Manhattan Project – Problem Set 4 – Solution

1 (a) $^4_2\alpha + ^9_4Be \rightarrow ^{13}_6C$

$Z_1 = 2 \quad Z_2 = 4$

$A_1 = 4 \quad A_2 = 9$

(b) $$E_{Coulomb} = \frac{e^2}{4\pi\epsilon_0 a_0}\frac{Z_1 Z_2}{A_1^{1/3} + A_2^{1/3}}$$

$$\frac{e^2}{4\pi\epsilon_0 a_0} = 1.2\,MeV$$

← the two formulas needed

Now just plug in

$$E_{Coulomb} = 1.2\,MeV \frac{2 \cdot 4}{4^{1/3} + 9^{1/3}} = 2.6\,MeV$$

(c) Yes. $(5.3\,MeV > 2.6\,MeV)$

(d) $Z_2 = 238$
$A_2 = 92$

(e) $$E_{Coulomb} = 1.2 \frac{2 \cdot 92}{4^{1/3} + 238^{1/3}} = 28.4\,MeV$$

(f) No

(2) (a) Momentum conservation says

$$p_{n,i} + 0 = p_{n,f} + p_{p,f}$$

Solve for $p_{n,f} = p_{n,i} - p_{p,f}$

(b) $$E_{n,i} + E_{p,i} = E_{n,f} + E_{p,f}$$

$$mc^2 + \frac{p_{n,i}^2}{2m} + mc^2 = mc^2 + \frac{p_{n,f}^2}{2m} + mc^2 + \frac{p_{p,f}^2}{2m}$$

All the mc^2 terms cancel! Then multiply through by $2m$ in what is left and you just have

$$p_{n,i}^2 = p_{n,f}^2 + p_{p,f}^2$$

(c) $$p_{n,i}^2 = (p_{n,i} - p_{p,f})^2 + p_{p,f}^2$$

$$\cancel{p_{n,i}^2} = \cancel{p_{n,i}^2} - 2p_{n,i}p_{p,f} + p_{p,f}^2 + p_{p,f}^2$$

$$0 = -\cancel{2}p_{n,i}p_{p,f} + \cancel{2}p_{p,f}^2$$

$$p_{n,i}\cancel{p_{p,f}} = p_{p,f}^{\cancel{2}} \Rightarrow p_{n,i} = p_{p,f}$$

(d) We can divide out $p_{p,f}$, but it might have been zero! That corresponds to the $p_{p,f} = 0$

$\Rightarrow p_{n,f} = p_{n,i}$

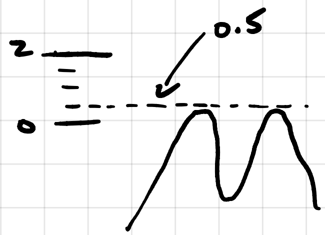
That is the case where nothing happened — the neutron missed the proton.

(e) $p_{n,i} = p_{p,f}$.

(f) All the energy and momentum is transfered.

3. Fission Products

(a) I get 0.5



(b) $10^{0.5} = 3.16$

(c) About 3% of the time you get A=95

(d) I get −2.2

(e) $10^{-2.2} = -0.006$

(f) About 0.01% of the time you get A=116

4. Relative Cross Sections

(a) $\log_{10} \frac{1}{40} = -1.6$

(b) $-6 - 1.6 = -7.6$

$(\log(\frac{1}{40} \times 10^{-6}))$

I got about 0.4

4(c) $10^{0.4} = 2.5$

So the plot is telling us that the absorption cross section of a thermal neutron hitting U-238 is 2.5 barns

(d) I get about 2.7

(e) $10^{2.7} = 501$

So the plot is telling us that the fission cross section for thermal neutrons hitting U-235 is 501 barns

(f) $\frac{e * 1}{e * 1 + c * 140} = \frac{501}{501 + 2.5 * 140}$

$= \frac{501}{501 + 350} = \frac{501}{851}$

$= 0.59$

The fissions can come from U-235 despite them being only 1 in every 140 atoms.