Manhattan Project – Assignment 2 – Solution

1. $E = mc^2$

$$E = mc^2 \qquad m = 1\,g = 0.001\,kg$$

$$c = 3\times10^8\ \frac{m}{s}$$

$$E = 0.001\,kg \times \left(3\times10^8\ \frac{m}{s}\right)^2$$

$$= 9\times10^{13}\ \underbrace{kg\frac{m^2}{s^2}}_{J}$$

That's a lot! A huge 1 GW nuclear reactor does

$$\underbrace{86{,}400}_{\#\text{ seconds in a day}} \times \underbrace{10^9}_{\#\text{ Joules / second}}\ J \text{ in a day}$$

1 gram of matter does a big nuke for 24 hours!

2. e^+e^- Annihilation

Total mass $= 2\times 9.1\times10^{-31}\,kg$

But the resulting energy is shared between two photons, so divide by 2.

More standard to write $8.2\times10^{-14}\,J$

$$E_\gamma = 9.1\times10^{-31}\,kg \times \left(3.0\times10^8\ \frac{m}{s}\right)^2 = 81.9\times10^{-15}\,J$$

3. The Coulomb and the electron-Volt

(a) One electron freely moving from the negative terminal to the positive terminal of a 9V battery releases

$$1.6\times10^{-19}\,C \times 9\,V = 14.4\times10^{-19}\,J$$

(because $1C \times 1V = 1J$)

More standard to write $1.44\times10^{-18}\,J$ or about $1.4\times10^{-19}\,J$

(b) If it were 1V, then we'd have

$$1.6\times10^{-19}\,C \times 1V = 1.6\times10^{-19}\,J$$

(c) The combination we found in (b) is called the eV.

$$1eV = 1.6\times10^{-19}\,J$$

4. Tiny Particles, Enormous Numbers, N_A

(a) mass = # of particles × mass per particle

So # of particles = $\dfrac{\text{mass}}{\text{mass per particle}}$

So it would take $\dfrac{1kg}{9.1\times10^{-31}\,kg/\text{electron}}$

Which is 1.1×10^{30} electrons

to make 1kg.

#4 is (CONT'D)

4(b) charge = # of particles × charge per particle

So # of particles = $\frac{\text{charge}}{\text{charge per particle}}$

So it would take $\frac{1C}{1.6 \times 10^{-19} C/\text{electron}}$

Which is 6.25×10^{18} electrons

to make 1C.

(c) $\#\text{of Carbon-12 atoms} = \frac{\text{mass}}{\text{mass per Carbon-12 atom}}$

$$= \frac{\cancel{12\text{ grams}}}{\cancel{12} \cdot 1.66 \times 10^{-24} \cancel{\text{grams}}/\text{Carbon-12 atom}}$$

$$= 6.02 \times 10^{23} \text{ Carbon-12 atoms}$$

(d) $1kg = 1000g$, so there are $\frac{1000}{12} N_A$ Carbon-12 atoms in 1 kilogram.

$$\frac{1000}{12} N_A = \frac{1000}{12} \times 6.02214076 \times 10^{23}$$

$$= 501.84063\bar{3} \times 10^{23}$$

$$= 5.0184063\bar{3} \times 10^{25} \text{ Carbon-12 atoms}$$

5 Measuring Mass in eV

(a) In Problem 2, we got 8.2×10^{-14} J.
In Part 3(c), we got $1.6 \times 10^{-19}\,J = 1\,eV$.

So we can convert what we got in Problem 2 to eV by doing

$$E_\gamma = 8.2 \times 10^{-14}\,J \,\frac{1\,eV}{1.6 \times 10^{-19}\,J} = 5.125 \times 10^{5}\,eV$$

If we had kept more decimal places at every step, we would have gotten $E_\gamma = 5.11 \times 10^{5}\,eV$

(b) Using $10^{6}\,eV = 1\,MeV$, this can be (and usually is) written as $E_\gamma = 0.511\,MeV$

6 Energy Released in Fusion

(a) Deuterium atomic mass $= 2.014102\,u$
Tritium atomic mass $= 3.016049\,u$
Total $= 5.030151\,u$

(b) Helium-4 atomic mass $= 4.002603\,u$
neutron mass $= 1.008865\,u$
Total $= 5.011468\,u$

(c) Difference (a)−(b) is

$$5.030151\,u - 5.011468\,u = 0.018683\,u$$

#6 is (CONT'D)

6(d) Convert 0.018683u to J using

$$1u = 1.66054 \times 10^{-27} kg$$

Of course we have to multiply by c^2.

I'm going to keep 5 significant figures at every step now. $c = 2.9979 \times 10^8 \frac{m}{s}$

$$0.018683 \times 1.66054 \times 10^{-27} kg \times \left(2.9979 \times 10^8 \frac{m}{s}\right)^2$$

$$= 0.27882 \times 10^{-11} J$$

(e) In 3(c) we found $1eV = 1.6 \times 10^{-19} J$, but the conversion factor to 5 significant figures is

$$1eV = 1.6022 \times 10^{-19} J$$

So we have

$$0.27882 \times 10^{-11} J \times \frac{1eV}{1.6022 \times 10^{-19} J}$$

$$= 0.17402 \times 10^8 eV$$

(f) Using $10^6 eV = 1 MeV$ we have

17.402 MeV

The agreed-upon value is 17.6 MeV. I am not sure how we got off by 1% when we were careful to keep 5 sig figs.