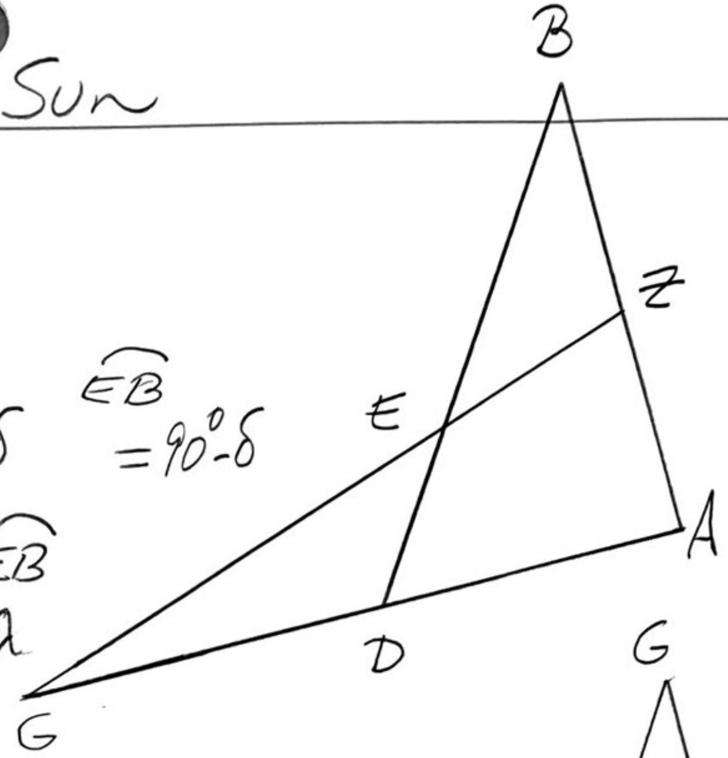
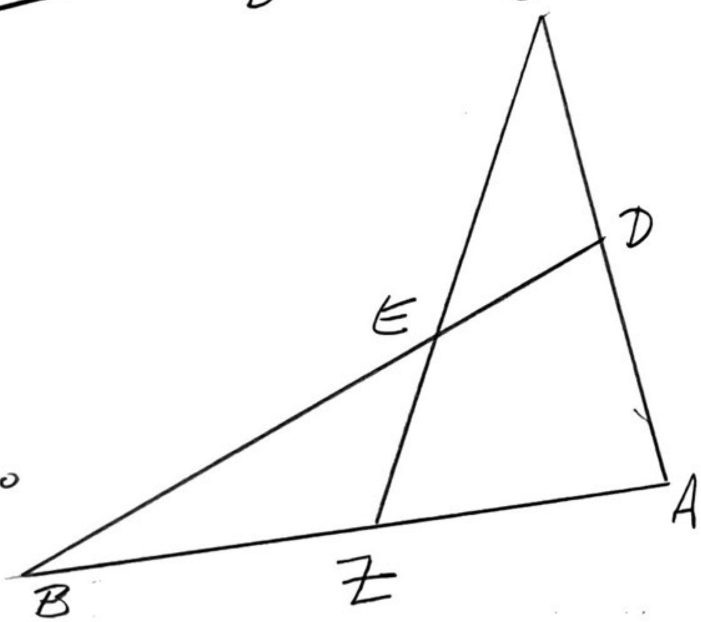
Position of the Sun



MTA
$$\frac{\sin \widehat{AZ}}{\sin \widehat{BZ}} = \frac{\sin \widehat{AG}}{\sin \widehat{GD}} \cdot \frac{\sin \widehat{DE}}{\sin \widehat{EB}}$$

$\widehat{AZ} = \epsilon$ $\quad \widehat{BZ} = 90^\circ - \epsilon$ $\quad \widehat{AG} = 90^\circ$ $\quad \widehat{GD} = \alpha$ $\quad \widehat{DE} = \delta$ $\quad \widehat{EB} = 90^\circ - \delta$

$\widehat{AZ} = 90^\circ - \alpha$ $\quad \widehat{BZ} = \alpha$ $\quad \widehat{AG} = 90^\circ$ $\quad \widehat{GD} = 90^\circ - \epsilon$ $\quad \widehat{DE} = 90^\circ - \lambda$ $\quad \widehat{EB} = \lambda$



MTB
$$\frac{\sin \widehat{AB}}{\sin \widehat{AZ}} = \frac{\sin \widehat{BD}}{\sin \widehat{DE}} \ \frac{\sin \widehat{GE}}{\sin \widehat{GZ}}$$

$\widehat{AB} = 90^\circ$ $\quad \widehat{AZ} = \epsilon$ $\quad \widehat{BD} = 90$ $\quad \widehat{DE} = \delta$ $\quad \widehat{GE} = \lambda$ $\quad \widehat{GZ} = 90^\circ$

$\widehat{AB} = 90^\circ$ $\quad \widehat{AZ} = 90^\circ - \alpha$ $\quad \widehat{BD} = 90^\circ$ $\quad \widehat{DE} = 90^\circ - \lambda$ $\quad \widehat{GE} = 90^\circ - \delta$ $\quad \widehat{GZ} = 90^\circ$

We have two theorems and two figures, we get four results:

MTA $$\frac{\sin\epsilon}{\sin(90^\circ-\epsilon)} = \frac{\sin 90^\circ}{\sin\alpha}\cdot\frac{\sin\delta}{\sin(90^\circ-\delta)} \quad\text{or}\quad \tan\epsilon = \frac{1}{\sin\alpha}\tan\delta$$

MTA $$\frac{\sin(90^\circ-\alpha)}{\sin\alpha} = \frac{\sin 90^\circ}{\sin(90^\circ-\epsilon)}\cdot\frac{\sin(90^\circ-\lambda)}{\sin\lambda} \quad\text{or}\quad \boxed{\cot\alpha = \frac{1}{\cos\epsilon}\cot\lambda}$$

MTB $$\frac{\sin 90^\circ}{\sin\epsilon} = \frac{\sin 90^\circ}{\sin\delta}\cdot\frac{\sin\lambda}{\sin 90^\circ} \quad\text{or}\quad \boxed{\frac{1}{\sin\epsilon} = \frac{1}{\sin\delta}\sin\lambda}$$

MTB $$\frac{\sin 90^\circ}{\sin(90^\circ-\alpha)} = \frac{\sin 90^\circ}{\sin(90^\circ-\lambda)}\cdot\frac{\sin(90^\circ-\delta)}{\sin 90^\circ} \quad\text{or}\quad \frac{1}{\cos\alpha} = \frac{1}{\cos\lambda}\cos\delta$$

The two boxed formulae are the most useful because α and δ are usually both unknowns. They can be rewritten as:

$$\boxed{\tan\alpha = \cos\epsilon\tan\lambda} \qquad \boxed{\sin\delta = \sin\epsilon\,\sin\lambda}$$

In agreement with p. 51 of Van Brummelen.

What about the other two formulae?

$$\tan\epsilon = \frac{1}{\sin\alpha}\tan\delta$$

$$\frac{1}{\cos\alpha} = \frac{1}{\sin\delta}\sin\lambda$$

These are useful if you have α and δ and you would like to know λ, or if you have α and δ and you would like to know ϵ.