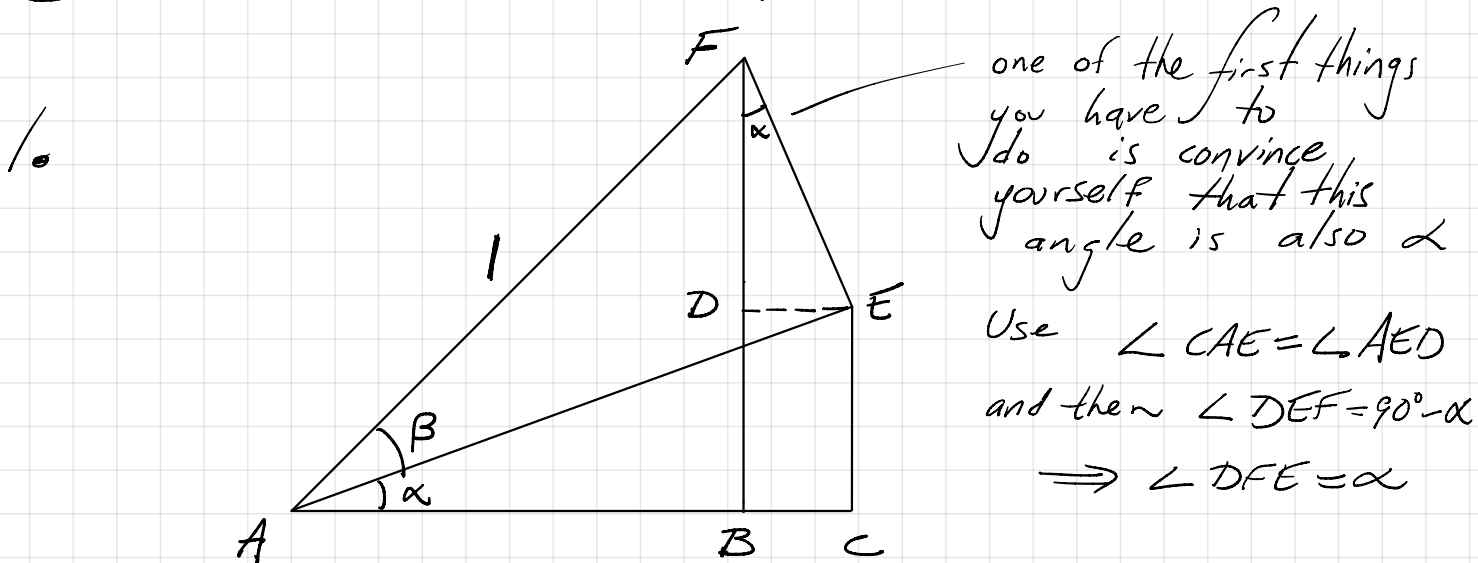


Numerical Analysis

Sep. 6, 2022

Solution to Problem Set 1



$\overline{AB} = \cos(\alpha + \beta)$ because it is adjacent to $\alpha + \beta$ on a triangle with hypotenuse 1

Here is the big thing: $\overline{AB} = \overline{AC} - \overline{BC}$
 $= \overline{AC} - \overline{DE}$

Then

$$\overline{FE} = \sin \beta \quad \text{so} \quad \overline{DE} = \sin \alpha \sin \beta$$

$$\overline{AE} = \cos \beta \quad \text{so} \quad \overline{AC} = \cos \alpha \cos \beta$$

$$\text{So, } \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$



$$2. \quad A = \frac{1}{2} b^2 \frac{\sin \theta \sin \phi}{\sin(\theta + \phi)}$$

$$\theta = 30^\circ$$

$$\phi = 45^\circ$$

$$b = 4 \text{ cm}$$

My HP-25 app gives

$$A = 2.93 \text{ cm}^2$$

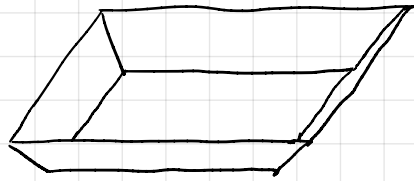
3. The Owner's Handbook recommends starting with the "innermost" quantity first. I guess I'll start with $\theta + \phi$.

3	
0	
<u>ENTER</u>	
4	
5	
<u>+</u>	75
<u>F</u> <u>SIN</u>	0.97
<u>g</u> <u>1/x</u>	1.04
3	
0	
<u>F</u> <u>SIN</u>	0.52
<u>x</u>	
4	
5	
<u>F</u> <u>SIN</u>	0.37
<u>x</u>	
4	
<u>g</u> <u>x²</u>	16
<u>x</u>	5.86
2	
<u>=</u>	2.93

4. and 5. were problems of your choice. In class I showed two formulae for the volume of the upper reservoir.

4. A naive way of getting the volume of the upper reservoir

The volume of a pyramid is $\frac{1}{3} l \cdot w \cdot h$.
Think of the reservoir as a truncated pyramid



result is in cubic feet

$$\text{It is } \frac{1}{3} l \cdot w \cdot h - \frac{1}{3} l' \cdot w' \cdot h' = 40000 \text{ ft}^3$$

units are in feet

5. A much more sophisticated estimate.
This formula was easy to get using calculus (an integral).

$$l'w'h + l's \cdot h + w's \cdot h + \frac{4}{3} s^2 \cdot h$$

$$l' = 75 \text{ ft} \quad w' = 50 \text{ ft} \quad s = 12.5 \text{ ft} \quad h = 8 \text{ ft}$$

$$\text{Result is } 44166.67 \text{ ft}^3$$

$$\text{or about } 44000 \text{ ft}^3$$

This formula also works if the reservoir water height, h , is less than 8 ft.