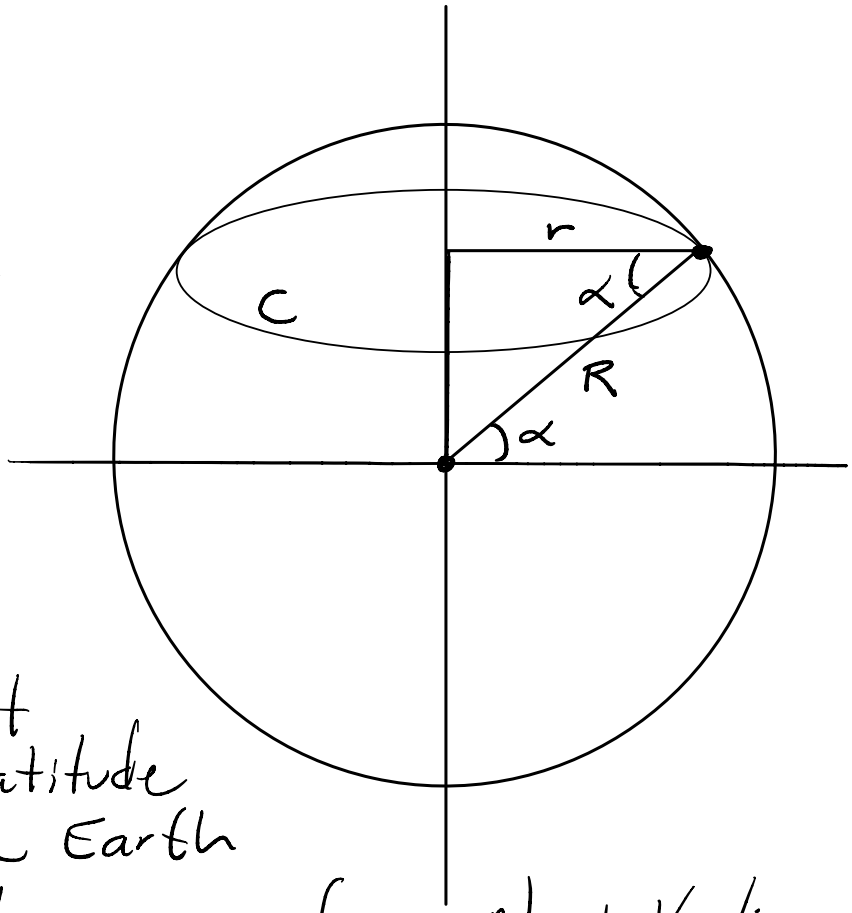
Problem Set 2 Solution

Problems 11, 12, and 13 from chapter 1 of Van Brummelen

Problem 11



α is the latitude of New York.

R is the radius of the Earth.

r is the radius of the circle that is the line of latitude going around the Earth at the latitude, α, of New York.

$$r = R\cos\alpha$$

C is the circumference of that circle.

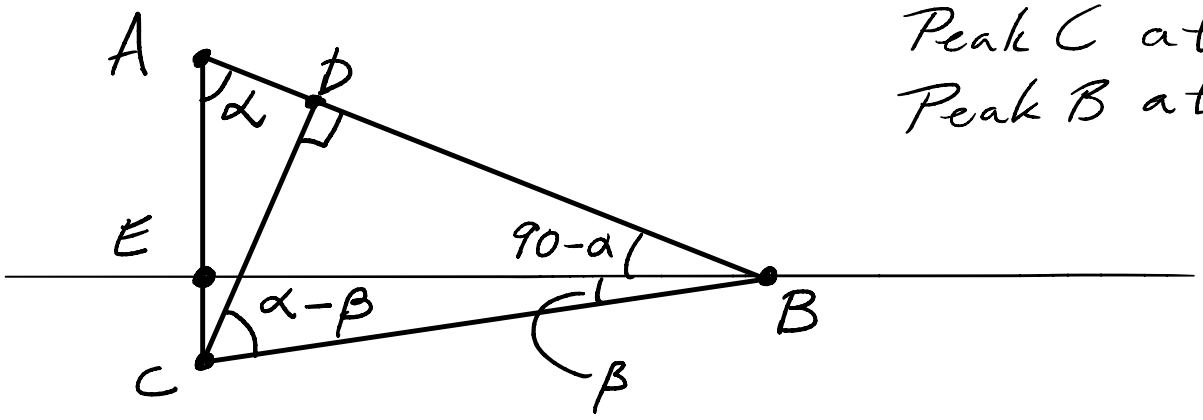
$$C = 2\pi r = 2\pi R\cos\alpha$$

$T = 23$ hr 56 minutes is the time for New York to go the distance C once. The speed is

$$s = \frac{C}{T} = 2\pi\cos 40.75^\circ \frac{6378\text{ km}}{(23+56/60)\text{ hr}} = 1268.5\ \frac{\text{km}}{\text{hr}}$$

Problem 12

Plane at A
Peak C at C
Peak B at B



α = known, given = $43.867°$

β = known, given = $5°$

$\overline{AC}$ = known, given = $8460ft - 4135ft = 4325ft$ (altitude of plane − elevation of peak C)

We need $\overline{EC}$. Once we have it, we add it to 4135ft, and we have the elevation of peak B.

$\overline{DC} = \overline{AC}\sin\alpha$

Since $\angle CBD = 90 - (\alpha - \beta)$, it must be that $\angle DCB = \alpha - \beta$.

$$\overline{CB} = \frac{\overline{DC}}{\cos(\alpha-\beta)} \qquad \overline{CE} = \overline{CB}\sin\beta$$

Put it all together....

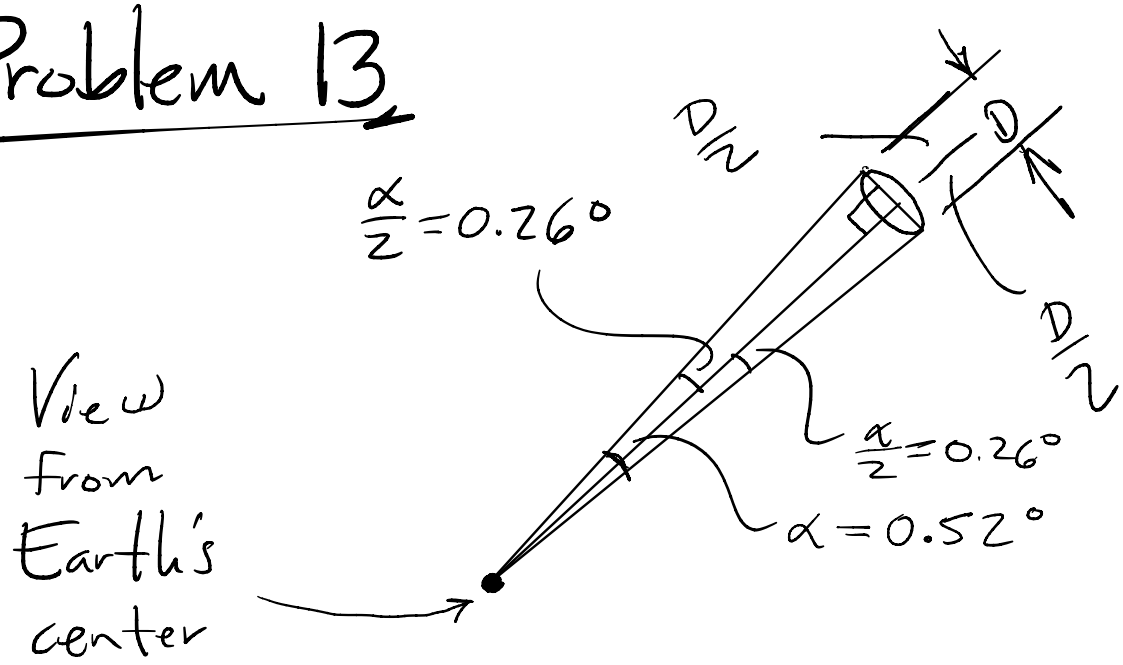
$$\text{Elevation of peak } B = 4135ft + \overline{CE}$$

$$= 4135ft + \overline{CB}\sin\beta = 4135ft + \frac{\overline{DC}}{\cos(\alpha-\beta)}\sin\beta$$

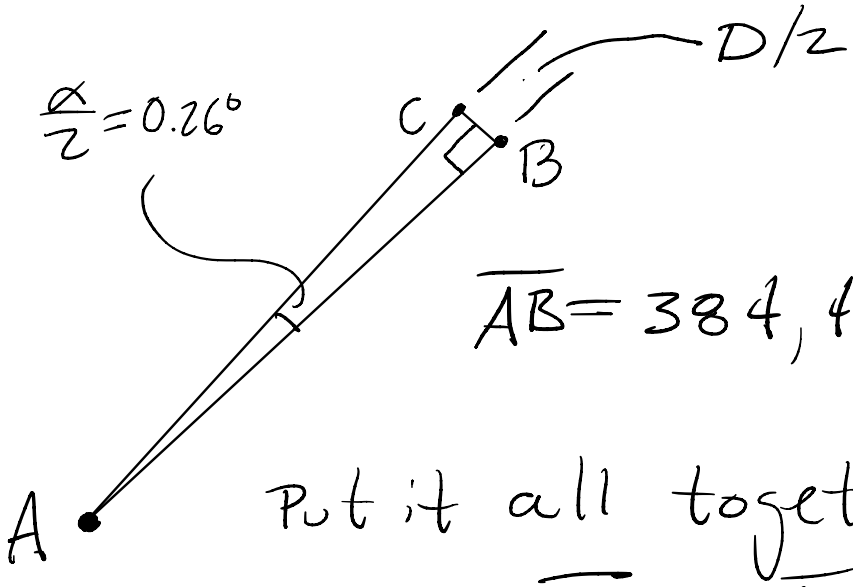
$$= 4135ft + \frac{\overline{AC}\sin\alpha}{\cos(\alpha-\beta)}\sin\beta$$

$$= 4135ft + \underbrace{\frac{4325ft \cdot \sin 43.867°}{\cos 38.867°} \cdot \sin 5°}_{335.5ft} = 4470ft$$

Problem 13



We bisect the angle α. That gives us two right triangles. Here is one of them:



$\overline{CB}=\frac{D}{2}$

$\overline{AB}=384,400\,km$

$\frac{\overline{CB}}{\overline{AB}}=\tan\frac{\alpha}{2}$

Put it all together:

$D=2\overline{CB}=2\overline{AB}\tan\frac{\alpha}{2}$

$=2\cdot 384,400km\cdot\tan 0.26°$

$=3489km$