Problem Set 5 Solution

Before doing either Problem 1 or Problem 2, let's recall the two major results from Friday:

$\sin\delta = \sin\lambda \cdot \sin\epsilon$
$\tan\alpha = \tan\lambda \cdot \cos\epsilon$

In these formulae,

$\delta \equiv$ declination of the Sun
$\alpha \equiv$ right ascension of the Sun
$\lambda \equiv$ ecliptic longitude of the Sun
$\epsilon \equiv$ obliquity of the ecliptic

In Van Brummelen, the derivation of those two results culminated at the top of p. 51.

Problem 1

```
In[398]:= ϵ = AstronomicalData["Earth", "Obliquity"];
```

The numerical value of ϵ is about 23.45°.

From Appendix A, p. 175, for Feb. 24:

```
In[399]:= λ = 336.3 °;
```

We would like α and δ. Solving the above equations for α and δ gives:

$\alpha = \arctan(\tan\lambda \cdot \cos\epsilon)$
$\delta = \arcsin(\sin\lambda \cdot \sin\epsilon)$

There are two issues with just sticking this into your calculator: (1) Your calculator might work in radians by default. That is the way Mathematica works. We have been working in decimal degrees or even in hms (which is a sexagesimal system) for right ascension. (2) The sine, cosine, and tangent functions do not have unique inverses. For example, $\sin\theta = \sin(180° - \theta)$. To make an even more explicit example, $\sin 15° = \sin(180° - 15°) = \sin 165° = 0.2588$. Thus there is no way in heck that arcsin(0.2588) can uniquely compute a value. It could be 15°. It could be 165°. Which one a particular calculator or program picks is up to the calculator manufacturer or the programmer. There are agreed-upon conventions, but the conventions don't change the fact that sin, cos, and tan do not have unique inverses!

```
In[409]:= α = ArcTan[Tan[λ] × Cos[ϵ]] / Degree
Out[409]= -21.9353
```

In[401]:=
```
δ = ArcSin[Sin[λ] × Sin[ϵ]] / Degree
```
Out[401]= -9.20426

Before considering ourselves done with this problem, we should understand the minus signs. One way of interpreting the minus sign in α is to just note that it can mean going to the left (west) along the ecliptic from the First Point of Aries. A second way of interpreting the minus sign in α, is to note that when describing a position on a circle, you can always add 360° to the angle and get to the same position. So an equally valid answer for α in degrees is α + 360° = 338.0647°. Also, it is common to write right ascensions in hours, minutes, and seconds, so we could convert and get α = 22h 32m 16s. Regarding the minus sign in δ, it just means that you go down from the celestial equator.

Problem 2

We are given α = 8.421h and δ = 19.22°. We can solve either of the first two formulae to get λ. We should use both and make sure we get the same answer:

$\lambda = \arcsin(\sin\delta / \sin\epsilon)$
$\lambda = \arctan(\tan\alpha / \cos\epsilon)$

First we'll get λ from δ (upper formula):

In[403]:=
```
δ = 19.22 Degree;
ArcSin[Sin[δ] / Sin[ϵ]] 180 / Pi
```
Out[404]= 55.8155

We have a problem! 55.8155° is nowhere even close to α (which is 8.421 × 15 = 126.315 °. Ah, but we expected this problem. Because sin(180°-θ) = sin θ we have an ambiguity. We need 180° - λ = 180° - 55.8155° = 124.18°. That is our value for λ using δ.

Now we'll get λ from α (lower formula):

In[405]:=
```
α = 8.421 × 15 °;
ArcTan[Tan[α] / Cos[ϵ]] 180 / Pi
```
Out[406]= -56.0093

Again, we have a problem! -56.0093° is not even close to α (even if we add 360°). Because tan(180°+θ) = tan θ, the arctan function also has an ambiguity. We need 180° + λ = 180° - 56.01° = 123.99°. That is our value for λ using α.

So what day is it?

We have gotten two values for λ which are pretty close: 124.18° and 123.99°. Consulting Van Brummelen's table on p. 177, we see that it must be July 28. By the way, if you put this into SkySafari Pro 6 (a very accurate program used to drive telescopes), you get July 26, 2022 at 5:30pm as the time this year that λ will be 124°.