Mathematical Analysis

The Foundation of Calculus

PRELIMINARY SYLLABUS

Unofficial/Short Course Title: Mathematical Analysis

Spring 2025, Deep Springs College, ***Prof. Brian Hill***

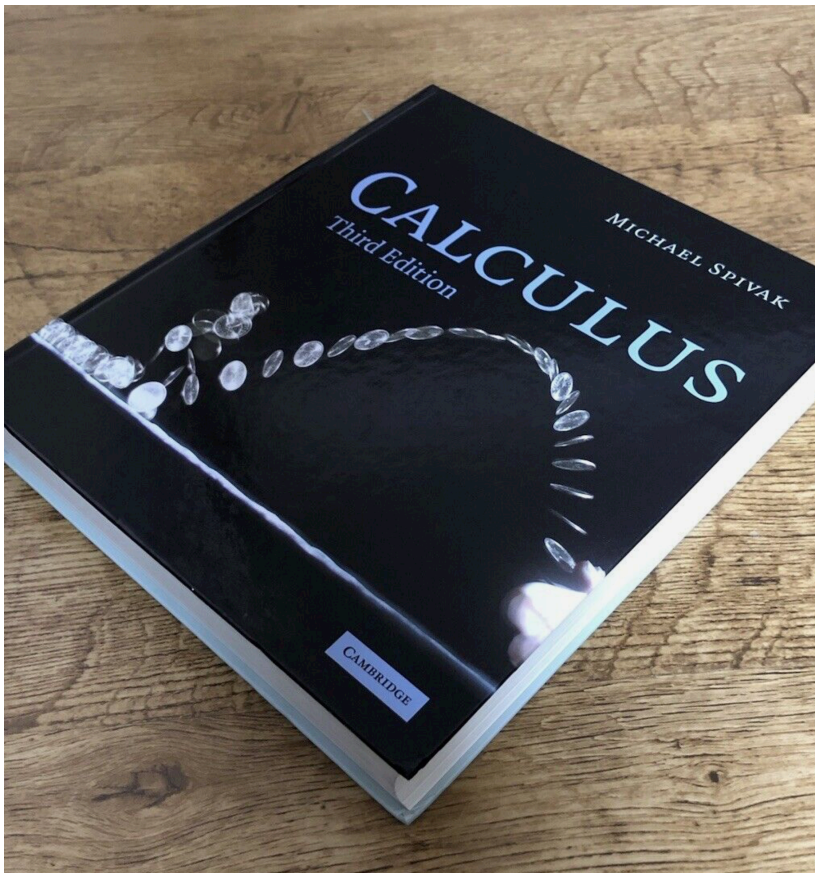
Daily Schedules (Kept Retrospectively)

- **Daily Schedule Term 4**

Required Text

Michael Spivak, Calculus, 3rd Edition, Cambridge University Press, 2006 (Amazon link)

Because there are a lot of editions, reprints, and knock-offs, to confirm that you are buying the correct edition from Cambridge University Press, check that what you are buying looks like:



The Mathematical Association of America review begins, "This is the best Calculus textbook ever written." Need more be said? Anyway, this is the principal text for the course, but we will be using it mostly for its Parts I and II (its first eight chapters). This book's Part III can stand alone as the basis of a standard course in derivative and integral calculus. However our topic is the foundation of calculus, not calculus itself, and we will only get to Chapter 9 in Part III, which introduces the derivative.

Overview

There is a wealth of material that logically fits in between standard high school mathematics and calculus. This material has been deemed too hard by most schools — both those with college prep tracks and with colleges themselves. It is simply skipped! Only those that continue on to the junior level as mathematics majors get to see it, and they see it in a course that is often titled "Introduction to Analysis" or "Introduction to Mathematical Reasoning." It is perfect for the Deep Springs curriculum insofar as it requires no prerequisites beyond very good high school mathematics, and yet it is both more advanced and provocative than what is generally found in lower-division math courses.

The material is called "mathematical analysis" or "real analysis" and it includes many theorems that are absolutely essential to calculus but are typically used as if they are obvious and without proof. What students usually move on to in their lower division courses, with the time freed up by skipping the foundations of calculus are all sorts of pragmatic results — like the chain rule as a method for simplifying derivatives, or integration by parts as a common trick for doing integrals, and even more mundane rules.

One might think that with only so much time to either obtain and apply practical results or to do mathematical proofs, that one should spend one's time on pedestrian problems. Michael Spivak, renowned for textbooks in mathematics ranging from the introductory level to textbooks used by mathematics graduate students, disagrees: "calculus ought to be the place in which to expect, rather than avoid, the strengthening of insight with logic." He goes on: "precision and rigor are neither deterrents to intuition, nor ends in themselves, but the natural medium in which to formulate and think about mathematical questions."

A course in the topics of mathematical analysis that undergird calculus prepares one to think rigorously, to understand real analysis the way mathematicians do, and to understand fundamental theorems that are beautiful and significant in their own right. As a treat we will even go beyond the confines of the real numbers and in our final unit introduce imaginary numbers (also known as complex numbers). This will allow us to conclude with the 1799 proof by Gauss of The Fundamental Theorem of Algebra.

Grading

Note: Part of the reason this syllabus is preliminary is that I would like to discuss with you, once we get about half way through the course, what you would like to do for the final unit.

- 40% assignments
- 50% total for three or four exams, dates to be determined, but coming approximately evenly through the course
- 10% preparation for class and leadership of course

Problem Sets / Handouts / Exams

There will be problem sets due at least every week, and sometimes every class, limited only by how quickly I can assign, write solutions, and grade. The more problems you do the better.

In addition to the problem sets and their solutions, there will be handouts, exams, and exam solutions to file. Locate a three-ring binder and a three-ring hole punch, and file everything chronologically. Actually, reverse-chronological is the most convenient, because you then naturally open your binder to what you are currently working on.

Problem sets should be *neat* and on standard 8 1/2 x 11 paper. Multi-page problem sets — and most will be multi-page — should be stapled. The nicest technical work is facilitated by engineering pads, such as these **Roaring Spring Engineering Pads at Amazon** (which are pretty expensive unless you buy by the case), and done with a mechanical pencil, a ruler, and an eraser at hand.

Absences (and late work)

The College's policies on absences (and late work) are applicable. Refer to the Academic Year 2024-2025 Deep Springs Handbook.