Assignment 3: Interference — Solution

This assignment is designed is a brief introduction to the effects caused by light having a wave nature. We focus on interference between two slits.

Problem 1 — Manual Calculation of Interference

First we analyze the problem manually, by actually measuring path differences, converting the path differences to phase differences, and then converting the phase differences to intensities.

It is extremely good to do this manually, as I had you do, the first time you do it. I am writing the solution in Mathematica since I have done it manually many times before.

S1 is 10mm above the centerline. S2 is 10mm below the centerline. The screen is 128mm away as measured along the center-line. The tick-marks on the screen go up to +77mm and down to -77mm and there are 15 tick-marks A-O above the centerline, and 15 tick-marks Q-AE below the center-line. Here are expressions for the coordinates of S1, S2, and the tick-marks. Using the preceding facts,

```
In[1]:= s1 = {0.0, 10.0};
        s2 = {0.0, -10.0};
        tickMarks = N[Table[{128.0, 77 y / 15}, {y, -15, 15}]];
        tickYs = Transpose[tickMarks][[2]]
```

Out[4]= {-77., -71.8667, -66.7333, -61.6, -56.4667, -51.3333, -46.2,
  -41.0667, -35.9333, -30.8, -25.6667, -20.5333, -15.4, -10.2667,
  -5.13333, 0., 5.13333, 10.2667, 15.4, 20.5333, 25.6667, 30.8,
  35.9333, 41.0667, 46.2, 51.3333, 56.4667, 61.6, 66.7333, 71.8667, 77.}

The formula for distance is

```
In[5]:= distance[p_, q_] := Sqrt[(p[[1]] - q[[1]])² + (p[[2]] - q[[2]])²]
```

```
In[6]:= distancesFromS1[p_] := distance[p, s1]
        distancesFromS2[p_] := distance[p, s2]
```

```
In[8]:= distances1 = Map[distancesFromS1, tickMarks];
```

```
In[9]:= distances2 = Map[distancesFromS2, tickMarks];
```

```
In[10]:= pathDifferences = Flatten[ Map[Differences, Transpose[{distances2, distances1}]]]
```

Out[10]= {10.2927, 9.77417, 9.22854, 8.65551, 8.05511, 7.42767, 6.77383, 6.09463,
  5.39149, 4.66624, 3.9211, 3.15868, 2.38197, 1.59424, 0.799012, 0., -0.799012,
  -1.59424, -2.38197, -3.15868, -3.9211, -4.66624, -5.39149, -6.09463,
  -6.77383, -7.42767, -8.05511, -8.65551, -9.22854, -9.77417, -10.2927}

```
In[11]:= phaseDifference6mm = 2 Pi #1 / 6 &;
        phaseDifference4mm = 2 Pi #1 / 4 &;

        phaseDifferences6mm = Map[phaseDifference6mm, pathDifferences];
        phaseDifferences4mm = Map[phaseDifference4mm, pathDifferences];
```
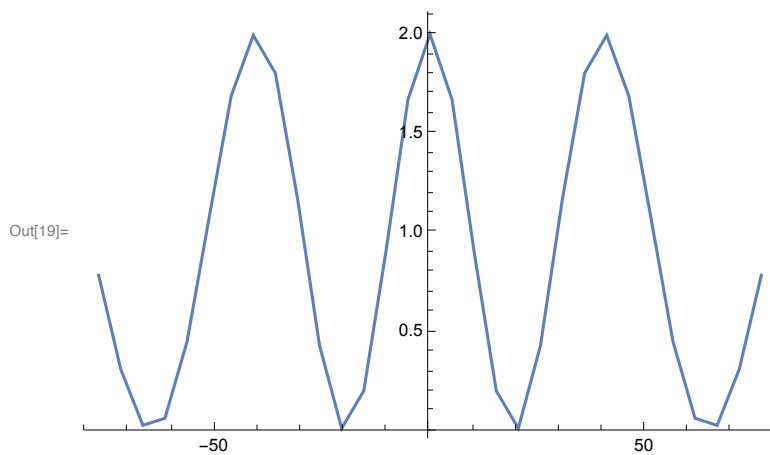
```
In[15]:= intensity = 1 + Cos[#1] &;
```

```
In[16]:= intensities6mm = Map[intensity, phaseDifferences6mm];
        intensities4mm = Map[intensity, phaseDifferences4mm];
```

```
In[18]:= Transpose[{tickYs, intensities6mm}];
```

```
In[19]:= ListPlot[Transpose[{tickYs, intensities6mm}], Joined → True]
```
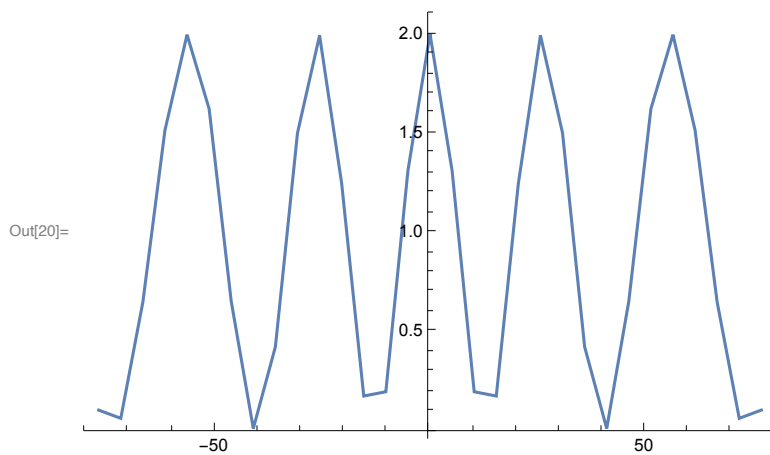
Out[19]=


```
In[20]:= ListPlot[Transpose[{tickYs, intensities4mm}], Joined → True]
```
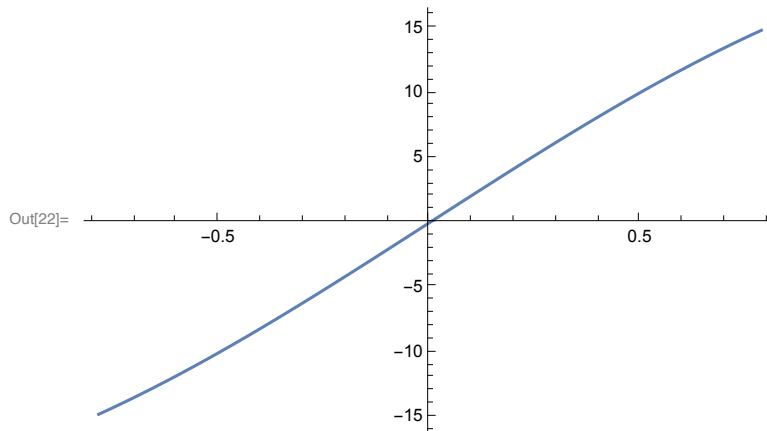
Out[20]=

Problem 2 — Mathematical Calculation of Interference

(a) 20mm * sin θ

(b) Phase difference is path difference / wavelength * 2 π. To put this slightly more pedantically in terms of the given values of a and λ: if wavelength is 6mm then 20mm / 6mm = 10/3. So the phase difference is $\Delta\phi$ = 20mm * sin θ / 6mm * 2 π = 10/3 * sin θ * 2 π = 20 π sin θ / 3.
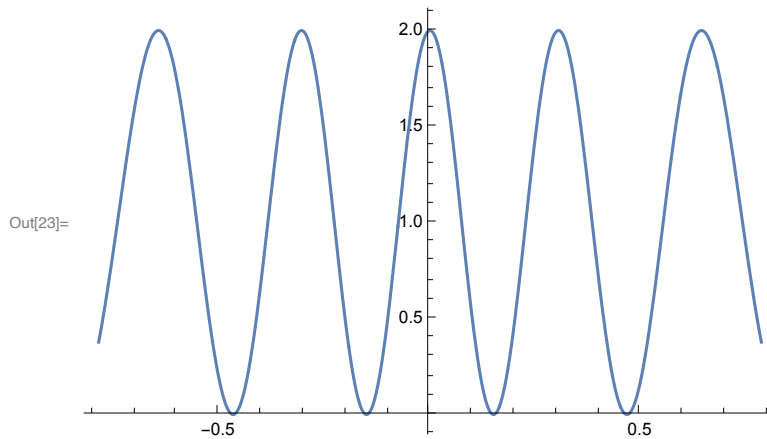
(c) Plot $\Delta\phi$ as a function of θ from -π/4 to +π/4.

In[21]:= `deltaPhi[θ_] := 20 Pi Sin[θ] / 3;`
`Plot[deltaPhi[θ], {θ, -Pi / 4, Pi / 4}]`

Out[22]=

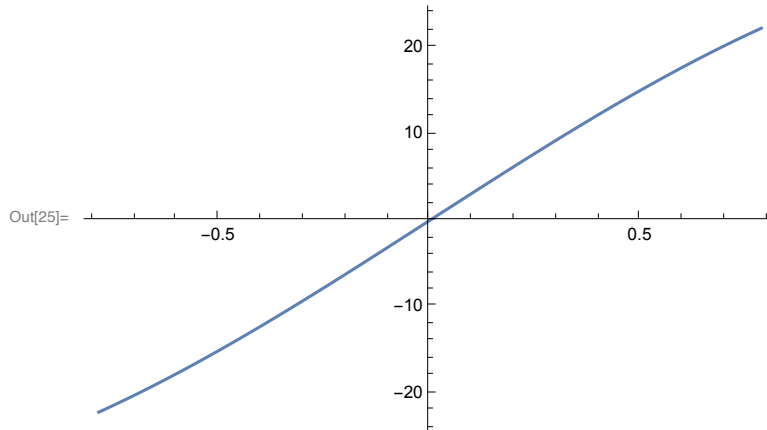(d) Plot I = 1 + cos($\Delta\phi$) as a function of θ from -π/3 to +π/3.

In[23]:= `Plot[1 + Cos[deltaPhi[θ]], {θ, -Pi / 4, Pi / 4}]`

Out[23]=

(d) Parts (d), (e), and (f) are to repeat parts (b), (c), and (d), but with $a / \lambda = 5$ instead of $a/\lambda = 10/3$. To put this slightly more pedantically in terms of the given values of a and λ: if wavelength is 4mm then 20mm / 4mm = 5. So the phase difference is $\Delta\phi = 20mm * \sin\theta / 4mm * 2\pi = 5 * \sin\theta * 2\pi = 10\pi\sin\theta$.
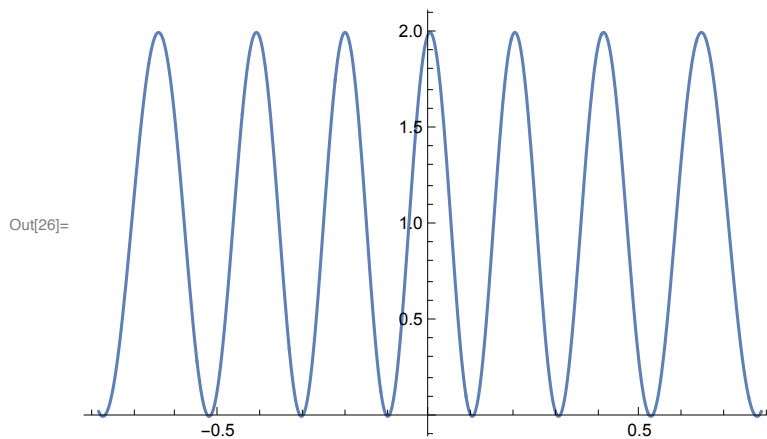
(e) Re-Plot $\Delta\phi$ as a function of θ from $-\pi/4$ to $+\pi/4$.

In[24]:= **deltaPhi[θ_] := 10 Pi Sin[θ];**
Plot[deltaPhi[θ], {θ, -Pi / 4, Pi / 4}]

Out[25]=

(f) Plot I = 1 + cos($\Delta\phi$) as a function of θ from $-\pi/4$ to $+\pi/4$.

In[26]:= **Plot[1 + Cos[deltaPhi[θ]], {θ, -Pi / 4, Pi / 4}]**

Out[26]=

Remarks on Problems 1 and 2

You found both manually and with equations that the patterns of light and dark becomes more closely spaced as the ratio of a/λ increases.

What happens in an actual circular aperture (which would require a double-integral, rather than a sum of just two wave sources), is that the central spot gets smaller and smaller, and less and less of the light is perceived as being in rings around the central spot because the radius of those rings gets smaller and smaller as the central spot gets smaller.

In other words, for sufficiently small wavelength or large aperture, the pattern of the focused light becomes more and more like a point. The wave nature of light becomes less and less important as the ratio of wavelength to aperture decreases. See Problem 3 on the Airy disk.

Problem 3 — Airy Disk and Diffraction Limit

The formula for the first minimum of the Airy Disk in the small-angle approximation, with the angle in degrees was given as:

In[27]:= $\texttt{firstMinimum}[\lambda\_,\ A\_] := \dfrac{360}{2\ \mathrm{Pi}}\ 1.22\ \dfrac{\lambda}{A}$

We have no way to derive that formula in this class. As mentioned above it requires a double-integral. We are just going to use the result.

(a) Plug in A = 254mm (that is 10 inches) and λ = 550 nm (that is green light):

In[28]:= $\theta = \texttt{firstMinimum}\left[550 \times 10^{-9},\ 254 \times 10^{-3}\right]$

Out[28]= 0.00015136

(b) Convert to arcseconds:

In[29]:= $\theta * 3600$

Out[29]= 0.544896

In other words, a 10-inch telescope becomes diffraction-limited at about 1/2-arcsecond. Or to put it differently, when seeing is at very best 1-arcsecond and at worst more like 5-arcseconds, and when working in the green, diffraction is generally negligible with a 10-inch telescope.