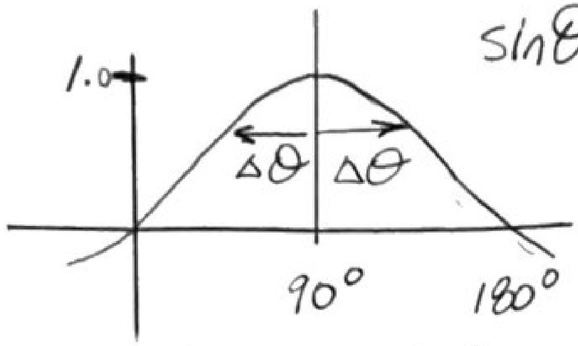
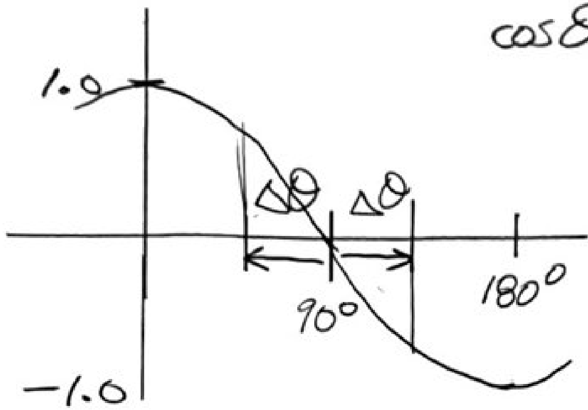
Problem Set 9 Solution

Problem 1(a)



The pleasing symmetry is that
$\sin(90^\circ + \Delta\theta) = \sin(90^\circ - \Delta\theta)$

Problem 1(b)



The pleasing symmetry is that

$$\cos(90^\circ + \Delta\theta) = -\cos(90^\circ - \Delta\theta)$$

Problem 1(c)

Let $90^\circ + \Delta\theta = \theta$

Then $90^\circ - \Delta\theta = 180^\circ - (90^\circ + \Delta\theta)$

$= 180^\circ - \theta$

So $\sin(90^\circ + \Delta\theta) = \sin(90^\circ - \Delta\theta)$

is equivalent to

$\sin\theta = \sin(180^\circ - \theta)$

Problem 1(d) Same argument as in 1(c).
Result is

$$\cos\theta = -\cos(180° - \theta)$$

Problem 1(e)

There is a triangle whose sides are

$$180° - A, \quad 180° - B, \text{ and } 180° - C$$

and whose angles are

$$180° - a, \quad 180° - b, \text{ and } 180° - c$$

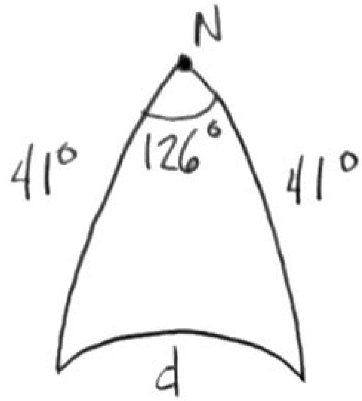
For this triangle, the Law of Cosines says

$$\overbrace{\cos(180° - C)}^{-\cos C} = \overbrace{\cos(180° - A)}^{-\cos A}\overbrace{\cos(180° - B)}^{-\cos B}$$
$$+ \underbrace{\sin(180° - A)}_{\sin A}\underbrace{\sin(180° - B)}_{\sin B}\underbrace{\cos(180° - c)}_{-\cos c}$$

Problem 1(f)

$$\cos C = -\cos A\cos B + \sin A\sin B\cos c$$

Problem 2(a)



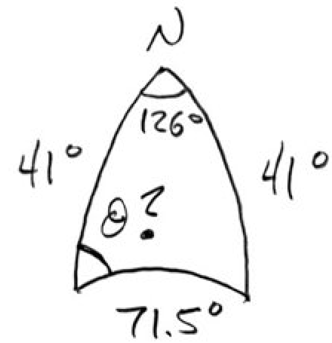
The Law of Cosines says

$$\cos d = \cos 41^\circ \cos 41^\circ + \sin 41^\circ \sin 41^\circ \cos 126^\circ$$

Or

$$d = \cos^{-1}\left(\cos^2 41^\circ + \sin^2 41^\circ \cos 126^\circ\right)$$
$$= 71.5^\circ$$

Problem 2(b)



$$\frac{\sin\theta}{\sin 41^\circ} = \frac{\sin 126^\circ}{\sin 71.5^\circ} \quad \text{or} \quad \theta = \sin^{-1}\frac{\sin 41^\circ \sin 126^\circ}{\sin 71.5^\circ}$$

Remarkable, eh? $\longrightarrow$ $= 34^\circ$ $\longleftarrow$ compass heading

That is more north than east.

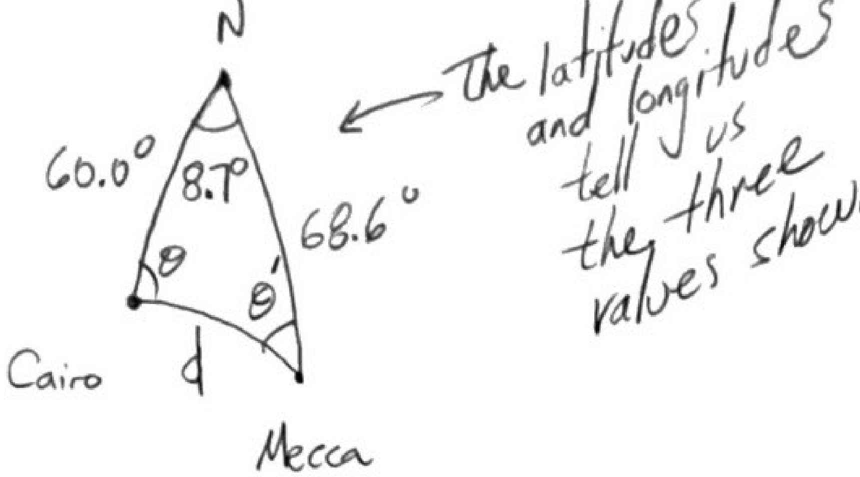
Problem 2(c)

$$71.5^\circ = 71.5^\circ \times \frac{60 \text{ nautical miles}}{1^\circ}$$

$$= 4290 \text{ nautical miles}$$

Problem 3

(a)



Exactly as in Problem 2 we must first get d.

$$d = \cos^{-1}(\cos 68.6° \cos 60° + \sin 68.6° \sin 60° \cos 8.7°)$$

$$= 11.6°$$

Then follow the method of Problem 2 again to get θ

$$\theta = \sin^{-1} \frac{\sin 68.6° \sin 8.7°}{\sin 11.6°} = 44.3°$$

However, the calculator's arcsin has failed us.

We want $180° - 44.3° = 135.7°$

(b) For the reverse, we need θ'.

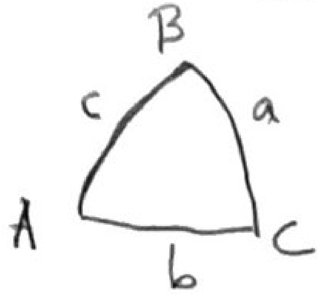
$$\theta' = \sin^{-1} \frac{\sin 60.0° \sin 8.7°}{\sin 11.6°} = 40.5°$$

This is how many degrees <u>west</u> <u>of</u> <u>north</u>. It is in the ballpark of (but not the same as) $135.7° - 180°$.

Problem 4

Van Brummelen p.106 #2

$A = 69^\circ$
$B = 84^\circ$
$C = 100^\circ$



First we need a, b, and c.

The Law of Cosines for angles says

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

Or,

$$c = \cos^{-1} \frac{\cos C + \cos A \cos B}{\sin A \sin B}$$

$$= \cos^{-1} \frac{\cos 100^\circ + \cos 69^\circ \cos 84^\circ}{\sin 69^\circ \sin 84^\circ}$$

$$= 98.4^\circ$$

Also,

$$b = \cos^{-1} \frac{\cos 84^\circ + \cos 69^\circ \cos 100^\circ}{\sin 69^\circ \sin 100^\circ}$$

$$= 87.4^\circ$$

And,

$$a = \cos^{-1} \frac{\cos 69^\circ + \cos 84^\circ \cos 100^\circ}{\sin 84^\circ \sin 100^\circ}$$

$$= 69.7^\circ$$

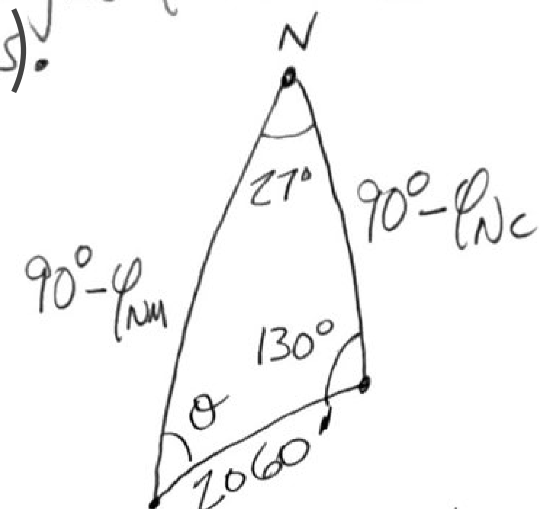
$$(a+b+c)\frac{2\pi}{360^\circ} \cdot 10\text{ inches} = 44.6''$$

Problem 5 Van Brummelen p.106 #4

Ceylon and Madagascar are close to the Equator. This problem may get tricky due to large angles (when measured from the poles).

φ_{NM} stands for latitude "near Madagascar"

φ_{NC} stands for latitude "near Ceylon"



Everything we have been told is captured in the above diagram. We would like to know φ_{NM}, φ_{NC}, and θ.

By the Law of Sines

$$\frac{\sin(90° - \varphi_{NM})}{\sin 130°} = \frac{\sin 2060'}{\sin 27°}$$

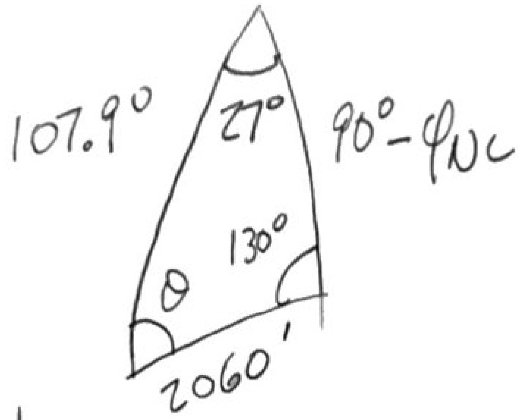
Also, $\sin(90° - \varphi_{NM}) = \cos \varphi_{NM}$. Therefore

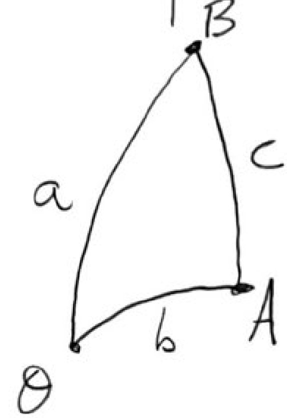
$$\varphi_{NM} = \cos^{-1} \frac{\sin 130° \sin 2060'}{\sin 27°} = 17.9°$$

<u>THIS ISN'T REASONABLE!</u> $-17.9°$ is reasonable.

So far this is what we've got:

We have two angles and the two opposing sides. Seems like it is time for Napier's analogies. See p. 105





$$\frac{\tan\frac{1}{2}(A+B)}{\cot\frac{1}{2}\theta} = \frac{\cos\frac{1}{2}(a-b)}{\cos\frac{1}{2}(a+b)}$$

also

$$\frac{\tan\frac{1}{2}(a+b)}{\tan\frac{1}{2}c} = \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}$$

$a = 107.9°$
$b = 2060'$
$A = 130°$
$B = 27°$

Or

$$\theta = 2\tan^{-1}\frac{\cos\frac{1}{2}(a-b)}{\tan\frac{1}{2}(A+B)\cos\frac{1}{2}(a+b)} = 53.4°$$

$$c = 2\tan^{-1}\frac{\tan\frac{1}{2}(a+b)\cos\frac{1}{2}(A+B)}{\cos\frac{1}{2}(A-B)} = 86.2°$$

To summarize, $\varphi_{NM} = -12.6°$
$\varphi_{NC} = 3.8°$

compass heading $\theta + 180° = 233.4°$

which Van Brummelen writes as S53.4°W

Problem 6

Part (a)

$$A = \tan^{-1}\frac{108}{81} = 53.13010235^\circ$$

Part (b)

$$B = 90 - A = 36.86989765^\circ$$

(1) Use the ordinary planar Pythagorean Theorem to get c (in mm) from a and b. c = 135 mm

(2) The scale of this map is 1000 ft = 18mm. Convert a, b, and c to feet:

a = 6500 ft b = 4500 ft c = 7500 ft

Part (c)

(3) The radius of the Earth is 20,900,000 feet. Convert a and b to radians. Keep all digits that your calculator displays.

a = $2.870813397 \times 10^{-4}$ b = $2.153110047 \times 10^{-4}$

or $a = 0.01644854 91^\circ$, $b = 0.01233641 19^\circ$

Part (d) $\sin b = \tan a \cot A$ or $A = \tan^{-1}\frac{\tan a}{\sin b}$

$$A = \tan^{-1} 1.33333338 = 53.13010332^\circ$$

Part (e) $\sin a = \cot B \tan b$ or $B = \tan^{-1}\frac{\tan b}{\sin a}$

$$B = \tan^{-1} 0.7500000219 = 36.86989845^\circ$$

Part (f) 7 figures of agreement Part (g) Same as (f).