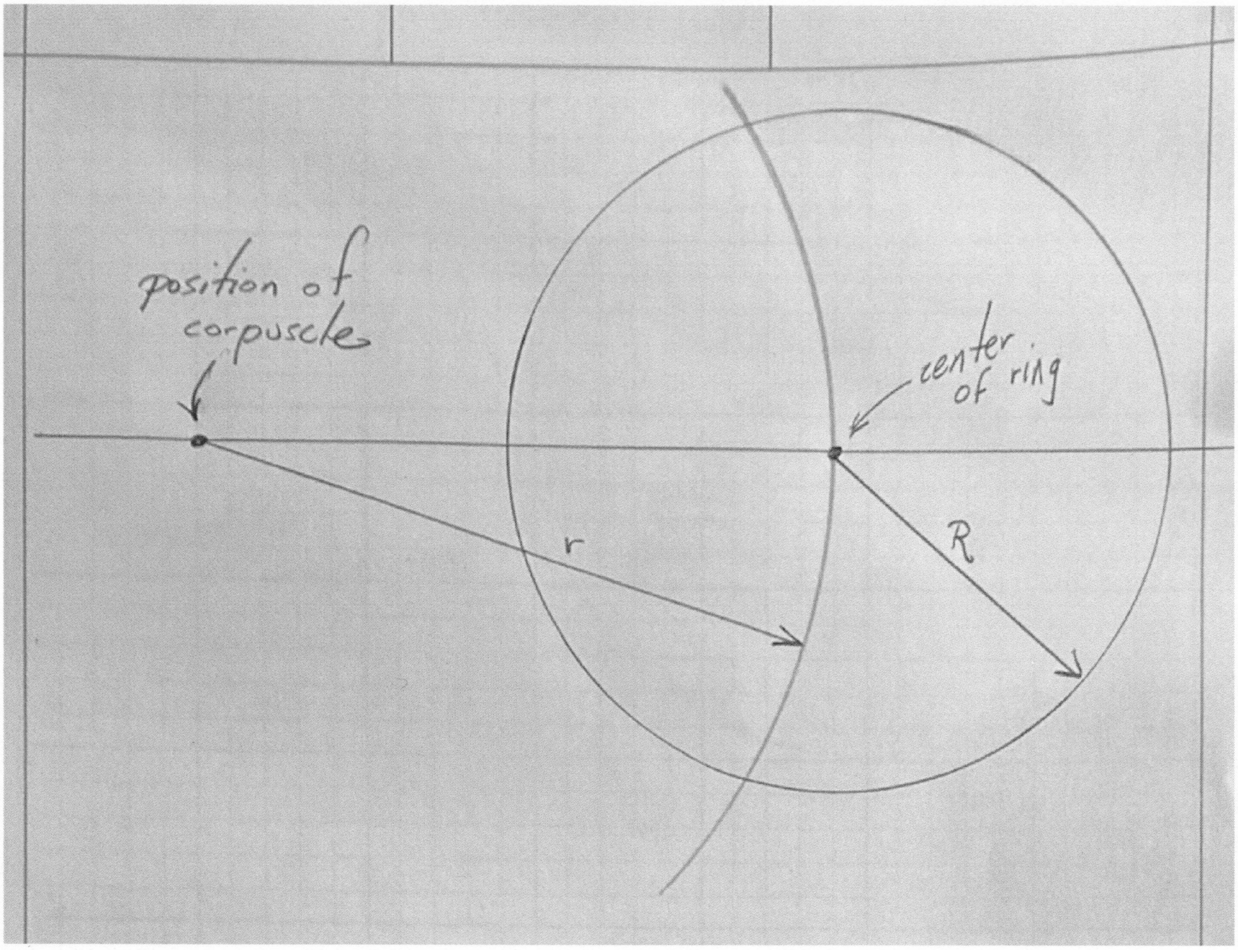
Problem Set 9 — SOLUTION



1 (a) A faint arc of radius r has been added to the figure above. Estimate as a percentage, what fraction of the ring is closer than r to the corpuscle? And what percentage is farther than r?

Note: Isn't it interesting and a bit magical that all the bits of the ring that are at a distance less than r (and compose less than half of the ring) are pulling just enough harder than "their share" to just balance all the bits that are farther than r (which compose more than half of the ring) and which are each pulling less than their share?!!

On the next page I have drawn a ring that has been broken up into 36 beads and made some measurements from the drawing.

~ 40% of the mass is closer than r

and ~ 60% of the mass is farther

1(b)

Out[89]//TableForm=

θ_{-170°	-3°	$\cos\theta_{-170^\circ}$	0.99863	r	138.5	$\frac{\cos\theta_{-170^\circ}}{r}$	0.0072
θ_{-160°	-7°	$\cos\theta_{-160^\circ}$	0.992546	r	137	$\frac{\cos\theta_{-160^\circ}}{r}$	0.0072
θ_{-150°	-11°	$\cos\theta_{-150^\circ}$	0.981627	r	135	$\frac{\cos\theta_{-150^\circ}}{r}$	0.0073
θ_{-140°	-14°	$\cos\theta_{-140^\circ}$	0.970296	r	131.5	$\frac{\cos\theta_{-140^\circ}}{r}$	0.0074
θ_{-130°	-17°	$\cos\theta_{-130^\circ}$	0.956305	r	127.5	$\frac{\cos\theta_{-130^\circ}}{r}$	0.0075
θ_{-120°	-20°	$\cos\theta_{-120^\circ}$	0.939693	r	122.5	$\frac{\cos\theta_{-120^\circ}}{r}$	0.0077
θ_{-110°	-23°	$\cos\theta_{-110^\circ}$	0.920505	r	117	$\frac{\cos\theta_{-110^\circ}}{r}$	0.0079
θ_{-100°	-26°	$\cos\theta_{-100^\circ}$	0.898794	r	110	$\frac{\cos\theta_{-100^\circ}}{r}$	0.0082
θ_{-90°	-29°	$\cos\theta_{-90^\circ}$	0.87462	r	103	$\frac{\cos\theta_{-90^\circ}}{r}$	0.0085
θ_{-80°	-31°	$\cos\theta_{-80^\circ}$	0.857167	r	95.5	$\frac{\cos\theta_{-80^\circ}}{r}$	0.0090
θ_{-70°	-32°	$\cos\theta_{-70^\circ}$	0.848048	r	87.5	$\frac{\cos\theta_{-70^\circ}}{r}$	0.0097
θ_{-60°	-32°	$\cos\theta_{-60^\circ}$	0.848048	r	79.5	$\frac{\cos\theta_{-60^\circ}}{r}$	0.0107
θ_{-50°	-31°	$\cos\theta_{-50^\circ}$	0.857167	r	71.5	$\frac{\cos\theta_{-50^\circ}}{r}$	0.0120
θ_{-40°	-29°	$\cos\theta_{-40^\circ}$	0.87462	r	64.5	$\frac{\cos\theta_{-40^\circ}}{r}$	0.0136
θ_{-30°	-25°	$\cos\theta_{-30^\circ}$	0.906308	r	56	$\frac{\cos\theta_{-30^\circ}}{r}$	0.0162
θ_{-20°	-19°	$\cos\theta_{-20^\circ}$	0.945519	r	50	$\frac{\cos\theta_{-20^\circ}}{r}$	0.0189
θ_{-10°	-10°	$\cos\theta_{-10^\circ}$	0.984808	r	45.5	$\frac{\cos\theta_{-10^\circ}}{r}$	0.0216
θ_0	0	$\cos\theta_0$	1.	r	44	$\frac{\cos\theta_0}{r}$	0.0227
θ_{10°	10°	$\cos\theta_{10^\circ}$	0.984808	r	45.5	$\frac{\cos\theta_{10^\circ}}{r}$	0.0216
θ_{20°	19°	$\cos\theta_{20^\circ}$	0.945519	r	50	$\frac{\cos\theta_{20^\circ}}{r}$	0.0189
θ_{30°	25°	$\cos\theta_{30^\circ}$	0.906308	r	56	$\frac{\cos\theta_{30^\circ}}{r}$	0.0162
θ_{40°	29°	$\cos\theta_{40^\circ}$	0.87462	r	64.5	$\frac{\cos\theta_{40^\circ}}{r}$	0.0136
θ_{50°	31°	$\cos\theta_{50^\circ}$	0.857167	r	71.5	$\frac{\cos\theta_{50^\circ}}{r}$	0.0120
θ_{60°	32°	$\cos\theta_{60^\circ}$	0.848048	r	79.5	$\frac{\cos\theta_{60^\circ}}{r}$	0.0107
θ_{70°	32°	$\cos\theta_{70^\circ}$	0.848048	r	87.5	$\frac{\cos\theta_{70^\circ}}{r}$	0.0097
θ_{80°	31°	$\cos\theta_{80^\circ}$	0.857167	r	95.5	$\frac{\cos\theta_{80^\circ}}{r}$	0.0090
θ_{90°	29°	$\cos\theta_{90^\circ}$	0.87462	r	103	$\frac{\cos\theta_{90^\circ}}{r}$	0.0085
θ_{100°	26°	$\cos\theta_{100^\circ}$	0.898794	r	110	$\frac{\cos\theta_{100^\circ}}{r}$	0.0082
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θ_{120°	20°	$\cos\theta_{120^\circ}$	0.939693	r	122.5	$\frac{\cos\theta_{120^\circ}}{r}$	0.0077
θ_{130°	17°	$\cos\theta_{130^\circ}$	0.956305	r	127.5	$\frac{\cos\theta_{130^\circ}}{r}$	0.0075
θ_{140°	14°	$\cos\theta_{140^\circ}$	0.970296	r	131.5	$\frac{\cos\theta_{140^\circ}}{r}$	0.0074
θ_{150°	11°	$\cos\theta_{150^\circ}$	0.981627	r	135	$\frac{\cos\theta_{150^\circ}}{r}$	0.0073
θ_{160°	7°	$\cos\theta_{160^\circ}$	0.992546	r	137	$\frac{\cos\theta_{160^\circ}}{r}$	0.0072
θ_{170°	3°	$\cos\theta_{170^\circ}$	0.99863	r	138.5	$\frac{\cos\theta_{170^\circ}}{r}$	0.0072
θ_{180°	0	$\cos\theta_{180^\circ}$	1.	r	139	$\frac{\cos\theta_{180^\circ}}{r}$	0.0072

TOTAL = 0.3911

÷ 36

0.0109

Spot on!

1(c)

Problem 2

All the different lengths referred to are in the diagram on the following page

$$\frac{F_1}{F_2} = \frac{m_1 \cos\theta_1 / i_1}{m_2 \cos\theta_2 / i_2}$$

In 2-d $\frac{m_1}{m_2} = \frac{a_1}{a_2} = \frac{c_1}{c_2}$

So

$$\frac{F_1}{F_2} = \frac{c_1/i_1}{c_2/i_2} \frac{\cos\theta_1}{\cos\theta_2}$$

But Newton also has

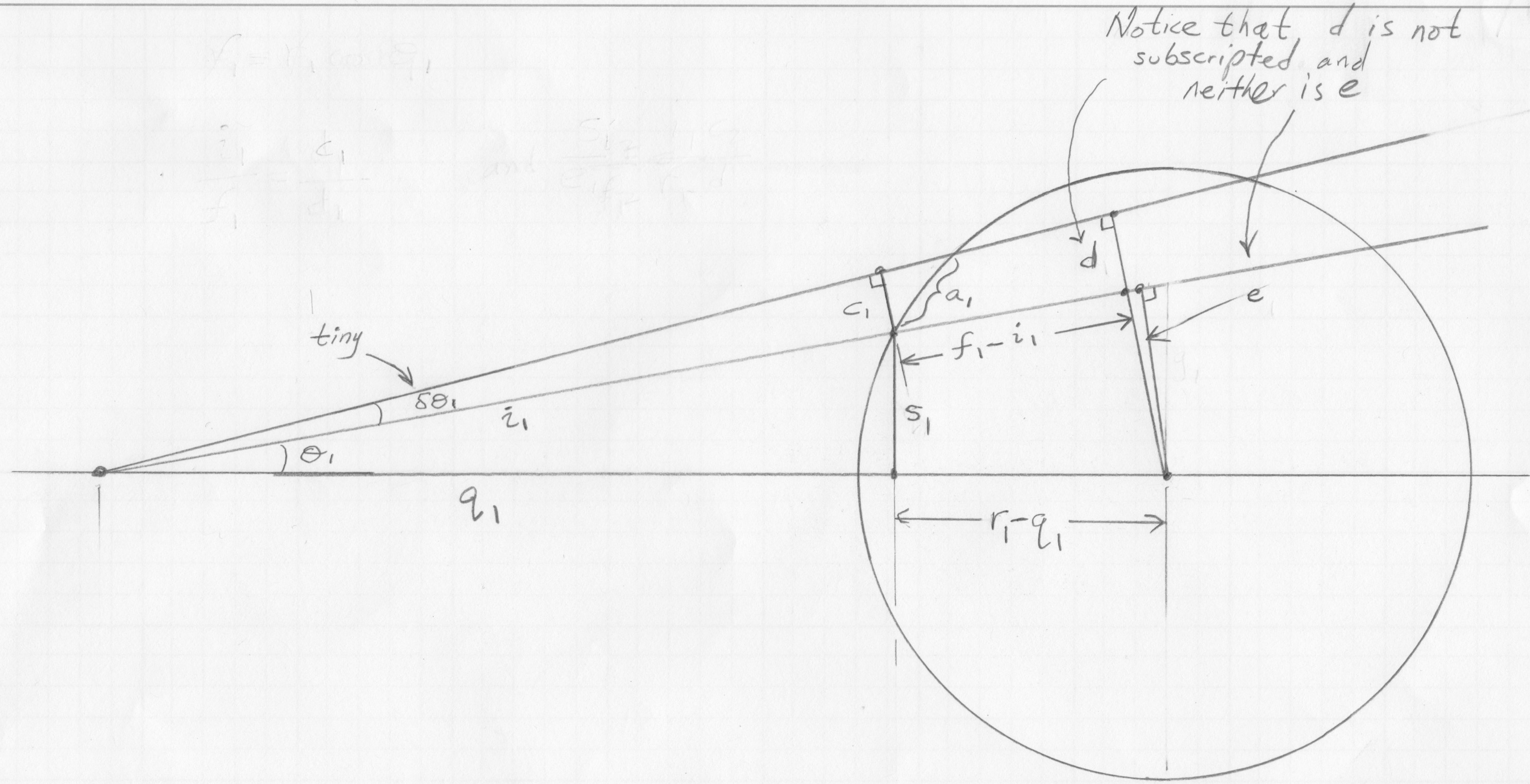
$$\frac{c_1/i_1}{c_2/i_2} = \frac{f_2}{f_1}$$

This he got from

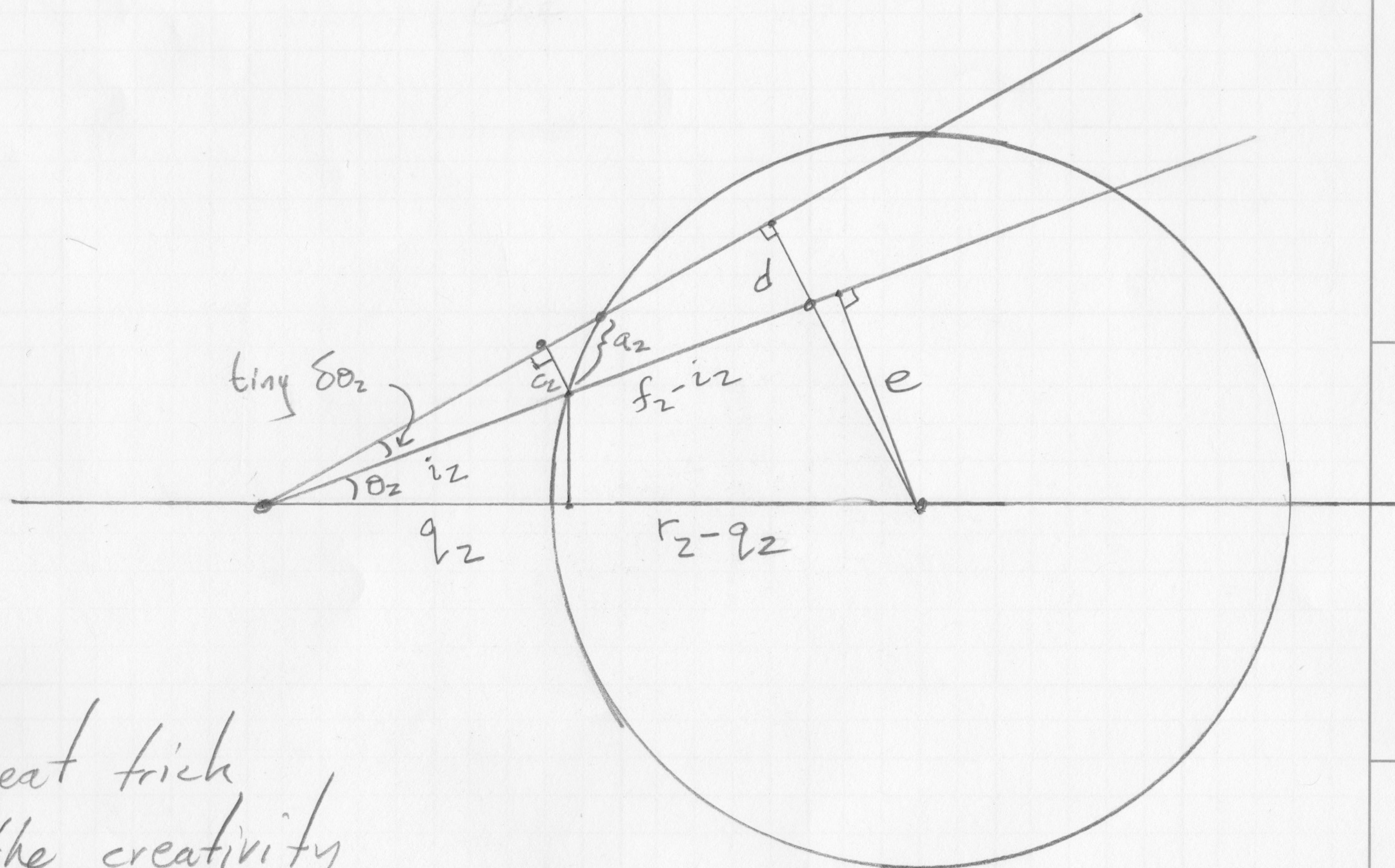
$d = \frac{f_1}{i_1} c_1$ and $d = \frac{f_2}{i_2} c_2$

So

$$\frac{F_1}{F_2} = \frac{f_2}{f_1} \cdot \frac{\cos\theta_1}{\cos\theta_2} = \frac{r_2}{r_1}$$



The diagram on the next page is trivially related to this one by changing all the "1"'s to "2"'s.



Newton's great trick here is the creativity of having a new construction for the corpuscle in the second position such that everything is changed except d and e which his construction involves keeping the same.

NUTS, A TYPO!!

10^{12} not 10^{11}

Problem 3 — The Tides

(a) The mass of the Moon is $M = 7.348 \times 10^{22}$ kilograms. The constant G in the gravitational force formula $\frac{GMm}{r^2}$ is $6.674 \times 10^{-11}\ \text{m}^3/\text{kg}/\text{s}^2$. So for the Moon, $GM = 4.904 \times 10^{11}\ \text{m}^3/\text{s}^2$. The average distance from the Moon to the Earth is $r = 3.844 \times 10^{8}$ m. Square r and calculate $\frac{GM}{r^2}$ to get the average acceleration of the Earth due to the Moon. The units are m/s^2.

$3.319 \times 10^{-5}\ \frac{m}{s^2}$

(b) The mass of the Sun is $M = 1.989 \times 10^{30}$ kilograms. So for the Sun, $GM = 1.327 \times 10^{20}\ \text{m}^3/\text{s}^2$. The average distance from the Sun to the Earth is $r = 1.474 \times 10^{11}$ m. Square r and calculate $\frac{GM}{r^2}$ to get the average acceleration of the Earth due to the Sun.

$6.110 \times 10^{-3}\ \frac{m}{s^2}$

(c) The near part of the Earth is about 59/60 as far from the Moon as the center of the Earth, and the far part of the Earth is about 61/60 as far from the Moon as the center. Using your answer to (a) and these distance ratios, find the acceleration of the part of the Earth nearest to the Moon and the acceleration of the part of the Earth farthest from the Moon. The Earth's crust is a thick pile of rock, and perhaps one wouldn't expect it to be very deformable. However, the water on the surface of the Earth is certainly deformable. How much more is the water on the near side of the Earth being accelerated toward the Moon than the water on the far side of the Earth?

nearest $= \left(\frac{60}{59}\right)^2 \times 3.319 \times 10^{-5}\ \frac{m}{s^2} = 3.432 \times 10^{-5}\ \frac{m}{s^2}$

farthest $= \left(\frac{60}{61}\right)^2 \times 3.319 \times 10^{-5}\ \frac{m}{s^2} = 3.211 \times 10^{-5}\ \frac{m}{s^2}$

difference $= 2.21 \times 10^{-6}\ \frac{m}{s^2}$

(d) The near part of the Earth is about 23140/23141 as far from the Sun as the center of the Earth and the far part of the Earth is about 23142/23141 as far from the Sun at the center. Repeat the methodology in part (c), this time using the acceleration calculated in part (b).

nearest $= \left(\frac{23141}{23140}\right)^2 \times 6.110 \times 10^{-3}\ \frac{m}{s^2} = 1.0000864 \times 6.110 \times 10^{-3}\ \frac{m}{s^2}$

farthest $= \left(\frac{23141}{23142}\right)^2 \times 6.110 \times 10^{-3}\ \frac{m}{s^2} = 0.9999136 \times 6.110 \times 10^{-3}\ \frac{m}{s^2}$

difference $1.05 \times 10^{-6}\ \frac{m}{s^2}$

Please contemplate (no need to write out an answer): Do you see why the tides that are raised by the Moon bulge on both the near and far side of the Earth? If not, please bring it up in class. By the way, the acceleration formulae we have used are entirely correct, but they aren't enough by themselves to tell you how high the tides are raised. They just tell you that they are raised — on both sides of the Earth — and they also tell you how much larger the effect of the Moon is than the Sun in raising tides.