Newton — Problem Set 1 — Galileo's Uniform Acceleration

Due Thursday, Sep. 8 (beginning of class)

1. Can Speed be Proportional to Distance, rather than Time?

In the definition of uniform accelerated motion, Sagredo, who represents a bright colleague of Galileo, suggests that "speed could be proportional to distance traversed," rather than in proportion to the time elapsed. Sagredo's suggestion is on p. 167. Salviati, who represents Galileo, is comforted to have a companion in this error, which he now recognizes as a fallacy. Simplicio, the buffoon representing the establishment scholars, not only accepts the proposition, but says it "ought to be conceded without hesitation or controversy."

Salviati ignores Simplicio and then says something even stronger: he not only claims it is not how nature is, he claims it is an impossible type of motion (p. 168). Galileo does not have calculus at his disposal. We do, and we know the solution to the differential equation $v = \alpha x$ exists and is x(t) = $x(0)\, e^{\alpha t}$. Below I have graphed this function for the time range 0 seconds to 1 second, with $x_0 = 0.1$ meters, and $\alpha = 3$/second. Is there anything completely unreasonable about this solution? In other words, is Galileo wrong in saying it is an *a priori* impossible type of motion, and not merely not how nature is? I suspect this could be argued either way. Say something clear and compelling that you believe. No matter what, your answer should reference this plot:

In[∘]:=
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Plot[x0 Exp[α t] /. {x0 → 0.1, α → 3} , {t, 0, 1}, GridLines → Automatic]
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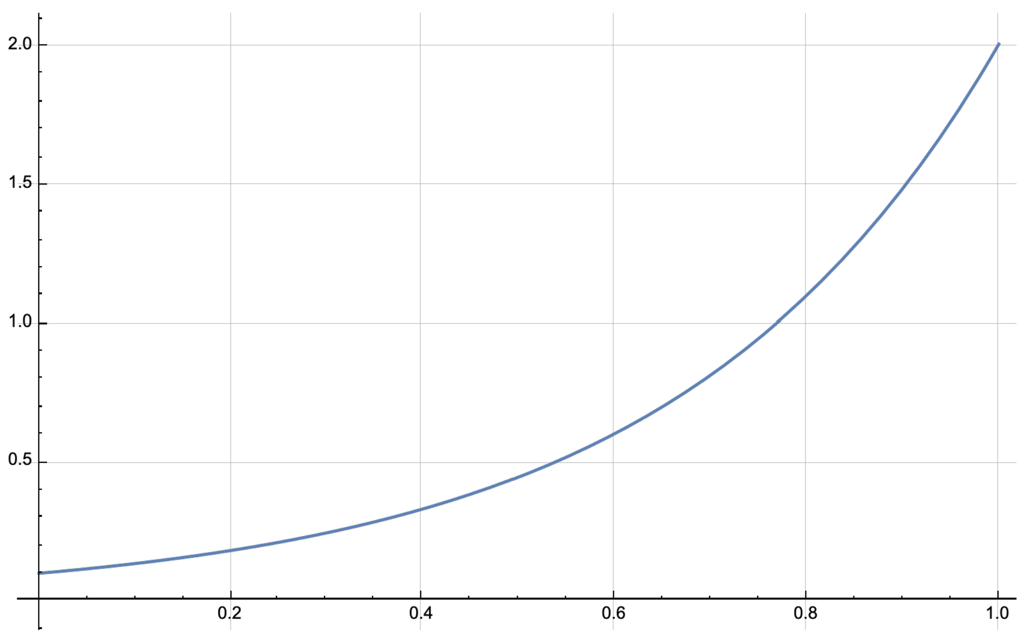
Out[∘]=



2. The Difference Between Rolling and Falling

(a) I will use the language of modern readers. If I throw a ball at 80 mph it has some kinetic energy. If I throw the same ball at 80 mph and put a spin on it, it has the kinetic energy of translation as before, but it now also has the kinetic energy of rotation. If a ball reaches the bottom of an extremely steep, nearly vertical inclined plane and is rolling, would one expect it to have more or less speed at the bottom than if it is sliding without friction and without rotation down the same plane? The idea here is to use our modern concept of energy conservation wherein the gravitational potential energy at the top of the plane has been converted to kinetic energy at the bottom.
(b) With (a) in mind, comment on Galileo's arguments surrounding Fig. 45.

3. 1, 3, 5, ... Sagredo is the king of straightforward arguments

(a) Copy Fig. 49 onto your solution. Locate the triangle ABC. What other triangles have the same size and shape as ABC? Sagredo and I see two. Identify them in your figure.
(b) By cutting up rectangles N?CI and QFIO into triangles, argue that the second row has three times the area as the first and the third row has five times the area as the first. Once you see how to do this, it is straightforward. You needn't look for some complex answer because your answer seems too simple. (Sagredo also advances pleasantly straightforward arguments on p. 186.)

4. Galilean Relativity

(a) In round numbers, the surface of the Earth is rotating eastward at a speed of 25,000 miles every 24 hours at the Equator (it is less than that away from the Equator). The radius of the Earth is 3960 miles. For precision and consistency, replace 25,000 in the above by $2\pi \cdot 3960$. The Earth's spin around its axis is not uniform motion. It is rotational motion. We need to see how much of a problem this is. If you could throw a ball a mile in the air (so that it was briefly at a distance of 3961 miles from the center of the Earth before coming back down), how much faster would the air molecules around the ball at the peak of the trajectory be moving eastward than the ball (which retains the eastward velocity it had when it left your hand).
(b) Would the ball land to the east of you, to the west of you, or right back on you? Ignore air resistance. The only reason we brought up air molecules in (a) is to have something we could visualize and compare with at the peak of the trajectory.
(c) To throw a ball $h = 1$ mile in the air requires a large initial speed, $v = \sqrt{2gh}$. In these units g, the acceleration of gravity, is about $80\,000$ miles/hour^2. What is the initial speed v in miles per hour?

I apologize that most of my questions were modern in nature and that I used Imperial units everywhere. It is a retreat to familiar terrain for me. On the other hand, I do think that the first two of these questions take you a little further than Galileo, thanks to several centuries of people building on his work.