Numerical Analysis on a Pocket Calculator

Terms 2-3, Deep Springs College, Prof. Brian Hill

Overview

Four subjects will emerge:

- 1. Operation and programming of a stack-based calculator, the Hewlett-Packard 25
- 2. General applications that were in the calculator's target market: games, finance, navigation, and surveying
- 3. Statistics: linear regression, exponential, logarithmic, and power law curve fitting, standard deviations and correlation coefficients, t-Test and X-squared hypothesis tests
- 4. Numerical analysis: Newton's root-finding method, Euler's method for first-order differential equations, numerical integration

There is a rich variety of background needed to deeply understand these subjects. Our classes will have a mix of developing the needed background and programming the Hewlett-Packard 25. In other words, we will constantly mix theory with concrete and pragmatic considerations.

The subjects are influenced by what Hewlett-Packard considered to be the HP-25's target market and by the capabilities of the calculator. This peculiar combination of constraints means that we will be taking a tour of a wide variety of subjects that mattered to practicing scientists and engineers in the mid-1970s. These subjects matter just as much today.

Daily Schedules

Detailed daily schedules will be kept retrospectively:

- Daily Schedule Term 2
- Daily Schedule Term 3

Texts

There will be no text to purchase for the above subjects. Instead I will be preparing handouts. I will draw from three books:

- The HP-25 Owner's Handbook
- HP-25 Applications Programs book
- If time, there will additionally be selections from Peter Henrici, *Computational Analysis with the HP-25 Pocket Calculator*, which is long since out-of-print

Additionally:

• We will cover some of *Bill & Dave: How Hewlett and Packard Built the World's Greatest Company* as a way of rounding out our historical knowledge of the company that is arguably the most influential company in what we now call Silicon Valley.

Resources

We will run the HP-25 app on smartphones. There is more than one developer offering these. They are inexpensive.

Grading

Five major areas:

- Active preparation for and engagement with the class: 15%
- Weekly problem sets involving both theory and programming (except during exam weeks and during the final project): 35%
- A midterm towards the end of Term 2: 20%
- A final exam towards the end of Term 3 (but before the final project): 20%
- A final project on a theory and programming problem of your choice: 10%

Miscellaneous Policies

There will be a lot of handouts. Get a three-ring binder to keep all the handouts and problem sets organized. Assignments should be on $8 1/2 \times 11$ paper (and not torn out from a bound notebook). Multi-page assignments should be stapled. Corrections should be erased (if done in pencil) or recopied (if done in pen).

The College's general policies on absences and late work are applicable. There was an email from the Dean on this September 8. The policies below are consistent with that email:

Whereas missed coursework affects both your classmates and professors by lowering the thinking and understanding you bring to a given class, and interrupts the course schedule that has been set up and is adjusted on an ongoing basis with substantial care. The same is true for absences — whereas a handful of absences might be "normal" at colleges with large lectures or less serious academics, at Deep Springs we expect students to miss *no classes* save for legitimate health issues or emergencies requiring also missing labor and governance obligations.

For a student wishing to submit a course assignment past its required deadline, the student may request an extension on the assignment directly from the professor 48 hours in advance. Within 48 hours of the due date, the student must request an extension directly from the Dean. Exceptions will be granted by the Dean only if the student faces unforeseen and unforeseeable circumstances. A student who misses the deadline will be penalized a number of points that is roughly equivalent to a whole letter grade for each 24-hour period the assignment is late. Assignments cannot be turned in after solutions and graded assignments have been passed back, which generally happens 1-2 classes after they were turned in.

Numerical Analysis Daily Schedule Term 2

Course **home page**

See also: Daily Schedule Term 3

Week 1 — Before Calculators — Calculating with the HP-25C

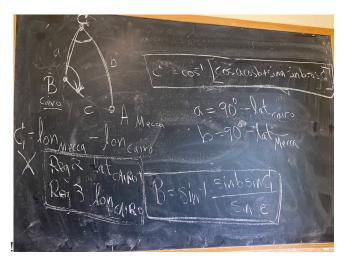
- Tuesday, Aug. 30 Prepare for class by following the directions in **Before Calculators**
- Friday, Sep. 2 Prepare for class by following the directions in **Getting Started** You will need to make a copy of **Sections 1-3 of the HP-25 Owner's Handbook**

Week 2 — More Functions of the HP-25C — The Stack — Registers — Beginning Programming

- Tuesday, Sep. 6 Prepare for class by following the directions in **Calculating with the HP-25** These directions include your first problem set due Sep. 6 You will need to make a copy of **Sections 4-5 of the HP-25 Owner's Handbook**
- Friday, Sep. 9 Prepare for class by following the directions in **The Stack**, **Registers**, and **Beginning Programming (Including Nimb)**

Week 3 — More Programming — Branching — Conditional Branching — Newtonian Physics

• Preparation for Tuesday, Sep. 13 — Prepare for class by following the directions in **Programming the HP-25** — These directions include your second problem set due Sep. 13 — The directions also include the **HP-25 Program Form** — You will probably find the formulas we had on the board last time very helpful in proceeding toward your working program:



- Tuesday, Sep. 13 Hand out **Solution to Problem Set 1**
- Preparation for Friday, Sep. 16 Prepare for class by following the directions in **Programming the HP-25 Part II**
- Friday, Sep. 16 Conditional Branching in Moon Lander

Week 4 — Applications: Graphing, Base Conversion, Dot Products, Simultaneous Equations

• Preparation for Tuesday, Sep. 20 — Prepare for class by following the directions in **Applications: Graphing and Base Conversion** — These directions include your third problem set due Sep. 20 — You will need to make a copy of **Chapters 1-2 of the HP-25 Applications Programs book** from which you will be keying in the first few programs, and finally we will read pp. 180-189 of *Bill & Dave*

- Tuesday, Sep. 20 Discussion of Reading The Uses of Dot Products Simultaneous Equations Return Problem Set 2 with Solution
- Preparation for Friday, Sep. 23 Prepare for class by following the directions in **Applications: Dot Products and Simultaneous Equations** Additionally, we will read pp. 264-282 of *Bill & Dave*
- Friday, Sep. 23 Discussion of Reading Solution to Problem Set 3 Start Finance

Week 5 — Start Finance Applications!

- Preparation for Tuesday, Sep. 27 Prepare for class by doing the problem set in Applications: Finance
- Tuesday, Sep. 27 Periodic Savings Formulas
- Preparation for Friday, Sep. 30 Prepare for class by following **Applications: Periodic Savings Program Modifications**
- Friday, Sep. 30 An HP-45 from David and Donna Sweetman LEDs and LCDs Analysis of Periodic Savings — Solution to Problem Set 4 — Introduce Wage Growth and Asset Inflation into Periodic Savings

Week 6 — Finish Finance — Term 2 Exam

- Preparation for Tuesday, Oct. 4 Prepare for class by doing the problem set in **Periodic Savings with Inflation**
- Tuesday, Oct. 4 Periodic Savings with Inflation Solution Use of Σ+ and Σ- Keys (Owner's Handbook, pp. 66-70) Least Squares as a Measure of Goodness of Fit
- Friday, Oct. 7 Term 2 (Midterm) Exam

Week 7 — Start Statistics Applications

- Preparation for Tuesday, Oct. 11 Here is a pretty straightforward assignment for Tuesday.
- Type in the program on pp. 87-91 of the Applications Handbook (PDF of those pages attached).
- Do the example on pp. 90-91 to make sure you understand how the program is used and that you have typed it in correctly.
- This is the part you turn in:

(a) Find *any* other dataset that you would like to do a linear regression on. (b) Make a table of the x and y values. (c) Document what those x and y values represent. (d) Run the program and record the slope a1 (*I usually call this m*), *the intercept a*0 (I usually call this b), and the r^2. (e) Plot the data, and the best fit line. (f) Make some predictions using your fit.

As an example of (f), suppose you know the price of Big Macs for 2000, 2005, 2010, 2015, and 2020. Predict the price for 2025 and 2030.

Also, if I were doing that example, I would subtract 2000 from each of the years. So I would record 0, 5, 10, 15, and 20, and predict 25 and 30. The intercept all the way back in the year Christ was born is not very interesting for a contemporary analysis!

Turn in (3) as your 6th assignment at the beginning of class. I think this is only 1-2 hours of work. If I haven't given you enough notice and you need one more day (until Wednesday dinner), email me before Tuesday's class. It would be best though to get it done so that you can more fully appreciate what we are going to do in class on Tuesday.

• Tuesday, Oct. 11 — Complete the Linear Regression Theory we started on Tuesday, Oct. 4

Numerical Analysis Daily Schedule Term 3

Course **home page**

See also: Daily Schedule Term 2

Week 8 — Best Fits of Various Types (Linear, Exponential, Logarithmic, Power Law)

- Tuesday, Oct. 25 Theory of Exponential Best Fit, Logarithmic Best Fit, and Power Law Best Fit Discussion of pp. 9-30 of Bill & Dave
- Preparation for Friday, Oct. 28 Read Applications Programs material on Exponential and Logarithmic Best Fit pp. 31-47 of Bill & Dave
- Friday, Oct. 28 Course Evaluation Discussion Developing r-squared intuition

Week 9 — Distributions and Statistical Tests

Friday's class canceled due to COVID outbreak (make-up Nov. 20)

 Preparation for Tuesday, Nov. 1 — Problem Set 7 on Standard Deviation and *r*-Value — Study Covariance and Correlation Coefficient pp. 101-102 of HP-25 Applications Programs — Finish Chapter 2 ("Apprentices") pp. 47-69 of *Bill & Dave*

Week 10 — Distributions and Statistical Tests, Continued

- Preparation for Tuesday, Nov. 8 **Problem Set 8** on Uniform and Binomial Distributions
- Tuesday, Nov. 8 The Binomial Distribution
- Preparation for Friday, Nov. 11 Study the **t-Test** (from **JMP** (maker of SAS)) and the program on pp. 121-123 of the *HP-25 Applications Programs* book Type in the program and do the example on p. 123 to make sure it works
- Friday, Nov. 11 Discussion of paired t-Test

Week 11 — Statistical Tests, Concluded — Numerical Methods: Root Finding (Newton's Method)

- Preparation for Tuesday, Nov. 15 Problem Set 9 Study the χ-Squared Test (also from JMP (maker of SAS)) and the program on pp. 118-120 of the *HP-25 Applications Programs* book Type in the program and do the example on p. 120 to make sure it works
- Tuesday, Nov. 15 Discussion of χ -squared Test
- Friday, Nov. 18 Class will prepare for and lead discussion of Newton's method pp. 71-99 of Bill & Dave (which takes us well into WWII)

Week 12 — Numerical Methods: Differential Equations (Euler's Method)

Makeup class on Sunday before Thanksgiving

- Sunday, Nov. 20 A look at some historical materials (calculators, Scientific American)
- Preparation for Tuesday, Nov. 22 In Chapter 5, study the program on pp. 83-84 **Problem Set 10**
- Tuesday, Nov. 22 Discussion of differential equations and the theory of Euler's method.

Week 13 — Numerical Methods: Integration (Simpson's Rule)

- Tuesday, Nov. 29 Introduction to Integrals and Simpson's Rule
- Preparation for Thursday, Dec. 1 Complete **Problem Set 11** on differential equations
- Thursday, Dec. 1 Proof of Simpson's Rule (Brian and Brandon) Applications of Simpson's Rule

Week 14 — Term 3 Exam — Start Special Projects

- Tuesday, Dec. 6 Term 3 Exam (directions follow)
 - There will be three problems. (In class I said four and that you would pick three of the four, but I am just going to make it three because I have not been creative enough to come up with four good problems.)
 - The first problem will be a statistics problem. Come to the exam with the power curve fit program on p. 99 already typed in. Make sure your program works on HP's example on p. 100.
 - The second problem will be another differential equation problem using Euler's method.
 - The third problem will use the parabola fit ideas we developed in Friday's class, but you don't have to study the theory, because I will have exactly the theory you need all written up. It might help to review the linear interpolation program on p. 85.
- Friday, Dec. 9 Choose, discuss, and get started on special projects

Week 15 — Present Special Projects

Special Projects: Choose a program we have not worked through or extend a program that we have already worked through. Explain the theory. Illustrate it with examples. Prepare 15-minute presentations for the last day.

• Tuesday, Dec. 13 — Special Project presentations