

Problem Set 4 - SolutionProblem 1

Before doing the solution to Problem 1, let's finish the theory. The Applications Programs Book has

$$BAL_K = \frac{1}{(1+i)^{-K}} \left[PMT \frac{(1+i)^{-K} - 1}{i} + PV \right]$$

In my derivation, instead of K , I had n . Instead of BAL , I just wrote B . Instead of PMT , I just wrote P . Finally, instead of PV , I had B_0 . So the equation we're trying to derive is:

$$B_n = \frac{1}{(1+i)^{-n}} \left[P \frac{(1+i)^{-n} - 1}{i} + B_0 \right]$$

Also, $\frac{1}{(1+i)^{-n}} = (1+i)^n$

So we are trying to show

$$B_n = B_0 (1+i)^n - P \frac{(1+i)^n - 1}{i}$$

In class, we already convinced ourselves that the B_0 term is right. So we had the P term still to understand. We had

$$-P \left[1 + (1+i) + (1+i)^2 + \dots + (1+i)^{n-2} + (1+i)^{n-1} \right]$$

Let $x = 1+i$. What is in square brackets is

$$1 + x + x^2 + \dots + x^{n-2} + x^{n-1} \equiv S_n(x)$$

This is a famous series. Take x times the series and subtract the series:

$$\begin{aligned} & xS_n(x) - S_n(x) \\ &= \cancel{x} + \cancel{x^2} + \cancel{x^3} + \dots + \cancel{x^{n-2}} + \cancel{x^{n-1}} + x^n \\ & \quad - (1 + \cancel{x} + \cancel{x^2} + \cancel{x^3} + \dots + \cancel{x^{n-2}} + \cancel{x^{n-1}}) \\ &= x^n - 1 \quad \text{wow!} \end{aligned}$$

$$(x-1)S_n(x) = x^n - 1$$

$$\Rightarrow S_n(x) = \frac{x^n - 1}{x - 1}$$

$$= \frac{(1+i)^n - 1}{1+i - 1} = \frac{(1+i)^n - 1}{i}$$

So the P term is

$$-P \frac{(1+i)^n - 1}{i}$$

And we have successfully derived

$$B_n = B_0(1+i)^n - P \frac{(1+i)^n - 1}{i}$$

Finally we know enough to trust the Applications Programs book. We use

$$i = 0.005 \leftarrow \text{that's } 6\% / 12$$

$$PV = \$500,300$$

$$PMT = \$3000$$

In Oct, Nov, Dec 2021 (Periods 1-3),
\$7997.01 of interest is paid.

The remaining balance is \$498,797.01

So all but \$1002.99 of your first \$3000.00 went to interest!

In 2022 (Periods 4-15), \$29,758.02 of interest is paid, and the remaining balance is \$492,555.03.

First full year

Interest

Balance

29,850.72

494,150.72

Second full year (24 periods)

59,322.17

487,622.17

Third full year (36 periods)

88,390.95

480,690.95

Fourth full year (48 periods)

117,032.23

473,332.23

Fifth full year (60 periods)

145,219.64

465,519.64

Problem 2

HP-25 Program Form

Title Two-Stock Portfolio Calculator Page 1 of 2

Switch to PRGM mode, press **[I]** **PRGM**, then key in the program.

DISPLAY		KEY ENTRY	X	Y	Z	T	COMMENTS	REGISTERS
LINE	CODE							
00								
01	2400	RCL 0						R0 AAPL Shares
02	2401	RCL 1						Initially 100
03	61	X						
04	2402	RCL 2						R1 AAPL Price
05	2403	RCL 3						Initially 132.00
06	61	X						
07	51	+						R2 KO Shares
08	74	R/S					Portfolio Value	Initially 200
09	01	1					displayed	
10	235104	STO+4					QTR Increment	R3 KO Price
11	2404	RCL 4					QTR Displayed	Initially 55.00
12	74	R/S					Key in AAPL	R4 QTR
13	21	X↔Y						Initially 0
14	2301	STO 1						
15	71	÷						R5
16	01	1						
17	51	+						
18	236105	STO X0						R6
19	2404	RCL 4					QTR Displayed	
20	74	R/S					Key in KO	
21	21	X↔Y						R7
22	71	÷						
23	01	1						
24	51	+						
25	23402	STO X2						
26	2404	RCL 4						
27	1474	FBASE					QTR Briefly Displayed	
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HP-25 Program Form

Title Two-Stock Portfolio Calculator Page 2 of 2
 Programmer Brian Hill

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Put initial values of shares and share prices in R0 to R3						
2	Put initial QTR (usually 0) into R4						
3	Start program		f	PRGM	R/S		
4	Program stops and displays total portfolio value						shown QTR
5	Key in AAPL values (price first, dividend second)						
6	Resume program		R/S				DO NOT HIT ENTER AFTER DIVIDEND
7	Key in KO values (price first, dividend second)						DO NOT HIT ENTER AFTER DIVIDEND
8	Resume program		R/S				
9	Program pauses and displays QTR						
10	Resume program		R/S				



you are shown QTR

DO NOT HIT ENTER AFTER DIVIDEND

DO NOT HIT ENTER AFTER DIVIDEND

Problem 3 — Table

QTR	QTR End Date	AAPL Stock #	AAPL Dividend	KO Stock #	KO Dividend	Portfolio Value
0	Dec. 31, 2020	Initial	Value of Portfolio			24,200.00
1	Mar. 31, 2021	122.15	0.205	52.71	0.42	23,323.15
2	Jun. 30, 2021	136.96	0.205	54.11	0.42	24,913.23
3	Sep. 30, 2021	141.50	0.22	52.47	0.42	25,480.13
4	Dec. 31, 2021	178.20	0.22	58.78	0.42	29,270.15
5	Mar. 31, 2022	174.61	0.23	62.00	0.44	29,012.65
6	Jun. 30, 2022	136.72	0.23	62.91	0.44	25,299.05
7	Sep. 30, 2022	150.01	0.23	58.58	0.44	26,749.62

ACTUALLY WE USED SHARE PRICES FOR SEPT. 23, 2022