Term 4 Exam Solution

Problem 1

This is an application of Problem 16 from Problem Set 3

(a) Yes, it sometimes is above the horizon. Any star whose declination is more than $37.4° - 90° = -52.6°$ is sometimes above the horizon at Deep Springs.

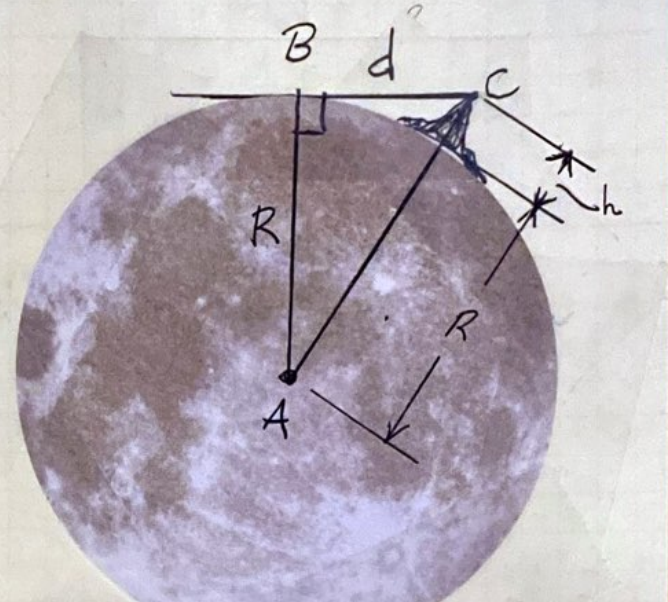
(b) $-47.5° - (-52.6°) = 5.1°$

Its maximum altitude is $5.1°$.

(c) $13h\ 30m = 13 \cdot 15° + \frac{30}{60} \cdot 15°$

$= 195° + 7.5° = 202.5°$

Problem 2



$\overline{AB} = R \qquad \overline{BC} = d$

$\overline{AC} = R + h$

$(R+h)^2 = R^2 + d^2$

$R + h = \pm\sqrt{R^2 + d^2}$

$h = -R \pm \sqrt{R^2 + d^2}$ ← only the + solution is positive

$h = -R + \sqrt{R^2 + d^2}$

Problem 3

apply $\sin(\alpha+\beta)$ identity

$$\sin 4\theta = \sin(2\theta+2\theta) = \sin 2\theta \cos 2\theta + \cos 2\theta \sin 2\theta$$

$$= 2\sin 2\theta \cos 2\theta$$

$\sin(\alpha+\beta)$ identity again

But $\sin 2\theta = 2\sin\theta\cos\theta$

and $\cos 2\theta = \cos^2\theta - \sin^2\theta$ ← $\cos(\alpha+\beta)$ identity

Therefore

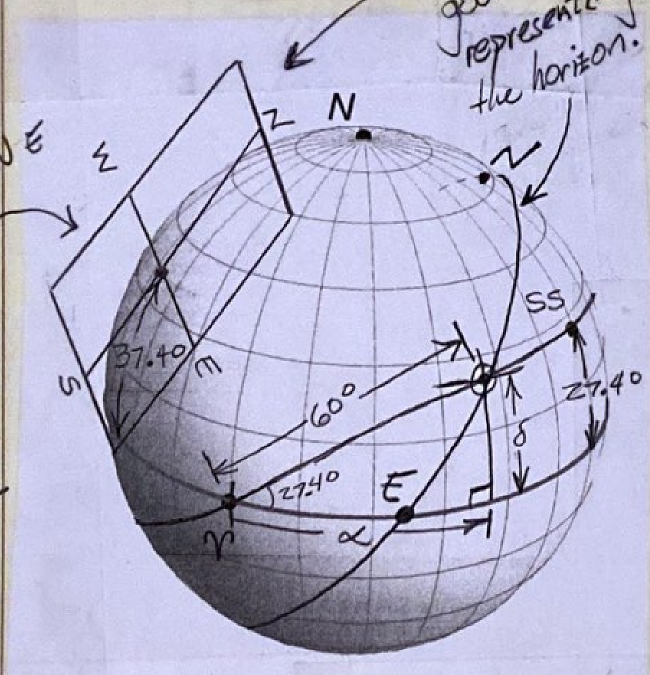
$$\sin 4\theta = 4\sin\theta\cos\theta\cdot(\cos^2\theta - \sin^2\theta)$$

$$= 4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta$$

Problem 4

There are two good ways of representing the horizon.

THIS ONE IS MORE VISUAL BUT LESS USEFUL



Two places with 27.4° should be 23.4°!

"RHS" is short for "right-hand side"

Problem 5

If a triangle's angles are α, β, and γ its polar triangle has sides $180-\alpha$, $180-\beta$, and $180-\gamma$. We know that

$$180^\circ-\alpha+180^\circ-\beta+180^\circ-\gamma < 360^\circ$$

Cancel $180^\circ+180^\circ$ with 360° leaving

$$-\alpha-\beta+180^\circ-\gamma < 0$$

Move α, β and γ to the RHS

$$180^\circ < \alpha+\beta+\gamma$$

:)