Manhattan Project — Assignment 4 — The Coulomb Barrier, The Neutron, Fission Products, Cross Sections

1. The Coulomb Barrier

To understand what you are calculating, you will want to review the Coulomb Barrier handout:

https://brianhill.github.io/manhattan-project/resources/TheCoulombBarrier.pdf

(a) First let's do alpha-particle bombardment on beryllium as shown in Fig. 2.23. Write down Z_1, A_1, Z_2, and A_2, assuming that particle 1 is the alpha particle and particle 2 is the most common form of Beryllium, which is Beryllium-9.

(b) Use the last two formulas in the Coulomb Barrier handout to find the Coulomb barrier for this collision (report your answer in MeV) .

(c) The alpha particles from polonium carry 5.3 MeV of kinetic energy. Is this enough to overcome the Coulomb barrier you found in (b)?

(d) Now let's do alpha-particle bombardment on uranium. Z_1 and A_1 will be the same as in part (a), but you need to write down Z_2 and A_2. (Let's assume that particle 2 is the most common form of Uranium, which is Uranium-238.)

(e) What is the Coulomb barrier for this collision?

(f) Assuming the alpha-particles from polonium are still being used, is this enough to overcome the Coulomb barrier you found in (d)?

2. The Neutron

In the Photon-Proton Collision handout,

https://brianhill.github.io/manhattan-project/resources/PhotonProtonCollision.pdf

it was derived why the "mystery radiation" in Fig. 2.23 cannot be a photon. In this problem, we want to see how supposing that the mystery radiation is a neutron fixes the problem.

The very first collision in this 1-minute YouTube video will help explain what is going on:

https://youtu.be/4v2RHtBTbj8

The first collision is a collision between two equal-sized billiard balls. The neutron and the proton have very nearly equal mass. For this problem, we'll treat them as exactly equal $m_p\,c^2 = m_n\,c^2 = 938$ MeV, and we'll call both masses m. (Only if you are wondering, the neutron is actually 1.3 MeV heavier.)

(a) Momentum conservation says:

$$p_{n,i} + 0 = p_{n,f} + p_{p,f}$$

In this equation, italic p is momentum, roman subscript "n" means neutron, roman subscript "p" means proton, roman subscript "i" means initial, and roman subscript "f" means final.

All we have done is write down momentum conservation for a neutron striking a proton that is initially at rest. Afterwards, both of them can carry momentum. Momentum conservation is more complicated if, after the collision, they come out at angles, but there is no need to go into that.

Rearrange the momentum conservation to solve for $p_{n,f}$. Box that.

(b) Energy conservation says:

$$E_{n,i} = \sqrt{(mc^2)^2 + (p_{n,i}\,c)^2}$$
$$E_{n,f} = \sqrt{(mc^2)^2 + (p_{n,f}\,c)^2}$$
$$E_{p,i} = \sqrt{(mc^2)^2 + (p_{p,i}\,c)} = mc^2 \text{ (because } p_{p,i} = 0)$$
$$E_{p,f} = \sqrt{(mc^2)^2 + (p_{p,f}\,c)^2}$$

However, the neutron and the proton are heavy and slow-moving in this reaction. They are not relativistic. Therefore, we can make the same approximation as was made for non-relativistic motion in the handout:

$$E_{n,i} = mc^2 + \frac{p_{n,i}^2}{2\,m}$$
$$E_{n,f} = mc^2 + \frac{p_{n,f}^2}{2\,m}$$
$$E_{p,i} = mc^2$$
$$E_{p,f} = mc^2 + \frac{p_{p,f}^2}{2\,m}$$

Put these nice simple formulas for $E_{n,i}$, $E_{n,f}$, $E_{p,i}$, and $E_{p,f}$ into the equation for energy conservation, which is

$$E_{n,i} + E_{p,i} = E_{n,f} + E_{p,f}$$

Simplify like heck. Not much will be left. Box it.

(c) Now substitute for what you got in part (a) into what you got in part (b) and simplify like heck again. Box that.

(d) If you did (c) right, you will see there is a common factor of $p_{\mathrm{p,f}}$ in every term in your equation, so you can divide it out. But wait! You can't divide something out if it is zero, so before you move on, explain the interpretation of the collision if $p_{\mathrm{p,f}}$ is zero.

(e) Moving on, now with the $p_{\mathrm{p,f}}$ cancelled, what is left of the equation is super-simple. Write that down and box it.

(f) Compare the equations in (a) and (e) and make the most simple statements you can about the collision.

DISCUSSION (no need to answer, just think about these comments): Do you see why the first collision in the YouTube video is relevant? Do you see why this solves the problem at the top of p. 57? The photon can only give a little of its energy to the proton and satisfy energy and momentum conservation. The neutron can give all of its energy and momentum to the neutron and come to a dead stop! If you have unequal-sized objects, different outcomes occur, and the 1-minute YouTube video illustrated those nicely too.

3. Fission Products

(a) Consult Fig. 3.7 on p. 89. Estimate the height of the high plateaus by drawing a horizontal line (with a ruler) across the plateaus to the vertical axis.

(b) The vertical axis is a log axis! Take $10^{\text{whatever you got in part (a)}}$.

(c) The caption says whatever you got in (b) is a percentage. So what percentage of the time do you get a fission product with mass number A=95?

(d) Estimate the height of the bottom of the low valley between the plateaus by drawing a horizontal line (with a ruler) across the bottom of the valley to the vertical axis.

(e) Take $10^{\text{whatever you got in part (d)}}$.

(f) The caption says whatever you got in (e) is a percentage. So what percentage of the time do you get a fission product with mass number A=116?

4. Relative Cross Sections

(a) Look at the figures on p. 100. The horizontal axis is a log axis. 10^{-6} MeV is just 1eV. Thermal neutrons are about $\frac{1}{40}$ of an eV. What is $\log_{10} \frac{1}{40}$?

(b) So on a plot where the axis is $\log_{10}$ of MeV, $\frac{1}{40}$ of an eV is at -7.6. Find -7.6 on the horizontal axis in Fig. 3.11. Use a ruler to draw lines and find the value on the vertical axis. Write that down.

(c) Take $10^{\text{whatever you got in (b)}}$. This is the absorption cross section of a thermal neutron hitting Uranium-238 in barns.

(d) Repeat step (b) on Figure 3.12.

(e) Take $10^{\text{whatever you got in (d)}}$. This is the fission cross-section for a thermal neutron hitting Uranium-235 in barns.

(f) In nature, there are about 140 Uranium-238 atoms for every Uranium-235 atom. So if you aim thermal neutrons at an unenriched sample, the fraction of the neutrons that will cause Uranium-235 fissions is

$$\frac{\text{what you got in e} * 1}{\text{what you got in e} * 1 + \text{what you got in c} * 140}$$

DISCUSSION (no need to answer, just think about this): Given what a small fraction of the sample is U-235, it is brilliant of Bohr to have suggested that that was where all the fissions were coming from! We have these wonderfully detailed cross-section graphs now, but he had nothing of the kind! Section 3.6 is about proving his suggestion to be correct by enriching the Uranium-235 in a sample and seeing that this dramatically increased the ratio of fissions to captures. These 1920’s and 1930’s physicists were unfathomably smart. It was an age of radically fast progress on many fronts, from quantum mechanics to the Big Bang.