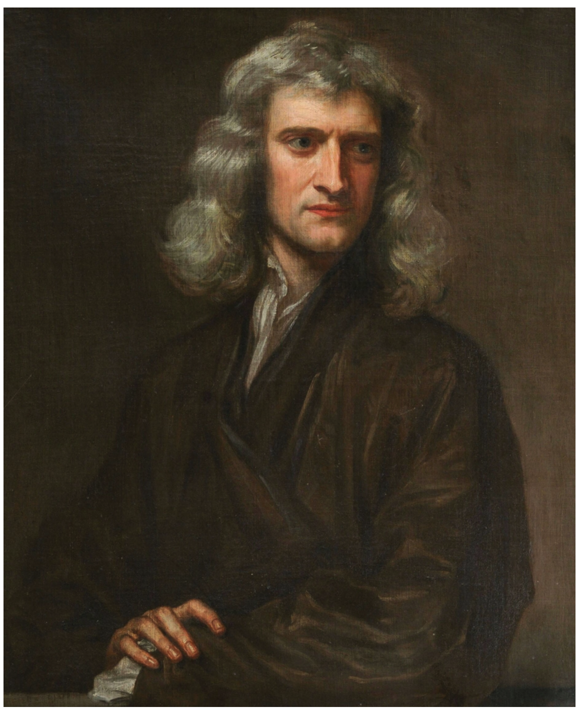
Newton and Leibniz

For context, Galileo's and Kepler's work comes early in the 1600s and the work of Newton and Leibniz comes late in the 1600s.

Newton

Newton's working style was that of the isolated genius, keeping notebooks from the time he was young filled with questions and progress he was making on those questions. Newton was poor and his father died before his birth. Things that might seem basic, like paper and candles by which to work, he had little of. He wrote very small and carefully to conserve paper.

Newton was born in 1642. Some of his most brilliant progress dates to 1665-1666 when the Great Plague (the same horrid bubonic plague that repeatedly ravaged many European cities in medieval times) was working its way around London.



Instead of spending all the downtime doom-scrolling and biting his nails, as we and most of our contemporaries did when a comparatively minor plague was working its way around the world, Newton retreated to a country manor and devoted himself to optics, calculus, gravitation, and what we now know as his three laws of motion — the second of which is probably the most famous equation in physics, $F = ma$, although Einstein's $E = mc^2$ has an equally good claim to being the most famous.

Working as an isolated genius with no contemporaries even approaching his caliber, Newton didn't bother to publish much. His mission was to build up his own understanding. Late in life he likened his endeavors to a child picking up shells at the seashore and examining them, and all in all, knowing very little of what was in total out there to be known. He occasionally corresponded or met with other scientists — not a lot, but enough that his breakthroughs from 1666 were surely being discussed by other scientists in England and France in the years before he finally published.

Not until 1687 does he finally bundle up the bulk of his work on gravitation and the three laws of motion into the book we call the *Principia*. Its full name is *Philosophiæ Naturalis Principia Mathematica* which translates to "Mathematical Principles of Natural Philosophy." It is a brilliant, sweeping, tightly argued, and systematic account of earthly gravitation and lunar, tidal, and planetary motion. It lays out his philosophical standards and approach, executes to his high standards upon that approach, and then concludes with questions that we still wonder about, even after the advances of Einstein two centuries later.

Newton develops calculus in order to solve the problems of planetary motion that he is concerned with. You would not recognize it as calculus, even though it most definitely is. It takes multiple months of study to see what Newton has done, appreciate its rigor, and recognize that calculus is within it. Later I will get to the differences between his calculus and modern calculus.

Leibniz

Leibniz publishes in 1686. By this point, although Newton still hasn't published, his ideas have been discussed for 20 years. It is extraordinarily hard for me to imagine as a scientist that has personally experienced how buzz about new ideas travels through a scientific community, that Leibniz did not have some exposure to Newton's ideas, even though Leibniz was in Paris and there were no telephones or telegraphs to quickly relay messages.

Now buzz travels very fast and is part of why we have such a modern obsession on submitting papers to journals. If you don't establish priority, the buzz will spread in days or weeks, and someone else will inevitably and quickly claim the idea. If you have been exposed to an idea, and you don't immediately understand it, but you mull it over, you can very, very easily come to the conclusion (after waking up one morning with clarity) that you had the idea.

Nonetheless the prevailing belief is that Leibniz discovered calculus independently of Newton, and nowadays, both are given credit, with the above highly plausible propagation of Newton's ideas generally ignored because there isn't sufficient specific evidence of it. Perhaps it is the "go along to get along" mentality that causes this.

I have seen the "go along to get along" mentality in many other discoveries. We have the Robertson-Walker metric, the Friedmann-Robertson-Walker metric, and nowadays the same thing is clumsily known as the Friedmann–Lemaître–Robertson–Walker metric. Why? Because it is socially and politically much easier to tack on more names than it is to strenuously argue who deserves the credit. Those that disagree with such a cynical view would point out, correctly, that often an idea goes through multiple iterations and formulations before coming into its most general and most compelling form. But this should not stop us from shirking from the question of who really made the biggest breakthrough. All of the above metrics describe the same thing — the expanding universe following the Big Bang — so the argument over the name of the metric (which you may have never heard of) is the proxy for the very substantial argument over who anticipated and gave the compelling and rigorous explanation of what Hubble observed experimentally in 1929.

Coming back to calculus, one related issue is that Leibniz's notation prevailed over Newton's. Many people, more often scientists than mathematicians, still use Leibniz notation today, although Spivak gives us a fine rant on why he doesn't use it. This brings me to the last point.

Newton's Calculus

You would not recognize Newton's calculus as calculus. It certainly is, but his language and methods are so different from what we now use that it is on the surface unrecognizable. The reasons are five-fold:

(1) Newton only deals in proportional relations. He never says "the second derivative of this curve is this," and then writes down a formula or number. Instead he says "the curvature of this curve is propor-

tional to the curvature of that curve" and then he proves and states the proportionality. It is as if he believes that curvature cannot be quantified. Instead, it can only be compared with other curvatures. This way of thinking (about scientific results as only being phrased in terms of proportionalities) pervades the science of the 17th century, even when speaking of more basic things, like areas and volumes, which we have no trouble quantifying, and even have a hard time understanding why anyone would only be able to speak them in terms of ratios.

(2) Newton does not use any notation that has survived to the present. Indeed, he mostly makes precise verbal arguments about what he is deriving, rather than assembling chains of equations as we typically do nowadays.

(3) Newton does not know about delta-epsilon proofs and limits (and neither does Leibniz). Both Newton and Leibniz deal with something we now call "infinitesimals" (Newton called them "fluxions"), and which we now have discarded as non-rigorous. An infinitesimal was a number that wasn't zero, but which was smaller than ***any*** ordinary number. It isn't until 1865 that Weierstrass brings us a version of calculus that is nowadays recognized as rigorous, and doesn't require infinitesimals. However, the whole machinery of limits is created and required by Weierstrass.

(4) Newton certainly knows about first derivatives and rates of change, but his focus is on second derivatives and curvature. That is what is at stake in the problems he is solving. It is not the straight line motion of a particle that he is interested in. He is interested in deflection, acceleration, and deceleration. These all require second derivative ideas (second derivatives are derivatives of derivatives). We will get to second derivatives, but we are carefully building up the ideas from first derivatives. Having read a bunch of the *Principia* myself, I can say that it feels as if Newton takes first derivatives for granted, and doesn't bother with pedantically introducing a theory of slopes and rates of change before launching into studies of curvature.

(5) Newton never uses Cartesian (x and y) coordinates. He instead makes geometric drawings. These drawings are precise enough to support calculations. They show all the relevant arcs, triangles, line segments, and lengths and their relationships (and as noted above, the relationships are very often phrased as proportionalities). But Newton never sets up a Cartesian coordinate system or draws any coordinate axes. It is kind of miraculous — given that nowadays we incessantly set down Cartesian, polar, or spherical polar coordinate systems before even starting on a problem — how much Newton gets done with geometrical drawings that look like the drawings of Euclid or Apollonius. When imagining how God thinks — if you believe there is a God commanding the motions of the planets — I can tell you, He certainly does not use the pathetic crutch of first laying down an ***arbitrary*** system of Cartesian coordinates. God surely thinks in a way that is free of coordinates, as Newton does, and indeed as modern general relativists think. But we peons depend on concrete coordinate systems, and calculus is always taught with x and y axes (or t and x axes) as foundational parts of its development (and multivariable calculus describing the three-dimensional world almost always begins by setting down t, x, y, and z axes and we do this despite the fact that all four of these axes are arbitrary).

Conclusions

Leibniz vs. Newton

Well, take all the preceding with a grain of salt. I am not a historian. However, I have been a direct observer of and participant in the behavior of a modern scientific community (the particle physics crowd in the 1980s and 1990s), and as such, I have my own viscerally-felt ideas of what is a plausible progression of scientific ideas in a community. I have also read enough Newton to know that he had no contemporary of his caliber in physics, and that even his bitter attacks were quite plausible and rational. Meanwhile the best 20th century physicists (e.g., Einstein) remain in awe and in gratitude of Newton's legacy. Einstein wrote (in his Foreword to Newton's *Opticks*):

Nature to [Newton] was an open book, whose letters he could read without effort. The conceptions which he used to reduce the material of existence to order seemed to flow spontaneously from experience itself, from the beautiful experiments which he ranged in order like playthings and describes with an affectionate wealth of detail. On one person he combined the experimenter, the theorist, the mechanic and, not least, the artist in exposition.

So perhaps the better question, rather than who invented calculus — Leibniz or Newton, or did they do it independently — is which of them had the better hair? Consult the portraits above and decide.

Our Class in Mathematical Analysis

And to bring this all back around to what we our class: we simply do not have time to follow a historical development, with its awkward ideas, its disputes over priority, and sometimes its entirely wrong turns. Instead, each generation of mathematicians and physicists digests, refines, and passes on what has gone before to the next generation.

We are learning the calculus as formulated by Weierstrass and digested for us by Spivak. It is the modern edifice which we study, admire, and attempt to add to, and few of us have the luxury of wading through 17th century documents to directly appreciate the geniuses of earlier centuries, which is a little sad, because it means we don't have time to get in touch with our scientific roots, just as you have not likely researched anything about your great-grandparents, and yet your existence depends entirely upon theirs.

As our education and life proceed, we encounter on a weekly or monthly basis yet more fields of research, any of which it would probably take us many years to meaningfully study. We have to pick and choose from among an embarrassingly rich set of subjects, and embrace the few that we choose, appreciate, and can develop ability for. Calculus in the formulation given to us in the second half of the 19th century, is a subject which is unusually widely studied because of its broad utility. Indeed, it is a gateway to almost every other field of scientific study, to most every field of engineering, and to all of the quantitative social sciences. It is also a beautiful edifice in its own right.

Most Classes in Calculus

As one final opinionated rant, I will add, that another thing that is sad is that nowadays calculus is very widely taught as a collection of useful formulas and rules. All derivation from first principles as we have been spending the better part of a semester on, is omitted. This is pragmatic. Few people end up finding the rigor beautiful. But I think it would be worse if we didn't even try to understand the foundation of calculus (which is mathematical analysis). In the remainder of our course (especially Chapter 10), we will finally start into some of the useful rules that other courses begin with. These go by the names of chain rule and product rule (and others), and are piled upon with rules applicable to specific functions like the logarithm or the cosine.