Problem Set 9: Applications of the Law of Sines, and the Law of Cosines

Due Monday, April 18

Problem 1 — Deriving the Law of Cosines for Angles

In this problem you will start with the Law of Cosines (top of p. 98) in and prove the Law of Cosines for angles (top of p. 100). This is what Van Brummelen did, but he did it in one line, leaving out all the intermediate steps. It is your job in this problem to convincingly fill in the intermediate steps.

(a) Graph sin θ from $\theta = 0^\circ$ to $\theta = 180^\circ$. Describe the pleasing symmetry of sin θ in this region.

(b) Graph cos θ from $\theta = 0^\circ$ to $\theta = 180^\circ$. Describe the pleasing symmetry of cos θ in this region.

(c) Capture the pleasing symmetry you described in part (a) as an identity for $\sin(180^\circ - \theta)$.

(d) Capture the pleasing symmetry you described in part (b) as an identity for $\cos(180^\circ - \theta)$.

(e) Apply the polar duality theorem to the Law of Cosines.

(f) Simplify your answer using the identities you found in (c) and (d).

Problem 2 — Distance and Direction, YVR to CDG

YVR Airport (Vancouver) has longitude -123° and latitude 49°.

CDG Airport (Charles de Gaulle) has longitude $+3^\circ$ and latitude 49°.

(a) Find d the distance your plane must travel to get from YVR to CDG. Answer in degrees.

(b) Find θ the direction that your plane should take off from YVR.

(c) Convert d to nautical miles using 1 nautical mile = 1’ and 1° = 60’.

Problem 3 — Mecca and Cairo

Mecca has longitude 39.9° and latitude 21.4°. Cairo has longitude 31.2° and latitude 30.0°.

(a) What is the Direction of Mecca from Cairo? (b) What is the direction of Cairo from Mecca? NB: this is not just 180° plus the answer from part (a).

Problem 4 — Perimeter of Triangle

Van Brummelen, p. 106 #2.

Problem 5 — Ceylon to Madagascar

Van Brummelen, p. 106 #4.

Problem 6 — The Locality Principle and Right Triangles in Deep Springs Valley

Consult the handout with the triangle in Deep Springs Valley: https://brianhill.github.io/heavenly-mathematics/resources/locality-principle/TrianglesInDeepSpringsValley.pdf

In class we already found that $a = 6000$ ft and $b = 4500$ ft.

Part (a) Use ordinary planar trigonometry (SOHCAHTOA stuff) to get A. You can assume the unlabeled angle is 90°. Write out all digits your calculator shows (probably 10 digits). Part (b) Use ordinary planar trigonometry or the fact that on the plane the angles of a triangle add up to 180° to get B. Again out all digits your calculator shows (probably 10 digits). Part (c) Convert a and b to degrees using R, the radius of the Earth, as 20,900,000 ft. Again keep all decimal places your calculator shows. Part (d) Use whichever of the the 10 identities on p. 86 that involves a, b, and A to get A. Part (e) Use whichever of the the 10 identities that involves a, b, and B to get B. Part (f) Compare the answer you got in (d) to the answer you got in (a). Part (g) Compare the answer you got in (e) to the answer you got in (b).

Problem 7 — Optional Challenge Problem — Deriving the Law of Sines from the Law of Cosines

p. 107 #11