Manhattan Project — Assignment 10 — Solution

1. Kinetic Energy at the Core of the Sun

(a) There is a formula that relates average kinetic energy to temperature and it is

$k_B T = \frac{3}{2} m v^2$

What appears as v^2 on the right-hand side is not the <average velocity> squared, but the average <velocity squared>. Anyway, what is $k_B T$ in the interior of the Sun? Use $k_B = 1.4 \times 10^{-23}$ J/K and T at the center of the sun of 15,000,000K.

ANSWER: $k_B T = 1.4 \times 10^{-23}\ \text{J/K} * 15,000,000\ \text{K} = 2.1 \times 10^{-16}\ \text{J}$

(b) ANSWER: $E_{\text{coulomb barrier}} = k \frac{q_1 q_2}{d} = 9 \times 10^9\ \text{J} \cdot \text{m}/\text{C}^2 * (1.6 \times 10^{-19}\ \text{C})^2 / (2 \times 10^{-15}\ \text{m}) = 1.15 \times 10^{-13}$

DISCUSSION: So the Coulomb barrier is about 500 times higher than the average kinetic energy. However, only a small fraction of the collisions have to result in a fusion.

2. Coal Powers the Sun — Not!

The Sun weighs 2×10^{30} kg.

(a) 1 kg of coal releases 30MJ when burnt. If the Sun were made entirely of coal, how many Joules would be released by burning it all?

ANSWER: $2 * 10^{30} * 30 * 10^6\ \text{J} = 60 * 10^{36}\ \text{J} = 6 * 10^{37}\ \text{J}$

(b) The Sun produces energy at the rate of 3.8×10^{26} Watts. At this rate of energy production, if it were burning coal, how many seconds will it take to burn all of its coal?

ANSWER: $6 \times 10^{37}\ \text{J} / (3.8 \times 10^{26}\ \text{J/s}) = 1.6 * 10^{11}\text{s}$

(c) Convert your answer to (b) to years.

ANSWER: $1.6 * 10^{11}$s / (24 * 60 * 60 * 365 seconds / year)=5100 years.

DISCUSSION: This is actually quite close to the estimate in Genesis!

3. Fusion Powers the Sun

(a) ANSWER: 10% * 2 * 10^{30} ∗ 1000 ∗ 6.02 ∗ 10^{23} = 1.2 ∗ 10^{56} Hydrogens

(b) ANSWER: 3 * 10^{55} Heliums will be produced

(c) ANSWER: 4 * 1.00784 - 4.00260 = 0.02876 u

(d) ANSWER: 0.02876 u * 1.66 x 10^{-27} kg/u * $(3.00 \times 10^{8}\ \text{m/s})^2$ = 4.3 ∗ 10^{-12} J

(e) 3 * 10^{55} Heliums ∗ 4.3 ∗ 10^{-12} J / Helium = 13 * 10^{43} J = 1.3 ∗ 10^{44} J

(f) 1.3 * 10^{44} / $(3.8 \times 10^{26}$ J/s) = 0.34 ∗ 10^{18} s = 3.4 ∗ 10^{17} s

(g)3.4 * 10^{17}s / (24 * 60 * 60 * 365 seconds / year) = 10,800,000,000 years

Subtracting 4.5 billion years from 10.8 billion years, we have 6.3 billion years before the Sun runs out of fuel.