Manhattan Project Midterm

Feb. 21, 2023 — Covering Reed through Chapter 5

Five Problems worth 4 points each, plus 2 points of extra credit, makes a maximum of 20 points.

Radioactivity, Half-Life

1. Iodine-131 in Radioactive Fallout

CONTEXT: Iodine-131 is one of the fission products of Uranium. If ingested in radioactive fallout, it is taken up by the thyroid, because that's what the thyroid does: it takes in iodine and produces iodine-containing thyroid hormones. Unfortunately, Iodine-131 is itself radioactive. It decays by β^- decay with a half-life of 8 days and these β-decays damage thyroid cells.

That's just context. Here is the problem:

(a) Complete the following reaction:

$$^{131}_{53}\mathrm{I} \xrightarrow[\substack{\beta^- \\ \text{8 days}}]{} {}^{A}_{Z}Q + \text{electron}$$

I just want you to answer with the numbers Z and A:

(b) The element Q above is Xenon, and the abbreviation is actually Xe. In 32 days (to make a nice round multiple of 8 days), if your thyroid started with 1600 Iodine-131 atoms, how many of those Iodine-131 atoms would be left at the end of those 32 days?

(c) Also, how many beta rays (electrons) was your thyroid bombarded with in this time?

SUGGESTION: If you aren't confident of your answer, it might be advisable to double-check it by making a table. How many are left after 8 days? How many after 16 days? How many after 24 days? And finally, how many are left after 32 days?

Cross Sections

2. Classical Cross Section of the Uranium-238 Nucleus

On Reed p. 50, Eq. 2.25 is a formula for the radius of a nucleus:

$r = a_0 A^{1/3}$

a_0 is some number that we will plug in later. A is the nucleon number of the nucleus.

(a) If a spherical nucleus has radius r, and you look at it from any angle, it appears to have the same area as if it were a disk. The area of a disk is,

$\sigma = \pi r^2$

Combine this formula for σ with the formula above for r to get the “cross section” of a nucleus. Don’t plug in yet. That will come at the next step.

(b) Plug in $A = 238$ and $a_0 = 1.2 \times 10^{-15}$ m to get σ for Uranium-238.

(c) You probably answered (b) in square meters. Nuclear physicists often use Fermis instead of meters for tiny lengths. Convert to ***square*** Fermis using 1 Fm = 10^{-15} m.

(d) Nuclear physicists also use barns. Convert your answer to (b) to barns using 1 barn ≡ 10^{-28} m^2.

DISCUSSION: Imagining that the nucleus presents itself to an oncoming neutron as a classical disk is hopelessly crude. It is a complex quantum-mechanical object whose properties are too complex to calculate. In the next problem, we deal with the actually-measured, quantum-mechanical cross sections.

Quantum-Mechanical Cross Sections

3. Relative Cross Sections — Fast Neutrons

On Assignment 4, we looked at the thermal neutron cross section for Uranium nuclei. Thermal neutrons have energy $10^{-7.6}$ MeV, so that put us at -7.6 on the horizontal axis in Figs. 3.11 and 3.12 on p. 100 of Reed.

In this problem, you will do the same thing, but for **fast neutrons**. For simplicity, let’s say a fast neutron is a neutron with 1 MeV of energy. 1 MeV = 10^{0} MeV, **so that puts us at 0 on the horizontal axes** of Figs. 3.11 and 3.12.

(a) Use a straight edge on Fig. 3.11 to get an accurate estimate of the log of the cross section in barns for fast neutrons to be captured by U-238.

(b) Take $10^{\text{whatever you got in Part (a)}}$ to find the cross section for capture in barns.

(c) Use a straight edge on Fig. 3.12 to get an accurate estimate of the log of the cross section in barns for fast neutrons to cause fission of U-235.

(d) Take $10^{\text{whatever you got in Part (c)}}$ to find the cross section for fission in barns.

(e) In unenriched uranium, there are about 140 Uranium-238 atoms for every Uranium-235 atom. So if fast neutrons are bouncing around in an unenriched sample, the fraction of the neutrons that will cause Uranium-235 fissions is:

$$\frac{\text{what you got in d} * 1}{\text{what you got in d} * 1 + \text{what you got in b} * 140}$$

Compute this ratio.

Molarity, Energy Released

4. Energy of a Critical Mass of Plutonium-239

Pure Plutonium-239, has atomic weight 239.05216 grams per mol. Let’s just round to 239 grams per mol.

(a) One critical mass of Plutonium-239 is about 10kg. How many mols of Plutonium-239 are in a critical mass?

(b) Knowing that Avogadro’s number, N_A, is about 6×10^{23} atoms/mol, how many atoms of Plutonium-239 are in a critical mass?

(c) If only 10% of these atoms fission during the explosion, how many of these atoms fission?

(d) If each fission of Plutonium-239 releases about 3×10^{-11}J, how many Joules do you expect a Plutonium bomb to release?

(e) The conversion between Joules and kilotons (equivalent) of TNT is quoted on p. 111 of Reed. Convert your answer to (d) to kilotons.

Mass Defect, $E = mc^2$, Power

5. Fusion in the Sun

CONTEXT: In the core of the Sun, 600 million tons of Hydrogen are fused to Helium every second. The basic process is that four Hydrogens, ${}_1^1H$, fuse to make one Helium, ${}_2^4He$.

That's just context. Here is the problem:

(a) Hydrogen has atomic mass of 1.008 grams/mol. Helium has atomic mass of 4.003 grams per mol. If four mols of Hydrogen fuse to make one mol of Helium, how many grams disappear into energy?

(b) Convert your answer to Part (a) from grams to kg, and while you are at it, round your result to one sig fig. Use scientific notation.

(c) Multiply your answer to (b) by the speed of light **squared** to get the energy released. The speed of light is 3×10^8 m/s. Thanks to the incredible convenience of MKS units, your result is in Joules, with no conversion factors needed.

(d) The Sun makes about 1.5×10^{14} mols of Helium per second. Combine that fact with your answer to (c) to get the rate of energy production in the Sun. REMINDER: Energy rates in MKS units are in Joules/second and an energy rate is called power, and the unit for power is so popular it has its own unit, the Watt, so report your answer in Watts (abbreviated W). Round to 1 sig fig.

Extra Credit

6. You Design a History Question

(I will share your answers.)

We have now studied a period of sweeping change from 1895, when Becquerel discovered natural radioactivity, to 1942, when the U.S. has started an ambitious wartime project to create two types of fission bombs, one using Uranium-235 and the other using Plutonium-239.

(a) In a few sentences, what stands out to you as the most significant development in this period? Is there one that made your jaw drop? Maybe not. I hope there is.

(b) If you were to add a history question for this exam, what would it have been?