Problem Set 7 Solution

(1) p. 92 #4, (2) p. 57 #6, (3) As we did in class on Monday, find the following quantities for Deep Springs at sunrise on July 4: (a) note the Sun's ecliptic longitude, (b) note the latitude of Deep Springs, (c) declination, (d) right ascension, (e) the ortive amplitude, (f) the ascensional difference, (g) the rising time — NB: July 4 is past the summer solstice, and this may affect your calculations (cf. Van Brummelen, p. 81) Also, solving your equation from p. 57 #6 part (a) for t and using your answers to the above, determine the amount of daylight at Deep Springs on July 4.

Problem 1

p. 92 #4.

(a) We are given A and c: we need B, a, and b, which we can get from I.2, II.1, and I.4, respectively:

```
In[254]:= A = 72.72 Degree;
          c = 109.8 Degree;

In[256]:= B = ArcCot[Cos[c] * Tan[A]] / Degree

Out[256]= -42.563
```

The arccot function is leading us astray. We want the solution that is 180° more than this:

```
In[257]:= B = ArcCot[Cos[c] * Tan[A]] / Degree + 180

Out[257]= 137.437

In[258]:= a = ArcSin[Sin[c] * Sin[A]] / Degree

Out[258]= 63.9503

In[259]:= b = ArcTan[Tan[c] * Cos[A]] / Degree

Out[259]= -39.5248
```

The arctan function is leading us astray. We want the solution that is 180° more than this:

```
In[260]:= b = 180 + ArcTan[Tan[c] * Cos[A]] / Degree

Out[260]= 140.475
```

(b) We are given a and b: we need c, A, and B, which we can get from II.5, I.1, and I.3, respectively:

```
In[261]:= a = 51.45 Degree;
          b = 78.73 Degree;

In[263]:= c = ArcCos[Cos[a] * Cos[b]] / Degree

Out[263]= 83.0044
```

```
In[264]:= A = ArcTan[Tan[a] / Sin[b]] / Degree
Out[264]= 51.9925
```

```
In[265]:= A = ArcTan[Tan[b] / Sin[a]] / Degree
Out[265]= 81.1419
```

(c) We are given a and B: we need b, c, and A, which we can get from I.3, I.5, and II.2, respectively:

```
In[266]:= a = 63.48 Degree;
B = 80.57 Degree;
```

```
In[268]:= b = ArcTan[Sin[a] * Tan[B]] / Degree
Out[268]= 79.4846
```

```
In[269]:= c = ArcTan[Tan[a] / Cos[B]] / Degree
Out[269]= 85.3259
```

```
In[270]:= A = ArcCos[Cos[a] * Sin[B]] / Degree
Out[270]= 63.8657
```

(d) We are given a and c: we need b, A, and B, which we can get from II.5, II.1, and I.5, respectively:

```
In[271]:= a = 69.72 Degree;
c = 78.42 Degree;
```

```
In[273]:= b = ArcCos[Cos[c] / Cos[a]] / Degree
Out[273]= 54.6097
```

```
In[274]:= A = ArcSin[Sin[a] / Sin[c]] / Degree
Out[274]= 73.2357
```

```
In[275]:= B = ArcCos[Tan[a] * Cot[c]] / Degree
Out[275]= 56.3217
```

(e) We are given A and B: we need a, b, and c, which we can get from II.2, II.3, and I.2, respectively:

```
In[276]:= A = 52.4 Degree;
B = 122.27 Degree;
```

```
In[278]:= a = ArcCos[Cos[A] / Sin[B]] / Degree
Out[278]= 43.813
```

```
In[279]:= b = ArcCos[Cos[B] / Sin[A]] / Degree
Out[279]= 132.367
```

```
In[280]:= c = ArcCos[Cot[A] * Cot[B]] / Degree
Out[280]= 119.096
```

Problem 2

p. 57 #6.

(a) We have established on multiple occasions that the ortive amplitude, η, satisfies:

$\cos n \cos \delta = \cos \eta$

and that the ascensional difference (or equation of daylight), n, satisfies:

$180^{\circ} + 2n = t$

(with the understanding that t, the amount of daylight, if measured in hours must be converted to an angle by multiplying by 15° / hour).

Therefore

$n = \frac{t}{2} - 90^{\circ}$

Putting that into the first equation gives

$\cos\left(\frac{t}{2} - 90^{\circ}\right) \cos \delta = \cos \eta$

or

$\sin \frac{t}{2} \cos \delta = \cos \eta$

One can of course take arccos of both sides so as to solve for η.

(b) 9.5 * 15° = 142.5°. Meanwhile, $\delta = -23.4°$ on the winter solstice. So η is

```
In[281]:= ArcCos[Sin[71.25 Degree] Cos[23.4 Degree]] / Degree
Out[281]= 29.6516
```

Which means Sun will rise 29.6516° south of east (a compass heading of 119.6516°).

(c) Atmospheric refraction makes the day longer, so we really should subtract several minutes from t to get the “true” length of the day, and that would make our answer to (b) a little further south. However, there is a second effect: the Sun is sweeping south in azimuth when it rises as well as rising in altitude. Since you saw it sooner you saw it when it was a little more north. I am really not sure which of these effects wins! Maybe they even cancel.

Problem 3

On July 4 at Deep Springs, with

(a) From Ptolemy’s table in Van Brummelen on p. 77.

```
In[282]:= λ = 100.9 Degree;
```

(b) Latitude of Deep Springs:

```
In[283]:= ϕ = 37.4 Degree;
```

(c) Declination (we also need $\epsilon = 23.4^o$):

```
In[284]:= ϵ = 23.4 Degree; δ = ArcSin[Sin[λ] Sin[ϵ]]; δ / Degree
Out[284]= 22.9534
```

(d) Right ascension:

```
In[285]:= α = ArcCos[Cos[λ] / Cos[δ]]; α / Degree
Out[285]= 101.85
```

is the angle we are looking for.

(e) The ortive amplitude:

```
In[286]:= η = ArcSin[Sin[δ] / Cos[ϕ]]; η / Degree
Out[286]= 29.4001
```

(f) The ascensional difference (or equation of daylight):

```
In[287]:= n = ArcCos[Cos[η] / Cos[δ]]; n / Degree
Out[287]= 18.893
```

(g) The rising time. If the Sun were at $\delta = 0$, it would rise α later than ♈. But it rises n earlier than that. So our answer, converted from degrees to hours at 15^o / hour, is:

```
In[288]:= risingTime = (α / Degree - n / Degree) / 15
Out[288]= 5.53048
```

So the Sun rises almost six hours after ♈ rises.

```
In[289]:= amountOfDaylight = 12 + 2 (n / Degree) / 15
Out[289]= 14.5191
```

Which is 14 hours and 31 minutes.