

Problem Set 2 Problem 1

S1 and S2 are 20 mm apart

S1

S2

You can see the situation is very symmetrical. Once we have analyzed points A-P, it is trivial to predict what happens for Q-AE.

A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
W
X
Y
Z
AA
AB
AC
AD
AE

TABLE TO FILL IN

DISTANCE FROM S2 (mm)	DISTANCE FROM S1 (mm)	DIFFERENCE (mm)
153	141	12
150	139	11
148	138	10

A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P

Using $\lambda = 6\text{mm}$

$$\frac{12\text{mm}}{6\text{mm}} \cdot 2\pi$$

$$\frac{11\text{mm}}{6\text{mm}} \cdot 2\pi$$

A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P

4π	
$11\pi/3$	

Using $\lambda = 4\text{mm}$

$$\frac{12\text{mm}}{4\text{mm}} \cdot 2\pi$$

$$\frac{11\text{mm}}{4\text{mm}} \cdot 2\pi$$

A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P

6π	
$11\pi/2$	

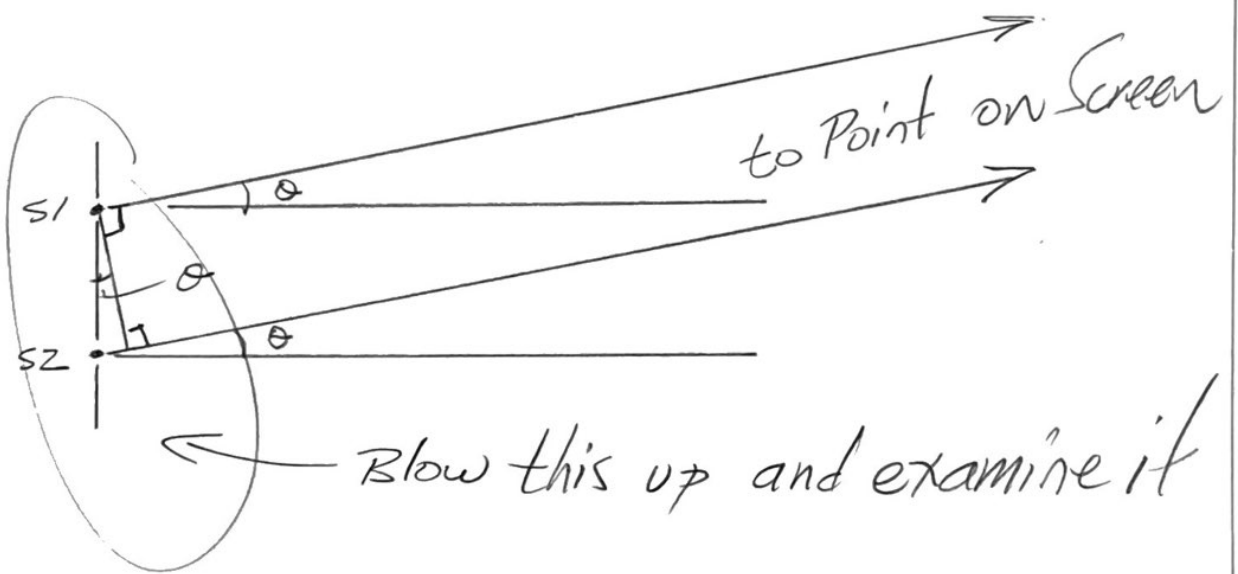
$\Delta\phi$ IN RADIANS

$$\Delta\phi = \frac{\text{PATH DIFFERENCE}}{\lambda} \cdot 2\pi$$

$$I = 2 \cos^2 \frac{\Delta\phi}{2}$$
$$= 1 + \cos \Delta\phi$$

Problem Set 2 Problem 2

An approximation often made is that the points A, B, C, \dots, AD, AE are very far away and all that matters is the angle toward them:



2 (a) Using trig, what is the path difference

2 (b) Assuming $\frac{a}{\lambda} = \frac{10}{3}$

Convert path difference to radians (just as in problem 1).

In the solution, I changed the plot ranges to $-\pi/4$ to $\pi/4$ to better match what was done in problem 1.

z(c) Graph

$\Delta\varphi$ as a function of θ
from $\theta = -\frac{\pi}{2}$ to $\theta = +\frac{\pi}{2}$

z(d) Graph

$I = 1 + \cos\Delta\varphi$ as a function
of θ from $\theta = -\frac{\pi}{2}$ to $\frac{\pi}{2}$

z(e), z(f), z(g)

Repeat z(b), z(c), z(d), but
assume $\frac{a}{\lambda} = 5$

Problem 3

Far from a circular aperture a star which you wish would focus to a point actually focuses to a diffraction pattern, called the Airy disk. The first minimum of the Airy disk is a circle of radius given by

$$\sin \theta = 1.22 \frac{\lambda}{A}$$

in radians

For very small angles $\sin \theta \approx \theta$

or $\sin \theta \approx \theta \cdot \frac{2\pi}{360^\circ}$

If θ is in degrees and you use the approximation just given, then

$$\theta \cdot \frac{2\pi}{360^\circ} = 1.22 \frac{\lambda}{A}$$

Solve for $\theta = \frac{360^\circ}{2\pi} 1.22 \frac{\lambda}{A}$

(a) Plug in $A = 254 \text{ mm}$ and $\lambda = 550 \text{ nm}$
what is θ ?

(b) Convert to arc-seconds.