Manhattan Project – Problem Set 6 – Solution

1. Reed Problem 5.1, p.235 – Air-Cooled Reactor

Operating at power P in a time Δt the reactor produces $Q = P\Delta t$ of heat.

The volume of air flowing through the reactor in this same time, Δt, is $V = f\Delta t$ where f is the flow rate. This air has mass $m = \rho V = \rho f \Delta t$.

(Don't confuse Δt, a time, with ΔT, a temperature rise.)

Reed gives us a thermodynamic formula:

$$Q = mc\Delta T$$

So,

$$P\cancel{\Delta t} = \rho f \cancel{\Delta t} c \Delta T$$

The time interval cancels out and you are only left with rates (power is a rate and air flow, f, is a rate). Solve for ΔT:

$$\Delta T = \frac{P}{\rho f c}$$

$$= \frac{1\,\frac{MJ}{\cancel{sec}}}{1\frac{\cancel{kg}}{\cancel{m^3}}\, 14.16\,\frac{\cancel{m^3}}{\cancel{sec}}\, \frac{1000\,J}{\cancel{kg}\cdot K}}$$

$$= \frac{1000}{14.16} K = 70.6\ K$$

$$\boxed{\begin{aligned} P &= 1 MW = \frac{1 MJ}{sec} \\ \rho &= \frac{1 kg}{m^3} \\ c &= \frac{1000\,J}{kg\cdot K} \\ f &= 30000\ \frac{ft^3}{min} \\ &= 30000 \left(12\ \text{inches} \cdot \frac{0.0254\,m}{\text{inch}}\right)^3 \frac{}{min} \\ &= 849.5\frac{m^3}{min} = 14.16\ \frac{m^3}{sec} \end{aligned}}$$

A change of 1K is the same as 1°C which is $\frac{9}{5}$ °F so that is 127°F. If it started at room temp (72°F) it comes out at about 200°F. That is a very hot and large blowdryer.

2. Reed Problem 5.2 – Uranium Load

We need the mass of N channels. Each channel is is a cylinder with mass $m_{channel} = \rho V = \rho \pi r^2 L$

So the total mass is

$$m_{total} = N m_{channel}$$

$$= N\rho\pi r^2 L$$

$$\boxed{\begin{aligned} N &= 1248 \\ \rho &= \frac{18.95 g}{cm^3} \\ r &= \frac{d}{2} = \frac{1.1\ \text{inches}}{2}\cdot\frac{2.54 cm}{\text{inch}} = 1.397 cm \\ L &= 24 ft\ \frac{12\ \text{inches}}{\text{foot}}\cdot\frac{2.54\,cm}{\text{inch}} = 731.5 cm \end{aligned}}$$

$$= 1248\cdot\pi\cdot(1.397cm)^2\ 731.5cm \cdot 18.95\,\frac{g}{cm^3}$$

$$= 1060671299\ g$$ ← a truly silly way of reporting the answer ☺

How about 106 metric tons as a useful way of reporting the answer.

3. Reed Problem 5.3 – Graphite Bricks

$$m_{total} = N\, m_{brick}$$

$$m_{brick} = \rho\, V_{brick}$$

$$V_{brick} = w\cdot w\cdot l$$

$$N = 40{,}000$$

$$\rho = 2.15\,\frac{g}{cm^3}$$

$$w = 4.125\ \text{inches}\cdot\frac{2.54\,cm}{\text{inch}} = 10.48\,cm$$

$$l = 16\ \text{inches}\cdot\frac{2.54\,cm}{\text{inch}} = 41.91\,cm$$

So

$$m_{total} = N\rho w^2 L = 40{,}000\cdot\frac{2.15 g}{cm^3}\cdot(10.48cm)^2\cdot 41.91\,cm$$

$$= 395857318\ g = 396\ \text{metric tons of graphite}$$

↙ silly being again ☺

← useful

4. Reed Problem 5.4, p. 235
Uranium Enrichment

(a) Build a table

Enrichment Round	Concentration
0	0.00720
1	0.00792
2	0.00871
3	0.00958
4	0.01054
5	0.01160
6	0.01276

(b) The idea of building the table was (i) to start seeing how painfully slow repeated enrichment would be, and (ii) to see the pattern:

$$\boxed{\begin{aligned} r &= 1.1 \\ f_0 &= 0.00720 \end{aligned}}$$

0	f_0
1	rf_0
2	$r^2 f_0$
3	$r^3 f_0$
⋮	
i	$r^i f_0$

$$\boxed{f = r^i f_0}$$

(c) If you see the pattern

$$f = r^i f_0 \quad \text{or} \quad \frac{f}{f_0} = r^i$$

Now comes the fancy algebra. Take $\log_{10}$ of both sides, and use

$$\log_{10} r^i = i \log_{10} r$$

$$\log_{10} \frac{f}{f_0} = i \log_{10} r \quad \text{or} \quad i = \frac{\log_{10} \frac{f}{f_0}}{\log_{10} r}$$

To find the needed number of enrichment rounds, put in the needed final concentration:

$$\boxed{\begin{aligned} f_{final} &= 0.9 \\ f_0 &= 0.0072 \\ r &= 1.1 \end{aligned}}$$

$$n = \frac{\log_{10} \frac{f_{final}}{f_0}}{\log_{10} r} = \frac{\log_{10} \frac{0.9}{0.0072}}{\log_{10} 1.1} = 50.65 = 51 \text{ enrichment rounds}$$

Note: It doesn't matter what logarithm base you use as long as you are consitent. You could use the natural log ($\ln$) if you preferred.

Another note: You can even use $\log_{1.1} x$ if your calculator supports logarithms with arbitrary base. In that case, because $\log_{1.1} 1.1 = 1$ the denominator in the expression for n goes away, leaving just $\log_{1.1} \frac{0.9}{0.0072}$.