Correcting Course at Sea

A Heavenly Mathematics Project

After building a boat in visiting professor Douglas Brooks' class in Term 6, you and several Deep Springers decide to sail the boat out along the California coast. After a pleasant few days at sea, you decide to bring the boat back to a dock in the San Francisco Bay. However, still a ways from the entrance to the Bay, the battery on your GPS equipment goes out, leaving you unsure of your precise latitude and longitude. You have a good guess about your coordinates based on the last reading taken, but are unsure how close you are to that location now given winds and currents. Fortunately, on the boat you have your Heavenly Mathematics book, a Lénárt sphere, a sextant, and a good calculator. As it nears dusk, you tell your shipmates that you are going to determine the correct compass heading to guide you safely into the San Francisco Bay simply by reading the stars. They are amazed. "Just wait!" you tell them. Now it's time to get to work...

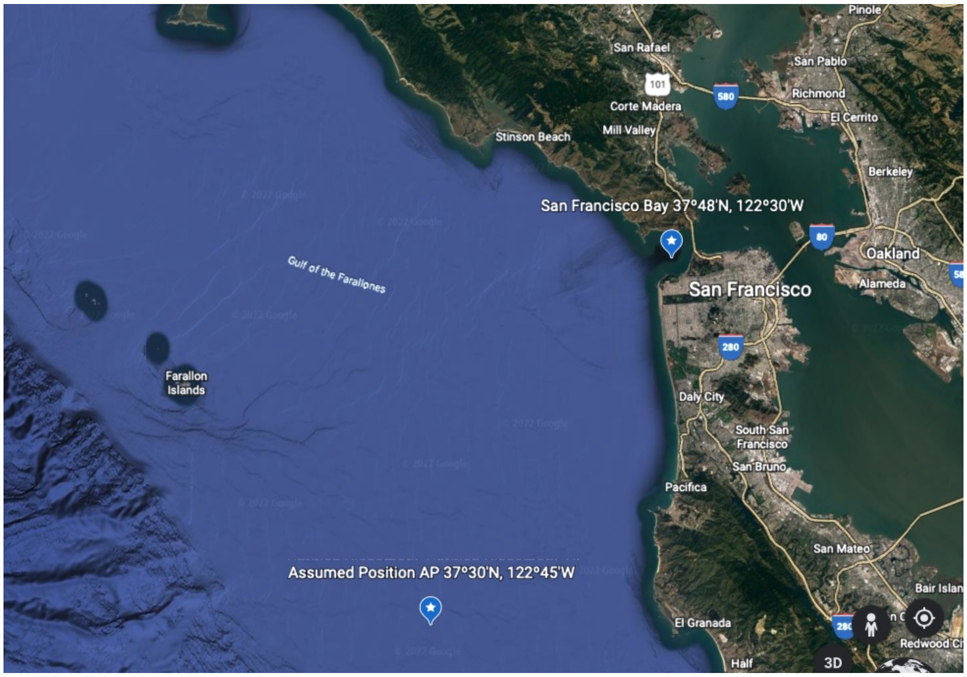
Project: Spherical Trigonometry and Celestial Navigation

For the final course activity, you are going to use spherical trigonometry for celestial navigation — using the stars to determine directions. The method of celestial navigation we are using for this project is called the *Method of Saint Hilaire.* It requires starting with a reasonable estimate of your current location (latitude and longitude), good to within 50 nautical miles or so. It then requires you to measure the altitude of two stars (or planets), and then, using the exact time of your measurements, compare those altitudes to their expected values at your estimated location using known information from a nautical almanac. Upon determining the expected azimuths of the the two stars, you can then use the discrepancies between your measurements and expected values to plot your actual position with respect to your initial guess. In the end, this allows you to correct your initial guess and find a much better approximation of your true location.

In the following, you will be carrying out the key steps of the Method of Saint Hilaire to safely guide your boat to your desired destination. Keep track of your steps, as you will need to write up your work as a report that includes a recommendation for how to direct the ship to ensure it reaches San Francisco Bay. Note that the method of Saint Hilaire is worked out for a different example using Venus and Spica rather than Mercury and Sirius in the text on pp. 154-67. Please reference the text as needed, as you will be recreating the method here.

Step 1 – Noting key information. Your last GPS reading suggests that your current location is latitude $\varphi = 37^o30'$ N and longitude $\lambda = 122^o45'$ W. Let's call this your "AP" for "assumed position." Now you use your sextant to determine the altitudes of two bright celestial objects. Let's assume you are making your measurements at exactly 8:00PM PDT on April 25, 2022 (this is 3:00AM on April 26, 2022 in Greenwich Mean Time.) You see two bright objects: Mercury, close to due west, and Sirius somewhat west of south. By your calculation, Mercury is at altitude $h_0 = 18^o20'$, and Sirius is at altitude $h_0 = 26^o25'$. (Good thing they are this high — objects below 15^o altitude are subject to visual distortion from the earth's atmosphere.)

Your job is to figure how far off of this assumed position you actually are — your "true position" — by comparing your observed altitudes of Mercury and Sirius with what you get from calculating their expected positions with spherical trigonometry.



Step 2 – Set up the astronomical triangle. See page 156 of Heavenly Mathematics to see what we call the astronomical triangle. You will want to use the astronomical triangle to compute the actual values of the altitudes h of Mercury and Sirius given known values of their positions at the given date and time. Those values can be determined from a nautical almanac. You will need to consult the almanac for the following values for Mercury and Sirius: declinations, and the local hour angles t (actually, the almanac only has values for the *Greenwich hour angle* for Mercury and the point of Aries, and the displacement of Sirius from the point of Aries, called its *sidereal hour angle*; you will have to use these values to compute the local hour angle for your longitude, using *hour angle diagrams*. See pp. 156-8 of the text.

For the purpose of this project, Ryan is your audio nautical almanac, and can provide you any information you need verbally.

Step 3 – Calculate the actual altitudes h of Mercury and Sirius.

Once you know δ, φ and the local hour angle t, can you find h?

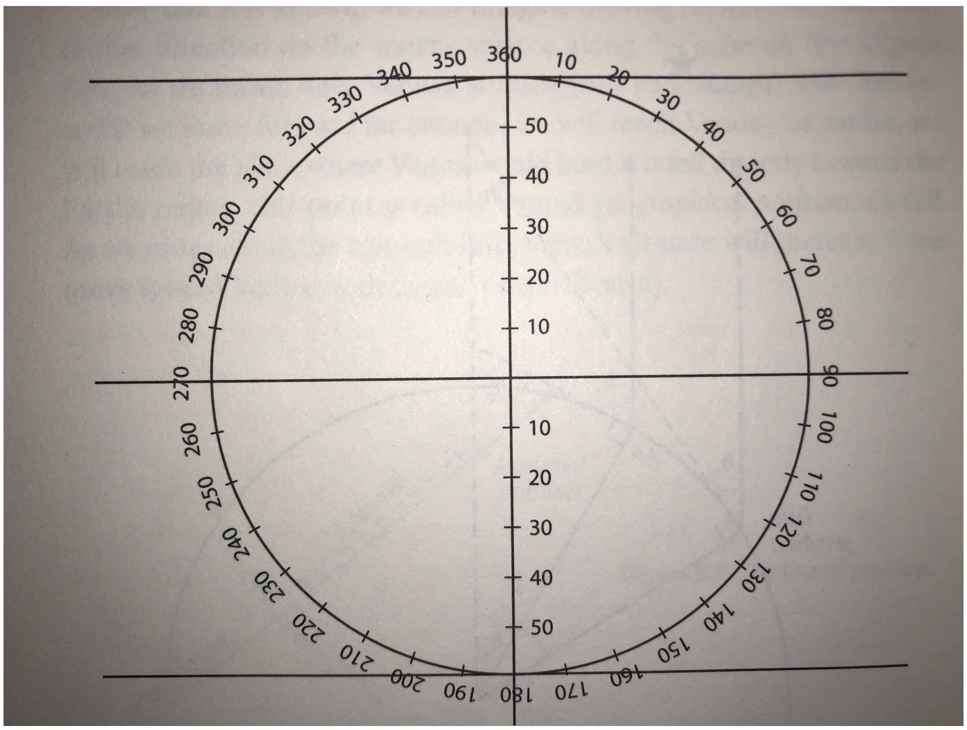
(**Extra Challenge!** A historic way to do this calculation lacking modern computing power is via a different trig function: the *haversine.* The haversine, which is just one minus the cosine, is not difficult to use, and simplifies the calculations somewhat by rewriting the equations you would otherwise use to solve this. If you are up for learning about and using the haversine, see pp. 157-62 of the text. You can also come back and recompute your values with the haversine if you finish and are still up for more math.)

Step 4 – Calculate the azimuths Z of Mercury and Sirius.

After finding h for each in Step 3, you have enough information to find the azimuth angles given the astronomical triangle. Important — once you find your angles, take care to note which directions your angles reference, e.g. west of south. You can check this by plotting Mercury and Sirius on your Lénárt sphere. Are they where they should be?

Step 5 – Find the *intercepts*. Consider h_0 to be the value of the altitude you observed for one of your celestial objects. Let h_c be correct value of your AP as per your calculation in Step 3. Calculate the difference between these two values. Now you need to figure out how far away from the AP you actually are. Note that if you travel along the *azimuth line* of Mercury, for example, as in directly toward it or directly away from it, Mercury's altitude will change relative to your position. (Indeed, what happens to the altitude of Mercury as you move towards it? Away?) Now, use the argument in Figure 9.10 and on pp. 163-4 to determine the value of your intercepts for both Mercury and Sirius. Hint: convert the difference between the observed altitude and calculated altitude to minutes.

Step 6 – Use a nautical plotting chart to find your true location. The final step involves setting up a plotting chart



Put your AP at the center. Then, considering the increments by 60 in either vertical direction as minutes, the top line and bottom line in the picture should be $\pm 1^o$ latitude from the AP, respectively. For longitude, we know that, unfortunately, lines of longitude get closer together the further one gets away from the equator. There is, however, a quick way to draw lines of longitude on the plotting chart: mark two places on the circle $37^o 30'$ up and down from the rightmost point of the circle, and draw a vertical line. Do the same on the left. These give lines of longitude $\mp 1^o$, respectively, from the AP's longitude.

Now use the azimuth calculations from Step 4 to plot an *azimuth line* for Mercury. This runs through the AP and uses the azimuth angle you calculated. Nest, using the value of the intercept from Step 5, plot a perpendicular line to the azimuth line called the *line of position* (LP) that meets the azimuth line at the intercept. (Be careful! Which direction from the AP do you want to put your intercept?) Note that along the LP, the altitude of

Mercury aligns with your observations. Thus you know that your ship is somewhere along the LP! But where?

Do the same in finding the LP for Sirius via the intercept along its azimuth line. Where the two LPs intersect is your corrected true position. You will need to estimate the values of the latitude and longitude from your plotting chart.

Step 7 – Write up your work along with a recommended direction for sailing! The San Francisco Bay is at latitude $37^o48'$ N, and $122^o30'$ W. Knowing this and your true position, you can calculate the appropriate compass heading to navigate safely to the Bay. Write up your derivations and recommendation clearly. Some questions to consider: How sure are you of your calculations? How might your observations and calculations been improved? Were Mercury and Sirius good choices for celestial objects to use? Why or why not?

You are welcome to consult your audio almanacs to check how close your true position calculations are to the original observed altitude measurements.