.4°, is Omega Centauri ever above the horizon?
(b) If your answer to (a) was "no," how much below the horizon is it when it is closest to being visible? Or, if your answer to (a) was "yes," how much above the horizon (the altitude) is it when it is most visible?
(c) The right ascension of Omega Centauri is about 13h 30m. Convert that to degrees.

Problem 2 (5 pts)

A stupendous use of the Pythagorean theorem is Galileo's calculation of the height of the largest mountains on the Moon.

In the figure, the Sun is actually extremely far off to the left, but it is drawn close by for practicality. Suppose AB (the radius of the Moon) is *R* which was already known (since Aristarchus).

The hemisphere of the Moon BGE is lit up. Galileo observed the distance BC when the tip of Peak C was just getting lit up, so that $\angle$ABC is 90°. The rest of the hemisphere BFE is dark. Let *d* stand for the distance BC. Since Galileo could observe *d* with his telescope, it is also known.



Call the peak's height *h*. Using just the Pythagorean Theorem, derive an equation that involves only the lengths *R*, *d*, and *h*.

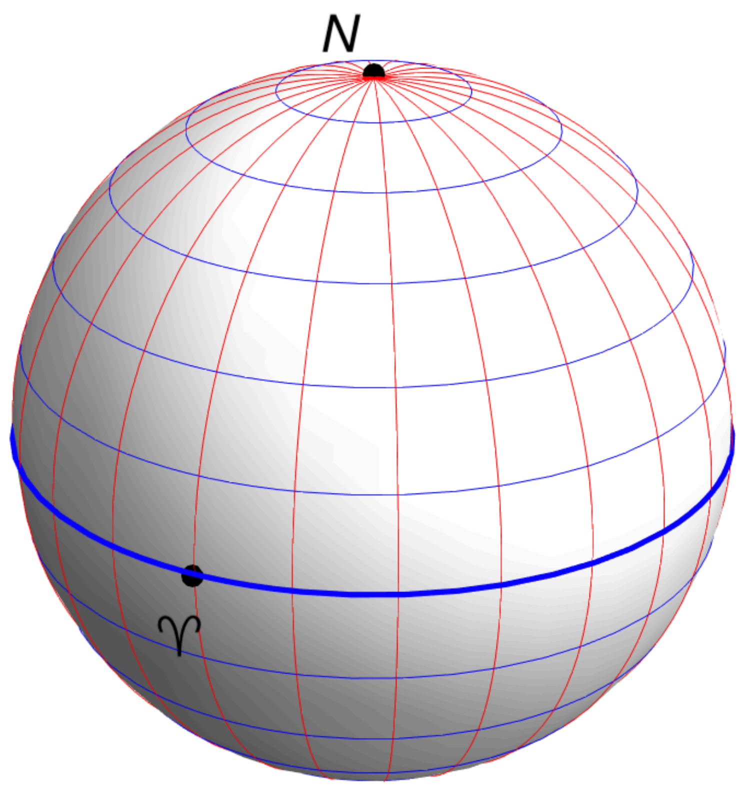
~~Use the quadratic formula to~~ solve for *h*. There will be two answers. Discard the bad one.

Problem 3 (5 pts)

Using sin(α+β) = sin α cos β + cos α sin β and also cos(α+β) = cos α cos β - sin α sin β, derive a formula for sin 4θ that only involves sin θ and cos θ.

Problem 4 (5 pts)

The figure at right shows the sphere with lines of latitude and longitude at 15° intervals. The “First Point of Aries” (the spring equinox point) and the North Pole are marked.



(a) Sketch the ecliptic (you will need the number ~~27.4º~~ 23.4° for its tilt). Utilize the lines so as to be fairly accurate. Mark the Summer Solstice as SS.
(b) Place the Sun 60° along the ecliptic from the First Point of Aries, labeling that distance in the figure.
(c) Given the location of the Sun in (b), label an arc α corresponding to its right ascension, and an arc δ corresponding to its declination.
(d) Deep Springs is located at 37.4° latitude. Sketch an approximate location for Deep Springs at sunrise given the Sun in the position of part (b). Label how 37.4° is used in the figure.
(e) Add the horizon plane for Deep Springs to your figure. Mark the directions N and E on your horizon plane that correspond to the directions north and east at Deep Springs.

If you mess your drawing up, ask for another copy of the figure. We brought spares.

Problem 5 (6 pts)

Starting with (1) the fact that the sum of the sides of any spherical triangle is less than 360°, and (2) the Polar Duality Theorem, show that the sum of the angles in a ~~polar~~ triangle must be greater than 180°. It might help to have Van Brummelen’s statement of the Polar Duality Theorem handy:

> **Polar Duality Theorem:** The sides of a polar triangle are the supplements of the angles of the original triangle, and the angles of a polar triangle are the supplements of the sides of the original.

Jargon reminder: The “complement” of θ is $90° - \theta$. The “supplement” of θ is $180° - \theta$.