Manhattan Project — Assignment 10 — Thermonuclear Fusion

This problem set is one last chance to use the formulae you learned early on. This time to understand fusion a little bit better. Fusion is completely different from fission. It makes heavier elements out of light elements instead of breaking up heavy elements.

Importantly, you don't need a chain reaction of neutrons (as in a fission bomb)—let alone slowed neutrons (as in a fission reactor). You just need ludicrously high temperatures. Fusion bombs are called "thermonuclear bombs." You get the ludicrously high temperatures by setting off a fission bomb next to the light elements. Deuterium, tritium, and lithium are the light elements that are used in practice. There are no practical fusion reactors. The high temperatures have (so far) proven to be completely impractical to contain.

The most interesting example of fusion in our life is the Sun, not thermonuclear bombs. Well, maybe thermonuclear bombs are interesting, but they certainly aren't cheery. So these final problems are about our cheery Sun.

It will take you much less time to complete this problem set than it took me to write it. I hope you enjoy the results!

1. Kinetic Energy at the Core of the Sun

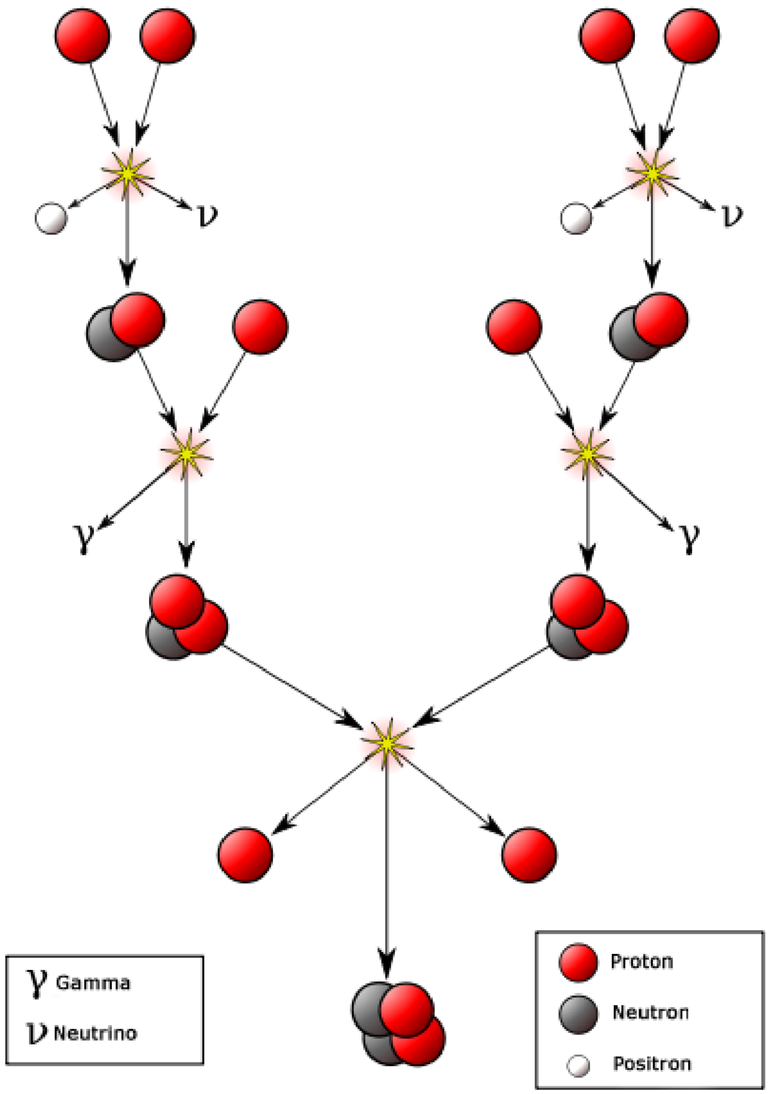
The temperature at the core of the Sun is pretty well known based on the neutrinos escaping the core of the Sun and coming directly to Earth. The gamma rays and kinetic energy produced at the core of the Sun takes something like 170,000 years to get to the surface of the Sun, but the neutrinos escape freely. The temperature of the core of the Sun is also pretty well known from theoretical models.

(a) There is a formula that relates average kinetic energy to temperature and it is

$$k_B T = \frac{3}{2} m v^2$$

What appears as v^2 on the right-hand side is not the <average velocity> squared, but the average <velocity squared>. Anyway, what is $k_B T$ in the interior of the Sun? Use $k_B = 1.4 \times 10^{-23}$ J/K and T at the center of the sun of 15,000,000K.

(b) For thermonuclear fusion to occur protons and neutrons in the form of Hydrogen nuclei and Deuterium nuclei have to collide hard enough to form Helium. There are actually several steps to this process. A diagram is on the next page.



Let's just look at the first step. If the radius of a proton is 1 fm, so colliding them requires bringing their centers 2 fm apart, and their charge on each proton is $q_1 = q_2 = 1.6 \times 10^{-19}$ C, what is

$$E_{\text{coulomb barrier}} = k\,\frac{q_1\,q_2}{d}$$

You will need the Coulomb constant, $k = 9 \times 10^9\ \text{J}\cdot\text{m}/\text{C}^2$

DISCUSSION: The kinetic energy you got in part (a) is smaller than the Coulomb barrier energy you got in part (b). You might think this means that the interior of the Sun isn't hot enough to cause fusion. However, it is, and this isn't actually a surprise. The upshot is that only a small fraction of the collisions have to result in fusions.

2. Coal Powers the Sun — Not!

The Sun weighs 2×10^{30} kg.

(a) 1 kg of coal releases 30MJ when burnt. If the Sun were made entirely of coal, how many Joules would be released by burning it all?

(b) The Sun produces energy at the rate of 3.8×10^{26} Watts. At this rate of energy production, if it were burning coal, how many seconds will it take to burn all of its coal?

(c) Convert your answer to (b) to years.

DISCUSSION: This is actually quite close to the estimate in Genesis!

3. Fusion Powers the Sun

(a) Assume that in its lifetime that just 10% of the Hydrogen in the Sun can be fused to make Helium. It takes four Hydrogens to make one Helium. How many Hydrogens will be fused in the Sun's lifetime? Use that 1 gram contains 6.02×10^{23} Hydrogens. Don't forget to convert grams to kg.

(b) How many Heliums will be produced? (Just divide by 4.) (As a checkpoint, I got 3×10^{55} Heliums.)

(c) Ignore the fact that fusion in the Sun is the multi-step process in the diagram on the previous page. Just pretend the reaction is:

$4 \text{ Hydrogen} \longrightarrow 1 \text{ Helium}$

The atomic weight of Hydrogen is 1.00784 u. The atomic weight of Helium is 4.00260 u. So how many atomic mass units are turned into energy when four Hydrogens are fused to make 1 Helium?

(d) Convert your answer to (c) to Joules using $E = mc^2$, 1 u = 1.66×10^{-27} kg, and $c = 3.00 \times 10^8$ m/s.

(e) Multiply your answer to (d) by the number of Heliums that the Sun will convert in its lifetime. This is (very roughly) how much energy the Sun can produce in its lifetime. (As a checkpoint, I got 1.3×10^{44} J.)

(f) Again using that the Sun is producing energy at the rate of 3.8×10^{26} Watts, how many seconds can the Sun last given your answer to part (e)?

(g) Convert your answer to (f) to billions of years. The Earth and Sun are 4.5 billion years old. How many more billion years do we have left before our Sun runs out of fuel?