

Day 2 - Ben and Brian's Presentation of Th./Prop. IV

First, an example

NB:
THIS EXAMPLE IS OF CIRCULAR MOTION, NOT STRAIGHT-LINE MOTION, SO IT ISN'T QUITE WHAT GALILEO HAS IN MIND.

An Earth-year is 365 days. A Mars-year (measured in Earth's days) is 687 days.

Earth goes around the Sun at a speed of 67,000 ~~mph~~ muff ←

I CHANGED THE UNITS TO MUFF SO THAT YOU ARE NOT TEMPTED TO USE 1 DAY = 24 HRS TO GET THE DISTANCE TRAVELED

$$\text{mph} = \frac{\text{miles}}{\text{hour}} = \frac{\text{miles}}{\text{hour}} \frac{24 \text{ hours}}{\text{day}} = \frac{\text{miles}}{\text{day}} \cdot 24$$

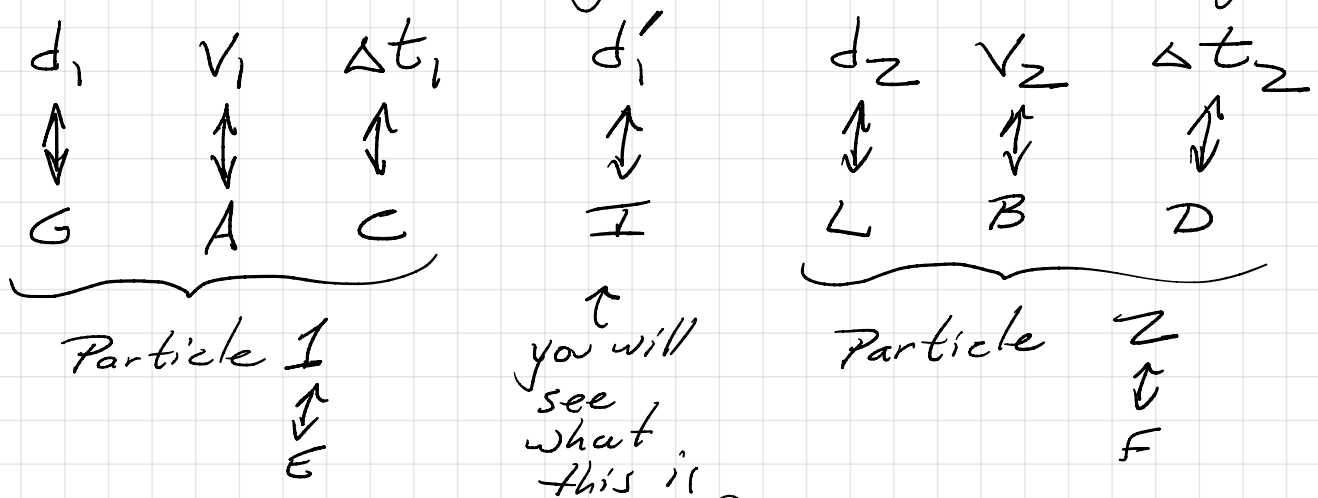
Mars goes around the Sun at 54,000 muff

Using compound ratios, as formulated in Th./Prop. IV, what is the ratio of distance traveled by Earth in an Earth-year vs. the distance traveled by Mars in a Mars-year?

$$\begin{aligned} d_E : d_M &:: \Delta t_E : \Delta t_M \times V_E : V_M \\ &= 365 : 687 \times 67,000 : 54,000 \\ &\approx 2 : 3 \end{aligned}$$

Now the Proof

Instead of Galileo's unmemorable letters, we make the following more memorable assignments



Particle 1 goes v_1 for Δt_1 , resulting in d_1

Particle 2 goes v_2 for Δt_2 , resulting in d_2

The proof has us consider, what if

Particle 1 went v_2 for Δt_1 , resulting in d_1' ?

In that case

$$d_1 : d_1' :: v_1 : v_2 \leftarrow \text{because the time is unchanged}$$

We also know

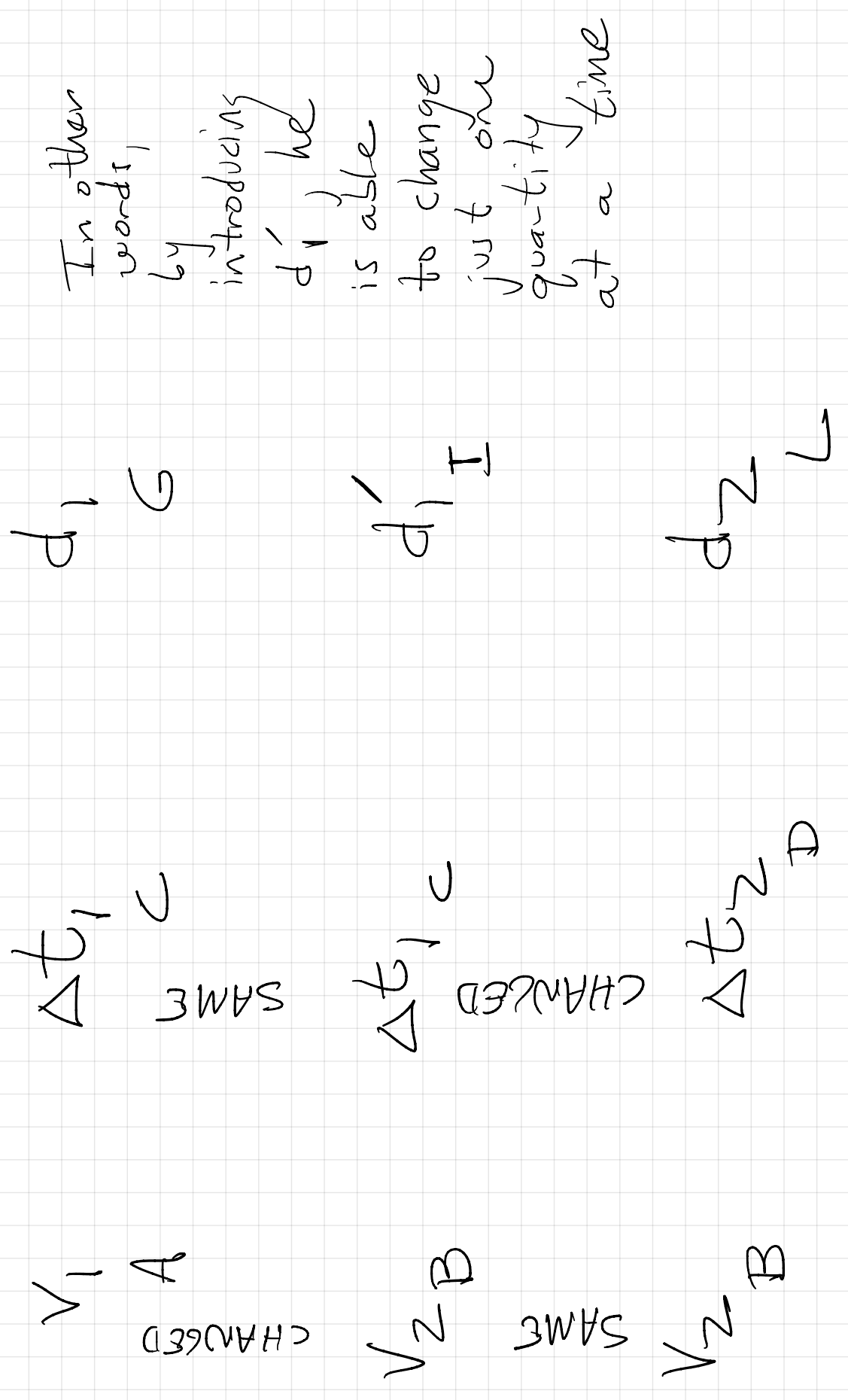
$$d_1' : d_2 :: \Delta t_1 : \Delta t_2 \leftarrow \text{because in this comparison, the speed is unchanged}$$

However

$$d_1 : d_2 :: d_1 : d_1' \times d_1' : d_2$$

$$:: v_1 : v_2 \times \Delta t_1 : \Delta t_2 \quad \text{Q.E.D.}$$

Perhaps this diagram makes Galileo's intermediate step clearer



Important Techniques/Properties Involving Ratios

Consider

$$d_1, d_2, d_3$$

← three distances or
three of anything
of the same type

$$d_1 : d_3 :: d_1 : d_2 \times d_2 : d_3$$

As a special case with $d_3 = d_1$

$$\underbrace{d_1 : d_1}_{1} :: d_1 : d_2 \times d_2 : d_1$$

$$\Rightarrow d_2 : d_1 = \frac{1}{d_1 : d_2}$$

$$\text{Or } d_1 : d_3 :: \frac{d_2 : d_3}{d_2 : d_1} \quad \text{Or } d_1 : d_3 :: \frac{d_1 : d_2}{d_3 : d_2}$$

A Summary of the First Six Theorems

- I** Distance is proportional to time (speed fixed)
- II** Distance is proportional to speed (time fixed)
- III** Time is inversely proportional to speed (distance fixed)
- IV** Distance is proportional to speed and time
- V** Time is proportional to distance and inversely proportional to speed
- VI** Speed is proportional to distance and inversely proportional to time