Mathematical Analysis Problem Set 13

Reading for Monday, June 14th

Continue with the complex numbers in Courant & Hilbert to the end of Section II.5.2.

Problem Set 13 for Monday, June 14, 2021

From Section II.5.1

Exercises 1, 2, 3, and 4 on p. 91.

Comments on the Section II.5.1 Exercises

A couple of the exponents are impossible to read. In Exercise 2, it should read:

$$\left(-\frac{1}{2} + i\,\frac{\sqrt{3}}{2}\right)^3$$

In Exercise 3, the third expression is $\frac{1}{i^5}$ and the last expression is $\frac{(4-5\,i)^2}{(2-3\,i)^2}$.

From Section II.5.2

Exercises 1 and 2 on pp. 93-94, and exercises 1, 2, 3, 5, and 6 on p. 97

Comments on the Section II.5.2 Exercises

In Exercise 3, by "absolute value," Courant & Hilbert mean the modulus, defined on p. 93.

For Exercise 6, this is easy if you start with Equation (8) on p. 95. You use $z_1 = \rho_1(\cos\phi_1 + i\sin\phi_1)$ and $z_2 = \rho_2(\cos\phi_1 + i\sin\phi_1)$, and simplify.

One More Important Problem is on the Reverse

A Problem on e^x, $\sin x$, and $\cos x$

One way of defining e^x is by:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots$$

Write out e^{iy} and separate your answer into a real part and imaginary part (*i.e.,* into the form $a + bi$).

By "write out" e^{iy}, I mean that you put put $x = iy$ into the formula for e^x and go crazy simplifying until you see the pattern. You can assume y is real.

One way of defining $\cos x$ and $\sin x$ is by:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

These expressions only work if x is in radians. They would have no end of factors of $\frac{2\pi}{360}$ in them if x were in degrees. Icchh.

Use the expressions for $\cos y$ and $\sin y$ to simplify your expression for e^{iy}. The simplification should be quick and dramatic. If it isn't, go back and check your your expression for e^{iy}.