Manhattan Project – Assignment 3 – Solution

1. Using $R(t) = -\lambda N(t)$ and $\lambda = \frac{\ln 2}{t_{1/2}}$

(a) Convert 138 days to seconds

$$138 \text{ days} = 138 \text{ days} \cdot \frac{24 \text{ hours}}{\text{day}} \cdot \frac{60 \text{ minutes}}{\text{hour}} \cdot \frac{60 \text{ seconds}}{\text{minute}}$$

$$= 138 \cdot 24 \cdot 60 \cdot 60 \text{ s} = 11923200 \text{ s}$$

$$= 1.19 \times 10^7 \text{ s}$$

(b) $R = -\lambda N = -\frac{\ln 2}{t_{1/2}} N$

$N = N_A$
$= 6.02 \times 10^{23}$

$t_{1/2} = 1.19 \times 10^7 \text{ s}$

$$= -\frac{\ln 2}{1.19 \times 10^7 \text{ s}} \cdot 6.02 \times 10^{23}$$

$$= -\left(\frac{\ln 2}{1.19} \times 6.02\right) \times 10^{16} / \text{s}$$

← this part I have to stick into a calculator

$$= -3.51 \times 10^{16} / \text{s}$$

The minus sign just means we are losing atoms, not gaining them

We are losing 3.51×10^{16} Polonium atoms per second

(c) $3.51 \times 10^{16} \frac{1}{\text{s}} \cdot \frac{1 \text{ Ci}}{3.7 \times 10^{10} / \text{s}} = 9.5 \times 10^5 \text{ Ci}$

2. Alpha Decay of Polonium-210

(a) The A and Z that balance

$$^{210}_{84}Po \rightarrow ^{4}_{2}He + ^{A}_{Z}X$$

are $A = 206$, $Z = 82$.

(b) That is Lead (symbol Pb). The isotope is Lead-206. We could also have written: $^{206}_{82}Pb$.

3. β^- and β^+ Decay

(a)

	A	Z	N
Polonium	210	84	126

If β^- decay, Z increases by 1 and N decreases by 1 and A stays the same

So we would have

A	Z	N
210	85	125

and Z=85 is Astatine ← I never heard of it

(b) If β^+ decay, Z decreases by 1, N increases by 1, A stays the same

So we would have

A	Z	N
210	83	127

and Z=83 is Bismuth

4. Energy Released in Polonium-210 α-Decay

$$^{210}_{84}Po \longrightarrow {}^{4}_{2}He + {}^{206}_{82}Pb$$

(a) Total mass on the left-hand side is simply 209.982874 u.

(b) Total mass on right-hand side is

Helium-4	4.002603 u
Lead-206	205.974449 u
Total	209.977052 u

Here we only have four significant figures

(c) $209.982874 - 209.977052 = 0.005822\,u$

(d) $E = mc^2 = 0.005822\,u \cdot \frac{1.66054 \times 10^{-27}\,kg}{1\,u} \cdot \left(2.99792458 \times 10^{8}\,\frac{m}{s}\right)^2$

$= 0.08689 \times 10^{-11}\,J$

$= 8.689 \times 10^{-13}\,J$

(e) $8.689 \times 10^{-13}\,J \cdot \frac{1\,eV}{1.602176634 \times 10^{-19}\,J}$

$= 5.423 \times 10^{6}\,eV$

(f) $5.423 \times 10^{6}\,eV = 5.423\,MeV$

(g) $0.005822\,u \cdot \frac{931.4\,MeV}{u} = 5.423\,MeV$

↖ we are 1/3 % off of the accepted value