Heavenly Mathematics Term 5 Exam

Thursday, Apr. 21, 2022

On the first page of this exam are 2 problems. Choose 1 of them to do.

On the second page are 3 problems. Choose 2 of them to do.

On the last page are 2 proofs. Choose 1 of them to do.

In other words, on each page, you get to skip one problem.

Problem 1 (4 pts) Right Triangles on the Sphere

A spherical right triangle (angle C = 90º) has side lengths a = 43º and c = 53º. What are A, B and b? Show clearly how you obtain your results.

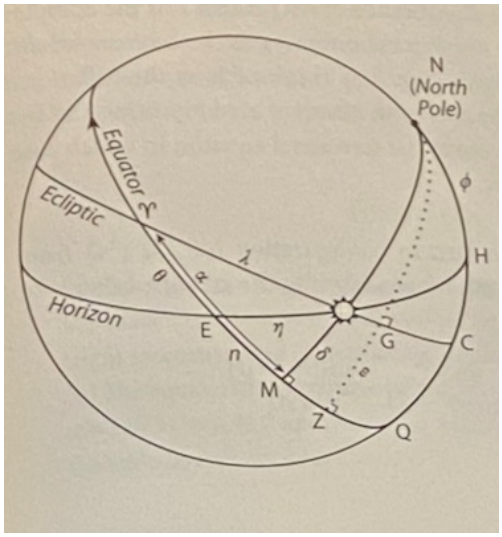
Problem 2 (4 pts) Spherical Version of the Pythagorean Theorem

(a) The Pythagorean Theorem in the plane says that the sides of a right triangle, a and b, with hypotenuse c have a relationship. Write the relationship down. Solve it for c.
(b) On the sphere, one of the 10 identities for right triangles involves a, b, and c. Write that identity down. Solve it for c.
(c) Does a = 90 ft, b = 90 ft, and c = 90 ft satisfy the identity you wrote down in (a)?
(d) Does a = 90º, b = 90º, and c = 90º satisfy the identity you wrote down in (b)?

Problem 3 (7 pts) Sunrise in Sitka

For today, April 21st, Ptolemy's tables say that the ecliptic longitude of the Sun is 30.7°. From this, and the inclination of the Ecliptic which is ε = 23.4°, one can get that the declination of the Sun is δ = 11.7°.

The latitude of Sitka, Alaska is ϕ = 57.1°. How far north of east does the Sun rise on April 21st in Sitka, Alaska.

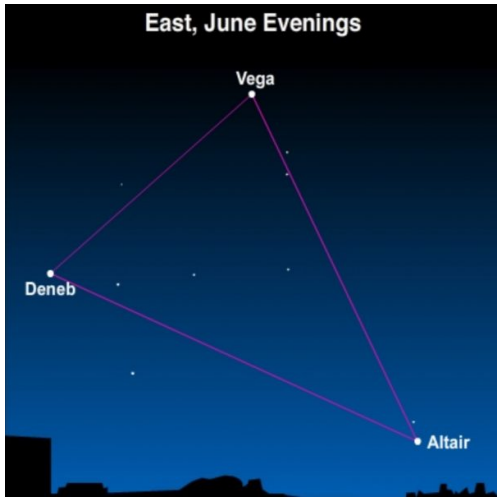


Problem 4 (7 pts) Flight from Las Vegas to Chicago

Las Vegas airport (LAS) has latitude 36°. Chicago O'Hare airport (ORD) has latitude 42°. When a plane takes off on a great circle route from Las Vegas, headed to O'Hare, its initial compass heading is 66° (that is 66° degrees east of north). What is the plane's compass heading upon arrival at O'Hare?

Problem 5 (7 pts) The Summer Triangle

The three stars known as the Summer Triangle are shown at right. The angular distance between Vega and Altair is 34.2°. The angular distance between Deneb and Altair is 38°. The angle at Altair is 40.5°. What is the angular distance between Deneb and Vega?



Problem 6 (7 pts) Theorem Doubler: Law of Cosines

Given the spherical Law of Cosines for triangle ABC:

$$\cos c = \cos a \cdot \cos b + \sin a \cdot \sin b \cdot \cos C$$

State and prove a corresponding law for the polar triangle to ABC.

Problem 7 (7 pts) Theorem Doubler: Quadrantal Triangles

A "quadrantal triangle" has one of its sides (not one of its angles) equal to 90°. Let's call the angles of such a triangle D, E, and F, and call the corresponding sides d, e, and f, with f = 90°.

(a) Imagine constructing the polar triangle corresponding to this quadrantal triangle. What would be its sides?
(b) What would be its angles? Be sure to use the fact that f = 90° to simplify.
(c) Take any one of the 10 identities for right triangles — your choice which one! — and apply it to the polar triangle to DEF. You now have an identity that applies to a quadrantal triangle!

HINT/HELP FOR BOTH PROBLEMS 6 AND 7:

If your formula sheet doesn't include the Polar Duality Theorem, here it is:

> **Polar Duality Theorem:** The sides of a polar triangle are the supplements of the angles of the original triangle, and the angles of a polar triangle are the supplements of the sides of the original.

Also, the "supplement of an angle θ" is just jargon for 180° - θ. Finally, for either problem, you will need to simplify using the formulas for sin(180° - θ) and cos(180° - θ).