Mathematical Analysis Problem Set 9

Reading for Thursday, May 20, 2021

Chapter 7, pp. 120 through the proof of Theorem 8 at the top of p. 123.

Comments: Theorems 1-3 are the “three hard theorems.” Try to appreciate how reasonable the results are. How could continuous functions behave anything but this way? But also appreciate that we have no way to prove them!

More comments: Theorems 4-7 are easy extensions of Theorems 1-3. The proofs are short. But keep in mind, we still don’t have Theorems 1-3, so we haven’t really yet proved Theorems 4-7. Then Theorem 8, is a bolt from the blue. It is so easy now to prove that there must be some positive x such that $x^2 - c = 0$ has a solution if $c > 0$. But we needed Theorem 4, which in turn needed Theorem 1, and we still can’t prove Theorem 1. After you have read and thought about Theorems 1-8, go back to Chapter 6, and take another look at Theorem 3 and its proof on p. 117.

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1 Chapter 6, Problem 1.

Comments: First make it very clear what the domain of f is for each part. For example, for Part (iii), the domain of f is the irrational numbers. So the problem for this part is to find a function F whose domain is all the real numbers that agrees with f in f’s domain, and it has to be continuous. Or if no such function can be found, then you state that. For Part (iv) you might be happy to graph f and then to graph F.

2 Chapter 6, Problem 2, but ignore the functions of problem 4-19.

3 Chapter 6, Problem 8

Comment: I think this follows straightforwardly from Chapter 6, Theorem 3.

4 Chapter 6, Problem 13

Comments: 13(a) seems a bit dull. It says that a function that is defined only on [a, b] and is continuous on that interval can be extended to be continuous on all the reals. However, 13(b) is so close to the same thing except the closed interval [a, b] is replaced by the open interval (a, b). Then you can’t necessarily extend the function to be continuous on all the reals. Spivak asks for a counterexample (an example to show that the assertion is false for the open interval.