Newton — Problem Set 2 — Solution

1. Quantity of Motion

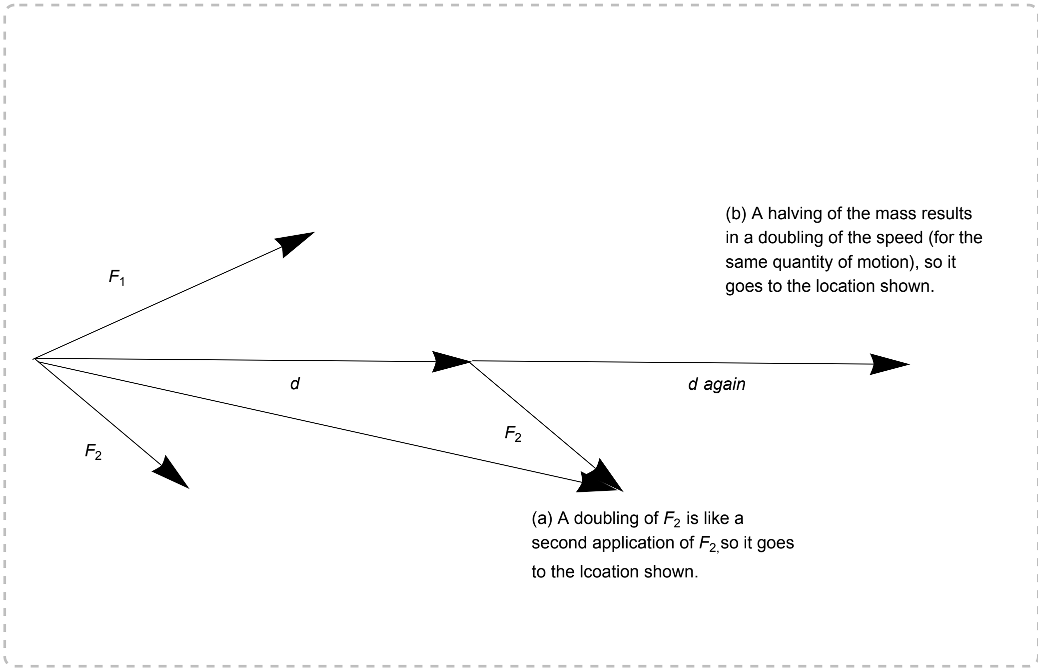
"The quantity of motion is the measure of the same arising from the velocity [or speed] and the quantity of matter [mass] conjointly."

The cloud's mass is 10x that of the droplets. The cloud's speed is 1/5 that of the droplets. Since quantity of motion is the compound of mass and speed, the cloud's quantity of motion is 2x that of droplets. They are also going different directions. The cloud is going westward. The droplets are going downward. We still don't know whether Newton (or his translator) intends to make a significant discussion between velocity and speed, so I am still using these terms interchangably.

2. The Three Laws of Motion

(a) The change in the SUV's quantity of motion in the eastward direction is its mass times 50 muff - 60 muff. So it is its mass times -10 muff.
(b) The change in the SUV's quantify of motion in the northward direction is its mass times -20 muff - 0 muff. So it is its mass times -20 muff.
(c) Change in motion is caused by equal and opposite impressed forces. The change in motion of the hay truck must be opposite of the SUV, so it is the SUV's mass times 10 muff.
direction.
(d) Similarly, it has motion northward that is the SUV's mass times 20 muff.
(e) It has gained SUV mass times 10 muff. But this is the same as its mass times 1/2 muff. However, it was going westward. So a gain in the eastward direction reduces the 60 muff speed to 59.5 muff.

3. Addition of Forces



(a) What motion will result if the impressed force F_2 is doubled in magnitude?

4. Center of Mass

(a) 0 A.U. x 2×10^{30} + 5.2 A.U. x 1.9×10^{27}kg / (2×10^{30}kg + 1.9×10^{27}kg) = 0.0049 A.U. along the line.

(b) This was an incompletely described question part. The issue is that I didn't say whether the Sun moves during this time. I was imagining that it did not move and I was just trying to get you to calculate the motion of the center of mass of the system.

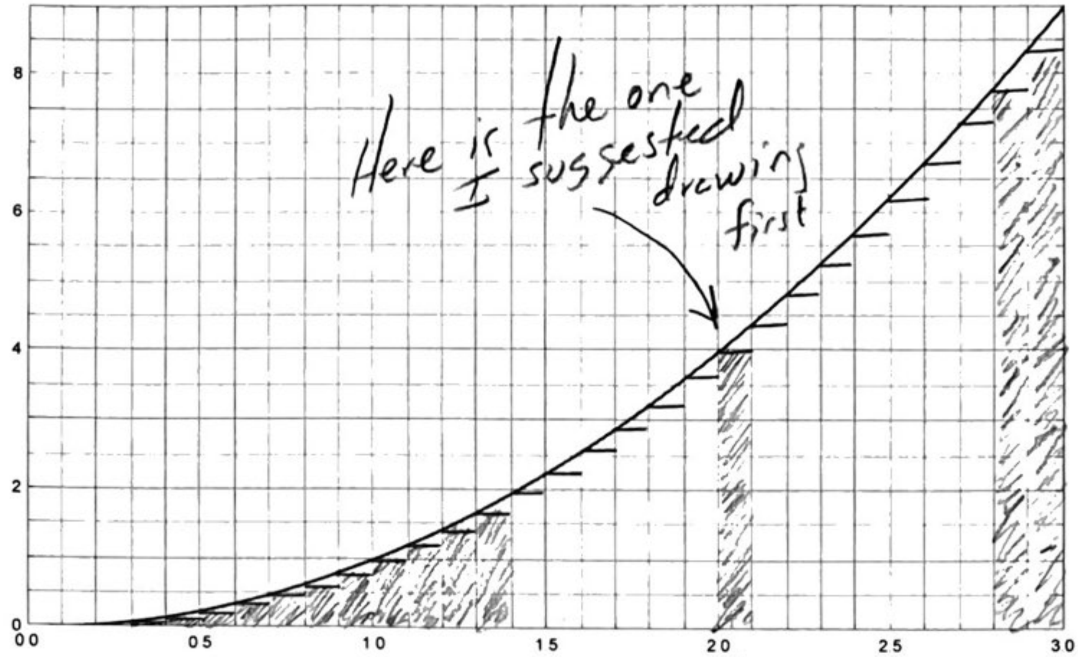
A perfectly good assumption would be that the Sun moves the opposite direction in this same time, but reduced by the ratio of the Sun's mass to Jupiter's mass, and that the center of mass does not move at all.

The other perfectly good assumption and answer (the one I had intended) would be to calculate 0 A.U. x 2×10^{30} + 6 A.U. x 1.9×10^{27}kg / (2×10^{30}kg + 1.9×10^{27}kg) = 0.0057 A.U. along the new line connecting the Sun and Jupiter, and draw that the center of mass spirals outward to this new point. By the way, the Sun is only 0.0046 A.U. in radius, so both the beginning and ending center of mass are slightly outside the Sun's surface.

5. Area Under a Curve

(a) Each little square is 0.1 wide and 0.5 tall, so each square represents 0.05 units of area. I counted something in the low 160s when I did it. I know that the right answer is 180 see part (d).

(b)



(c) These have width Δx

They have height $(k\Delta x)^2$

Their area is $k^2(\Delta x)^3$

$$\sum_{k=0}^{n-1} k^2(\Delta x)^3 = (\Delta x)^3 \sum_{k=0}^{n-1} k^2$$

this is a number independent of k, so it can be factored out of the sum

$$= \left(\frac{3}{n}\right)^3 \frac{(n-1)(n)(2n-1)}{6} = \frac{9}{2}\frac{1}{n^2}(2n^2-3n+1)$$

(d) $\lim_{n\to\infty}$ of this $\longrightarrow$ only $\frac{9}{2}\frac{2n^2}{n^2}$ term survives

limit is 9. $\frac{9}{0.05} = 180 \leftarrow$ best answer for part (a)