Heavenly Mathematics
Team 5 Exam Solution

$a = 43^\circ$
$c = 53^\circ$

Problem 1

By II.1 $\quad A = \sin^{-1}\dfrac{\sin a}{\sin c} = \sin^{-1}\dfrac{\sin 43^\circ}{\sin 53^\circ} = 58.6^\circ$

I.5 $\quad B = \cos^{-1}\dfrac{\tan a}{\tan c} = \cos^{-1}\dfrac{\tan 43^\circ}{\tan 53^\circ} = 45.3^\circ$

II.5 $\quad b = \cos^{-1}\dfrac{\cos c}{\cos a} = \cos^{-1}\dfrac{\cos 53^\circ}{\cos 43^\circ} = 34.6^\circ$

Problem 2

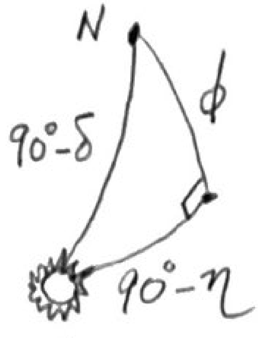
(a) $c^2 = a^2 + b^2 \qquad c = \sqrt{a^2+b^2}$

(b) $\cos c = \cos a \cdot \cos b \qquad c = \cos^{-1}(\cos a \cdot \cos b)$

(c) No, $8100\,ft^2 \neq 8100\,ft^2 + 8100\,ft^2$

(d) Yes, $\cos 90^\circ = 0 = \cos 90^\circ \cdot \cos 90^\circ$

Problem 3



$\delta = 11.7°$

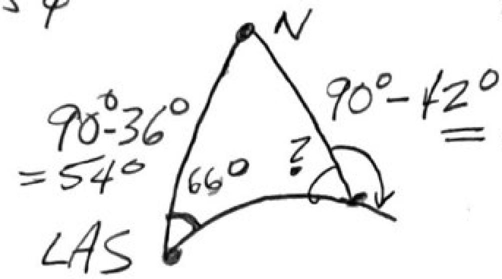
$\phi = 57.1°$

If you don't have the ortive amplitude formula written down then this part of the figure tells you

$$\underbrace{\cos(90°-\delta)}_{\sin\delta} = \cos\phi \underbrace{\cos(90°-\eta)}_{\sin\eta}$$

Or $\eta = \sin^{-1}\dfrac{\sin\delta}{\cos\phi} = 21.9°$ ← that is our answer

Problem 4



By the Law of Sines

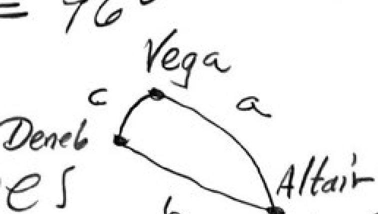
$$\frac{\sin ?}{\sin 54°} = \frac{\sin 66°}{\sin 48°} \quad \text{or} \quad ? = \sin^{-1}\frac{\sin 54° \cdot \sin 66°}{\sin 48°} = 84°$$

But 84° is not the compass heading upon arrival.

It is $180° - 84° = 96°$

Problem 5

By the Law of Cosines



$a = 34.2°$

$b = 38°$

$C = 40.5°$

Deneb ↙ Vega

$$c = \cos^{-1}(\cos a \cos b + \sin a \sin b \cos C)$$

$$= \cos^{-1}(\cos 34.2° \cos 38° + \sin 34° \sin 38° \cos 40.5°) = 24.0°$$

Problem 6 This was Problem 1 on Problem Set 9.

Problem 7

(a) Its sides would be

$a = 180^\circ - D$, $b = 180^\circ - E$, and $c = 180^\circ - F$

(b) Its angles would be

$A = 180 - d$, $B = 180 - e$, and $180^\circ - f = 90^\circ$

(c) Take identity I.1

$$\sin b = \tan a \cot A$$

$$\underbrace{\sin(180^\circ - E)}_{\sin E} = \underbrace{\tan(180^\circ - D)}_{\frac{\sin(180^\circ - D)}{\cos(180^\circ - D)}} \underbrace{\cot(180^\circ - d)}_{\frac{\cos(180^\circ - d)}{\sin(180^\circ - d)}}$$

$$= \frac{\sin D}{-\cos D} = -\tan D \qquad = \frac{-\cos d}{\sin d} = -\cot d$$

$\therefore \sin E = \tan D \cot d$