Continuity Implies Uniform Continuity

Of course this only applies on a closed interval

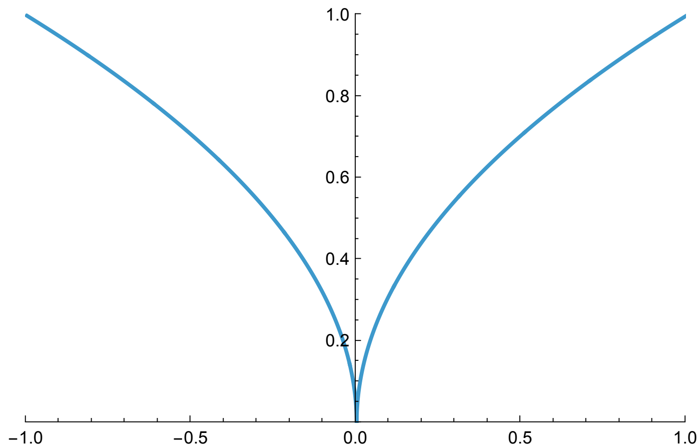
Theorem 1 on p. 143 is kind of a shock: If f is continuous on $[a, b]$ then f is uniformly continuous on $[a, b]$.

Couldn't the function get steeper and steeper?

Consider the function $f[x] = \sqrt{|x|}$

In[ ]:=
```
Plot[Sqrt[Abs[x]], {x, -1, 1}, PlotRange → {{-1, 1}, {0, 1}}, PlotPoints → 1000]
```

Out[ ]=



This function gets arbitrarily steep near 0. Therefore an ϵ that works for a given δ near say $x = \frac{1}{2}$ isn't going to work at $x = \frac{1}{4}$ or $x = \frac{1}{8}$ because at some point, that δ is just not small enough.

Isn't this a counter-example to Theorem 1 on p. 143?

The answer is no or it wouldn't be a theorem. For definiteness, consider the interval $[0, 1]$.

The trick is to go straight to the worst point, where the function is steepest, which is at $x = 0$. Whatever δ works there works at all the places where the function is less steep.

The δ that works there, we already have found in prior problem sets, it is $\delta = \epsilon^2$.