

Numerical Analysis — Problem Set 8 — Probability Distributions

Due Tuesday, Nov. 8 (beginning of class)

In this problem set, you will study two simple and important probability distributions, the uniform distribution, the binomial distribution, which is an approximation to the Gaussian distribution (colloquially known as the bell curve).

1. The Uniform Distribution

(a) Look at the random number generation program on p. 116 of the *HP-25 Applications Programs* book. Modify it so that after step 07, where the number is stored, the number is multiplied by 8, then add 0.5 to it, and then call f INT. The last two steps round the number to the nearest integer. None of these modifications change what is stored in R0. They only change what is displayed to be an integer between 0 and 8.

You don't need to turn this program in.

So that we all get the same answer in what follows, let's all start with $u_0 = 0.1917$.

(b) Run the program 25 times. Fill out the following table:

```
In[ ] = MatrixForm[Transpose[Table[{i, " "}, {i, Range[1, 25, 1]}]]]
Out[ ] // MatrixForm =
( 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 )
( 4 2 7 6 )
```

(c) Google the word "histogram" and look at a few of the images so that you know what you are trying to create. Create a histogram of the above data by first filling in the following table with tally marks:

1	
2	
3	
4	
5	

In case, you don't know, what is on the left are tally marks. It is just a convenient way of counting.

```
In[ ] = MatrixForm[Transpose[Table[{i, " "}, {i, Range[0, 8, 1]}]]]
```

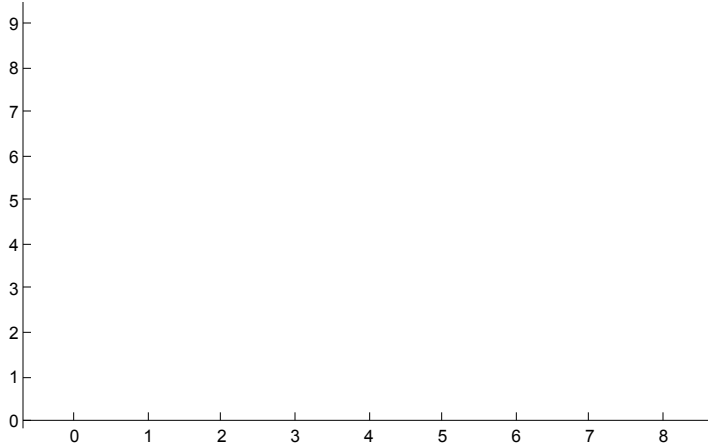
I have made the first few tallies:

```
Out[ ] // MatrixForm =
( 0 1 2 3 4 5 6 7 8 )
( / / / / / / / )
```

Now plot the data:

```
In[ ]:= Histogram[{}, 9, PlotRange -> {{-0.5, 8.5}, {0, 9}},
  Ticks -> {Range[0, 9, 1], Range[0, 9, 1]}
```

Out[]:=



(d) There were 25 data points in total. The sum of all the histogram bins is 25. Create a normalized histogram by dividing each of the values in part (c) by 25.

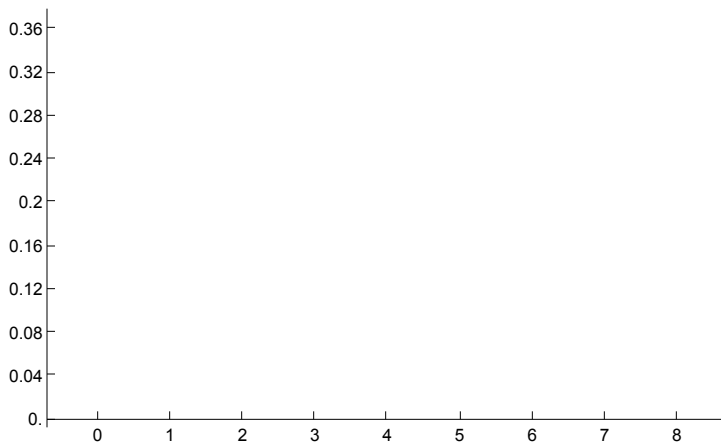
Out[]//MatrixForm=

$$\left(\begin{array}{cccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \right)$$

Plot those values:

```
In[ ]:= Histogram[{}, 9, PlotRange -> {{-0.5, 8.5}, {0, 0.36}},
  Ticks -> {Range[0, 9, 1], Range[0, 0.36, 0.04]}
```

Out[]:=



This is called a normalized histogram because the sum of all the bins is 1.0.

Discussion: you have made something that if you had done it hundreds or thousands of times would slowly become the uniform distribution. The value in each bin would become closer and closer to 0.111 (which is $1/9$).

2. The Binomial Distribution

(a) Go back to the original program on p. 116. Modify the program — this won't be as easy as in 1(a)! — to generate 8 random numbers that are either 0 or 1 and sum them up. This will always be an integer between 0 and 8. When you hit R/S again, it should generate 8 more random numbers that are either 0 or 1 and sum those up. The way to generate a random number that is either 0 or 1 is to use the program on p. 116, modify it to add 0.5 to the result and then do f INT.

Turn this program in on one of the HP-25 Program Forms.

Again, so that we all get the same answer in what follows, start with $u_0 = 0.1917$.

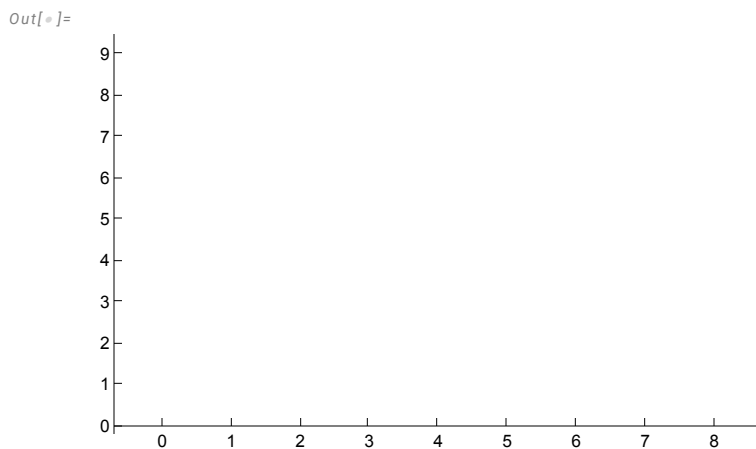
(b) Run the program 25 times. Fill out the following table.

```
In[ ]:= MatrixForm[Transpose[Table[{i, "  "}, {i, Range[1, 25, 1]}]]]
Out[ ]//MatrixForm=
( 1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 )
```

(c) Create a histogram of this data by filling in the following table with tally marks:

```
In[ ]:= MatrixForm[Transpose[Table[{i, "    "}, {i, Range[0, 8, 1]}]]]
Out[ ]//MatrixForm=
( 0    1    2    3    4    5    6    7    8 )
```

```
Histogram[{}, 9, PlotRange -> {{-0.5, 8.5}, {0, 9}},
  Ticks -> {Range[0, 8, 1], Range[0, 9, 1]}
```



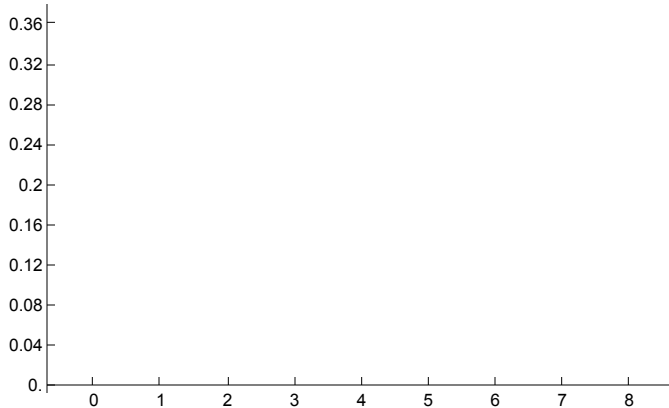
(d) Create a normalized histogram by dividing each of the values in part (c) by 25.

```
Out[ ]//MatrixForm=
( 0    1    2    3    4    5    6    7    8 )
```

Plot those values:

```
Histogram[{}, 9, PlotRange -> {{-0.5, 8.5}, {0, 0.36}},
  Ticks -> {Range[0, 8, 1], Range[0, 0.36, 0.04]}
```

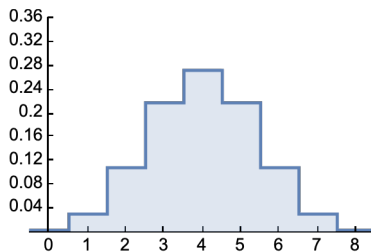
Out[*]=



Discussion: you have made something that if you had done it hundreds or thousands of times would slowly become the binomial distribution. Here is what you would get if you did it so many times that the non-uniformities smoothed out. Please ignore all the jiggery I had to do to get a nice-looking plot and just look at the plot below.

```
n = Table[{i-0.5, N[Binomial[8, i]/2^8]}, {i, 0, 8}];
n2 = Table[{i+0.5, N[Binomial[8, i]/2^8]}, {i, 0, 8}];
woven = Flatten[Transpose[{n, n2}], 1];
ListLinePlot[woven, PlotRange -> {{-0.5, 8.5}, {0, 0.36}}, Ticks -> {Range[0, 8, 1], Range[0, 0.36, 0.04]}, Filling -> Axis]
```

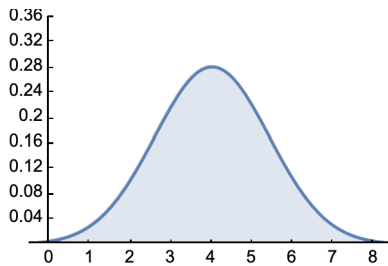
Out[52]=



I hope you can see the classic bell curve emerging:

```
In[63]:= Plot[PDF[NormalDistribution[4, Sqrt[2]], x] // Evaluate, {x, -0.5, 8.5},
  PlotRange -> {{-0.5, 8.5}, {0, 0.36}}, Ticks -> {Range[0, 8, 1], Range[0, 0.36, 0.04]}, Filling -> Axis]
```

Out[63]=



HP-25 Program Form

Title _____ Page _____ of _____

Switch to PRGM mode, press **[F]** **[PRGM]** , then key in the program.

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