Day 2 — Ben and Brian's Presentation of Th./Prop. IV

NB: THIS EXAMPLE IS OF CIRCULAR MOTION, NOT STRAIGHT-LINE MOTION, SO IT ISN'T QUITE WHAT GALILEO HAS IN MIND.

First, an example

An Earth-year is 365 days. A Mars-year (measured in Earth's days) is 687 days.
Earth goes around the Sun at a speed of 67,000 ~~mph~~ muff ← I CHANGED THE UNITS TO MUFF SO THAT YOU ARE NOT TEMPTED TO USE 1 DAY = 24 HRS TO GET THE DISTANCE TRAVELED

$$\cancel{mph = \frac{miles}{hour} = \frac{miles}{hour}\,\frac{24\,hours}{day} = \frac{miles}{day}\,24}$$

Mars goes around the Sun at 54,000 muff

Using compound ratios, as formulated in Th./Prop. IV, what is the ratio of distance traveled by Earth in an Earth-year vs. the distance traveled by Mars in a Mars-year?

$$d_E : d_M :: \Delta t_E : \Delta t_M \times V_E : V_M$$

$$= 365 : 687 \times 67{,}000 : 54{,}000$$

$$\approx 2 : 3$$

Now the Proof

Instead of Galileo's unmemorable letters, we make the following more memorable assignments

d_1	V_1	Δt_1	d_1'	d_2	V_2	Δt_2
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
G	A	C	I	L	B	D

Particle 1 $\leftrightarrow$ E   (I: you will see what this is)   Particle 2 $\leftrightarrow$ F

Particle 1 goes V_1 for Δt_1, resulting in d_1

Particle 2 goes V_2 for Δt_2, resulting in d_2

The proof has us consider, what if Particle 1 went V_2 for Δt_1, resulting in d_1'?

In that case

$$d_1 : d_1' :: V_1 : V_2$$ ← because the time is unchanged

We also know

$$d_1' : d_2 :: \Delta t_1 : \Delta t_2$$ ← because in this comparison, the speed is unchanged

However

$$d_1 : d_2 :: d_1 : d_1' \times d_1' : d_2$$

$$:: V_1 : V_2 \times \Delta t_1 : \Delta t_2 \qquad \text{Q.E.D.}$$

Perhaps this diagram makes Galileo's intermediate step clearer

V_1 A	Δt_1 C	d_1 G
CHANGED	SAME	
V_2 B	Δt_1 C	d_1' I
SAME	CHANGED	
V_2 B	Δt_2 D	d_2 L

In other words, by introducing d_1', he is able to change just one quantity at a time

Important Techniques/Properties Involving Ratios

Consider ← three distances or three of anything of the same type

$d_1 \,,\, d_2 \,,\, d_3$

$$d_1 : d_3 :: d_1 : d_2 \times d_2 : d_3$$

As a special case with $d_3 = d_1$

$$\underbrace{d_1 : d_1}_{1} :: d_1 : d_2 \times d_2 : d_1 \implies d_2 : d_1 = \frac{1}{d_1 : d_2}$$

Or $d_1 : d_3 :: \dfrac{d_2 : d_3}{d_2 : d_1}$ Or $d_1 : d_3 :: \dfrac{d_1 : d_2}{d_3 : d_2}$

A Summary of the First Six Theorems

I Distance is proportional to time (speed fixed)

II Distance is proportional to speed (time fixed)

III Time is inversely proportional to speed (distance fixed)

IV Distance is proportional to speed and time

V Time is proportional to distance and inversely proportional to speed

VI Speed is proportional to distance and inversely proportional to time