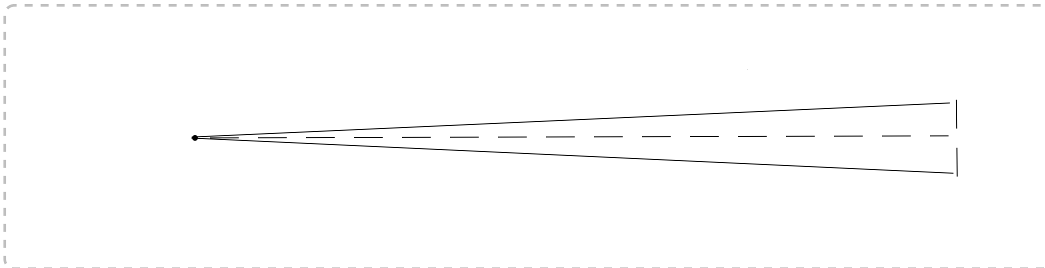
Assignment 2: Telescope Optics — Solution

Problem 1 — Degrees, Arc-Minutes, and Arc-Seconds



(a) The Andromeda Galaxy is 2.5 million light-years away. It is about 250,000 light-years across. The situation drawn above has the same proportions. The vertical line is 10 cm away the dot on the left, and is 1 cm high. If you measure the angle subtended with a protractor you will get 5 or 6°.

(b) Symbolically, let h be the height of the vertical line. Let d be the distance to the object. Let θ be the angle subtended. You can see that there are two right triangles in the drawing. Each has a very small angle, $\frac{\theta}{2}$, a base d, and a height $h/2$. This is a situation where the tangent function applies: $\tan \frac{\theta}{2} = \frac{h/2}{d}$. Then you solve that for θ. You get $\theta = 2 \arctan \frac{h/2}{d}$. With our numbers, that is $2 \arctan \frac{1}{20} = 5.725\,°$.

(c) $\frac{h}{d} = \frac{1}{10}$. That is it! Our answer in radians.

(d) To get degrees, multiply $\frac{1}{10}$ by $\frac{180\,°}{\pi}$. Get $\frac{18\,°}{\pi} = 5.730°$.

(e) OMG, the approximation is awfully close to the exact answer, and this is for an enormous object (very few objects take up 6° in the sky). The approximation only gets better as the angular size of the object is smaller. Because it works so well, from here on, we will only use the method in (c) and convert to degrees as in (d).

Problem 2 — Focal Length, f

(a) 1° degree of a circle of 2540mm in radius is $\frac{1\,°}{360\,°}\, 2\,\pi$ radians of that circle. The arc length is $\frac{1\,°}{360\,°}\, 2\,\pi\, * 2540\text{ mm} = 44.3\text{ mm}$.

(b) We get $\frac{1\,°}{360\,°}\, 2\,\pi\, * 1600\text{ mm} = 27.9\text{ mm}$.

Problem 3 — Focal Length and Camera Size

(a) The sensor area is 23.5mm by 15.7mm.

(b) Well if 27.9mm is 1° then 23.5mm is $\frac{23.5}{27.9} 1°$ = 0.842°, and 15.7mm is $\frac{15.7}{27.9} 1°$ = 0.563°.

(c) Just multiply by 60 arc-minutes / 1° and get 50 arc-minutes by 34 arc-minutes.

(d) 6248 x 4176 = 26,091,648 pixels. That is about 26 Megapixels.

(e) You get 0.485 arc-seconds for the width per pixel if you use 23.5mm as the total width and 6248 as the number of pixels and don't round at any of the intermediate steps. You get the same result as the height per pixel using 17.7mm and 4176. This is because the pixels are square.

(f) Assuming 2 arc-seconds of smearing, there are really only 50 arc-minutes / 2 arc-seconds = 1500 pixels of resolution in the width direction and 34 arc-minutes / 2 arc-seconds = 1020 pixels of resolution in the height direction. In other words you can at best get 1500 x 1020 = 1.5 Megapixels of image.

Problem 4 — Magnification and Apparent Field of View

(a) 2540mm / 40mm = 63.5 power.

(b) 63.5°.

(c) If 1° appears to be 63.5° then the whole eyepiece apparent field of view of 67° can show an object that is $\frac{67°}{63.5°} 1°$. We looked at the Moon with this eyepiece and it appeared that we were getting only about 3/4° of actual field of view out of this eyepiece.

Problem 5 — f/ Ratio and Exit Pupil

(a) 2540mm / 254mm is quoted as f/ = 10. Opticians make mighty strange use of the division sign. They also write this as simply f/10.

(b) 40mm / 10 = 4mm

(c) Yes, a 4mm shaft of light will fit through a 7mm opening. Yes, you have to have your eye quite well centered (to within +/- 1.5mm) to have the whole shaft of light pass through the pupil.

(d) My glasses lenses are about 15mm from my eye. An eyepiece eye relief of 15mm would be needed.